



A Capstone Course in Engineering Analysis for Mechanical Engineers

Dr. David R. Sawyers Jr., Ohio Northern University

David R. Sawyers, Jr. is an Associate Professor of mechanical engineering at Ohio Northern University, where he teaches courses in general engineering and in the thermal sciences. He received a B.S.M.E. degree from Rose-Hulman Institute of Technology and the M.S. and Ph.D., both in mechanical engineering, from the University of Notre Dame.

A Capstone Course in Engineering Analysis for Mechanical Engineers

Abstract

Much of the undergraduate mechanical engineering curriculum is designed around subject matter courses such as Statics, Control Systems, or Thermodynamics. While significant effort may be applied to the development of problem solving skills within these courses, the perspective of the students is typically bound to the subject being taught. As a result, students often have difficulty in applying methods learned in one class to the solution of problems in another class, just as they often have difficulty in seeing the interconnections between different subjects.

This paper describes a course on problem solving across a wide range of (math-based) problems encountered by mechanical engineers. The course includes some lectures, including a review of ordinary differential equations and an introduction to numerical methods - but a significant component is the assignment of “analysis problems” which require more effort to solve than the typical engineering homework problem. In some cases, these problems require students to apply familiar concepts, such as Newton’s Second Law of Motion or the First Law of Thermodynamics, but in a way that goes beyond the problems typically seen in an introductory Dynamics or Thermodynamics course. Other problems require students to apply concepts from multiple courses. Some of the problems are ambiguous or poorly defined, requiring additional assumptions or clarification in order to obtain a well-posed mathematical problem. Throughout the course, emphasis is also placed on adequately citing references, validating solutions, and communicating results. Student work was assessed primarily through evaluation of written reports.

Introduction

Engineering analysis - used here to denote the application of mathematical techniques to obtain the solutions to problems defined from physical principles - is a fundamental component of the engineering curriculum. Analysis is usually an essential part of the design process, especially when comparing the performance of potential designs or in validating proposed solutions. It is also the basis for a large proportion of the engineering curriculum, in engineering science courses such as Statics, Circuits, Kinematics, and Heat Transfer. Its importance is also reflected in several of the ABET criteria for accreditation of engineering programs (Criterion 3), as shown below¹:

- (a) an ability to apply knowledge of mathematics, science, and engineering
- (e) an ability to identify, formulate, and solve engineering problems
- (k) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

While the Capstone Design project usually provides a mechanism for applying engineering analysis beyond the context of a topical course, it also often highlights the difficulty students have in applying prior knowledge in new situations. In addition, the open-ended nature of design problems, as well as the amount of time devoted to project management, economics, and other aspects of the design process, means that there is little time left within the Capstone Design

course to systematically improve the students' ability to solve analysis problems. To address this issue, a new course was developed to focus on the application of problem solving skills first introduced in the engineering science courses. However, the course is not simply a repetition of problems seen previously - an attempt has been made to introduce problems which are not always straightforward extensions of a standard or typical problem. To a certain extent, these problems include at least some of the components identified by Jonassen, et. al. in what they call "workplace problems."² This has proven to be a difficult task, and it may only have been accomplished to a limited degree, but continued effort will be applied to improve the pool of problems to be assigned.

ME-4511 Engineering Analysis is taken by mechanical engineering students during the fall semester of their final year. The course is intended to serve as a one-semester "analysis capstone" course to complement the year-long design capstone that has long been required for mechanical engineering students at the author's university. The course also serves as a partial replacement for two courses previously taught under the quarter system: an engineering problem-solving course taught in the fall of the junior year, and a numerical methods course taught in the spring of the junior year. While the compression of two courses into one has necessitated some reduction in content and expectations, it has also offered greater opportunities since students in their final year of study have a wider range of prerequisite courses from which to draw.

In addition to lectures and homework on a variety of analytical and numerical problem-solving tools, students are assigned several "analysis problems." These extended problems are intended to draw upon knowledge gained both in ME-4511 and in previous courses. Some assignments require the solution of ambiguously-defined problems; some of the problems also cross the boundaries of traditional mechanical engineering courses. The development of effective analysis problems is one of the most significant challenges in developing the course.

Course Content

ME-4511 was taught for the second time in the fall of 2013. The prerequisite course requirements are structured so that students will have completed multi-course sequences in calculus and differential equations, physics, the thermal sciences, dynamic systems modeling and analysis, and mechanical design. Course development has been complicated by a recent transition from quarter to semester calendar, resulting in occasional (but sometimes significant) differences in the prior experience of individual students. This has required reorganization of the content (including a primer on Matlab programming, for example) and has limited the potential pool of analysis problems. Beginning in the fall of 2014, the calendar transition will be complete and all students will have taken all of the desired prerequisites, including a structured programming course.

The course outcomes stated on the syllabus include the following:

Upon completion of the course, students will be able to:

1. *solve engineering problems using a variety of analysis methods and software tools.*

2. *apply numerical techniques such as Runge-Kutta methods and finite-difference methods to obtain solutions to differential equations that apply to engineering practice.*
3. *solve problems which are not well-defined, or do not have an obvious closed-form solution.*
4. *solve problems which cross the traditional boundaries of mechanical engineering courses.*

The traditional course content (lectures and homework) included the following topics:

- A review of the analytical solution of ordinary differential equations.
- Numerical solution of ODEs: Euler's Method, Runge-Kutta methods, finite-difference methods; reduction of higher order equations to a system of first-order ODEs; initial and boundary conditions.
- Fourier analysis and numerical application of the fast-Fourier transform.
- Constrained optimization
- Numerical solution of PDEs by finite difference methods.
- Structured programming in Matlab.

The first offering of the course in 2012 also included a module on numerical integration; this was dropped in 2013 in order to allow more time for analytical and numerical solution of ordinary differential equations. Some variation in lecture material is anticipated from year to year in order to support the assigned analysis problems, which will be discussed in the following section.

Analysis Problems

The analysis problems are a key component of the course. Students work in self-selected teams of two, and have approximately two weeks to complete each assignment. In 2013 the following problems were assigned:

1. Acceleration of a falling chain.
2. Aerodynamic loading of a tapered radio mast.
3. Viability of a small-scale pumped storage facility.
4. Optimization of a vehicle-portable water tank.
5. Pipe diameter optimization for a fluid network.
6. Analysis of a two-mass spring-mass-damper system.

The problem statement for each problem can be found in Appendix 1. A brief qualitative evaluation of each problem is provided below.

Problem 1: Acceleration of a falling chain. This problem requires students to apply their knowledge of Dynamics and Differential Equations to a problem that is an extension of concepts seen previously. It is possible to obtain an analytical solution, although most students used Euler's Method to solve the problem numerically. While not exceptionally challenging, the problem did serve as an effective introduction to the analysis problem component of the course, including the requirements for validation and documentation that are emphasized throughout.

Problem 2: Aerodynamic loading of a tapered radio mast. This problem combined concepts previously seen in Fluid Mechanics (aerodynamic drag), Statics (distributed loading), and Calculus (differential segments, integration). It should be noted that the problem statement intentionally avoided explicitly mentioning these concepts, or providing the drag coefficient for the specified geometry. In spite of this, almost all of the students were able to easily identify the

relevant physical phenomena involved. However, many students had difficulty recalling their knowledge of calculus to develop a suitable equation. This can be done quite easily by considering the force on an infinitesimal segment and then integrating to find the total moment, a process which should have been familiar to the students. The resulting equation can then be integrated numerically, a procedure that was also introduced in the prerequisite calculus courses.

Problem 3: Viability of a small-scale pumped storage facility. This problem combined concepts from Fluid Mechanics and Engineering Economics. In order to obtain a reasonable solution, students needed to independently learn about the concept of pumped storage, determine what defines viability for such a system, and find reasonable data for peak and off-peak electricity pricing. Several groups clearly did not grasp the basic concept (storing off-peak electricity and then recovering it to sell at a premium when demand is high), while others ignored capital costs or other major factors. Several groups became so focused on obtaining a detailed solution to the conservation of energy (head loss) equation that they lost track of the big picture. On the other hand, two groups produced convincing conclusions with only a minimal amount of calculation.

Problem 4: Optimization of a vehicle-portable water tank. This was a fairly straightforward constrained optimization problem, which can easily be solved using the “Solver” feature of Microsoft Excel. The students were required to develop specific constraints from the problem statement. The most difficult aspect of the problem for most students was in validation - particularly in evaluating the potential uncertainty associated with parameters that were not clearly defined, such as material cost or water density.

Problem 5: Pipe diameter optimization for a fluid network. This problem combined a discrete optimization problem with a pump/pipe network flow problem. If the concepts from Fluid Mechanics are applied correctly, it is fairly straightforward to solve this problem using the “Solve from Table” feature of the Engineering Equation Solver (EES), which students have used in previous classes. However, in future the problem can be easily modified to make it much more challenging - if the pressure drop is specified instead of the flowrate, several numerical issues arise. The problem can still be solved using EES, but care must be taken in assigning initial guesses and in controlling the iterative solver in order to obtain an accurate, converging solution.³

Problem 6: Analysis of a two-mass spring-mass-damper system. This problem was an extension of concepts previously seen in two courses on Dynamic Systems analysis and control. Due to a lack of previous programming experience, the problem definition was fairly straightforward. In future, more interesting cases can be assigned to demonstrate the complex behavior of this type of system.

Assessment

Table 1 shows which course outcomes are addressed by each analysis problem. Obviously the degree to which an outcome is addressed varies from problem to problem - the table does not attempt to quantify this effect. However, even the qualitative analysis can be useful. Outcomes 2 and 4 seem to be least addressed. The apparent lack of coverage of Outcome 2 is a reflection of the fact that Table 1 only includes the analysis problems; significant coverage of this outcome

was provided by traditional homework problems. The limited coverage of Outcome 4 is not surprising since developing problems that cross traditional boundaries is not easy. This was exacerbated in 2013 due to the transition to semesters mentioned earlier - several returning co-op students had not taken either Fluid Mechanics or Heat Transfer (the strongest subjects of the instructor). However, this is clearly an area where continued effort is required.

Table 1: Correlation of analysis problems to the stated course outcomes (fall semester of 2013). An “X” indicates the outcomes which each problem addresses.

Problem	Outcome 1: solve engineering problems using a variety of analysis methods and software tools.	Outcome 2: apply numerical techniques ... to obtain solutions that apply to engineering practice.	Outcome 3: solve problems which are not well-defined, or do not have an obvious closed-form solution.	Outcome 4: solve problems which cross the traditional boundaries of mechanical engineering courses
1. Falling Chain	X	X		
2. Aerodynamic Loading	X		X	X
3. Pumped Storage	X		X	X
4. Water tank optimization	X			
5. Pipe diameter optimization	X		X	
6. Two-mass dynamic system	X	X		

Table 2 shows a rating of student performance on each analysis problem. This rating is based solely on the professional opinion of the instructor, and obviously does not reflect an impartial, absolute standard of accomplishment. However, in a senior-level required course, it would be surprising if most students were not able to perform at an acceptable level or higher. It is perhaps more useful to compare the relative numbers of students performing at the exceptional and acceptable levels. Those problems with a high proportion of “Exceptional” performances (#3, #4, and #6) will be reevaluated for next year. While some of these problems may not be sufficiently challenging and will be replaced, it may also be useful to rearrange the problems so that they become progressively more difficult as the students gain confidence in their problem solving abilities.

Table 2: Student performance on Analysis Problems

Problem	Exceptional	Acceptable	Marginal	Unacceptable
1. Falling Chain	15	18	0	2
2. Aerodynamic Loading	6	24	4	1
3. Pumped Storage	17	14	2	2
4. Water tank optimization	27	7	0	1
5. Pipe diameter optimization	10	24	0	1
6. Two-mass dynamic system	26	8	1	0

Table 3 shows the results of a self-assessment survey conducted by the students upon completion of the course. Past experience suggests that values below 3.5 indicate cause for concern. The values in Table 3 are consistent with typical results for upper-level courses within the mechanical engineering department.

Table 3: Student self-assessment of course outcomes.

Outcome	Average Response (0 = No achievement 5 = Achieved Very Well)
Outcome 1: solve engineering problems using a variety of analysis methods and software tools.	4.2
Outcome 2: apply numerical techniques ... to obtain solutions to differential equations that apply to engineering practice.	4.0
Outcome 3: solve problems which are not well-defined, or do not have an obvious closed-form solution.	3.9
Outcome 4: solve problems which cross the traditional boundaries of mechanical engineering courses	3.9

In addition to the self-assessment survey given in class, 34 of the 36 enrolled students completed the standard online course evaluation administered by the university. The evaluation included fifteen specific questions related to the course and the instructor with responses provided on a scale from 1 to 5 (with 5 corresponding to the most positive response). Excluding questions that were not applicable to this course (textbook, software, etc.), the lowest average response on any question was a 4.0, and none of the average responses were significantly lower than the mean response for all mechanical engineering courses.

This evaluation also included space for free-form responses related to (a) strengths of the course, (b) weaknesses, and (c) suggestions for improvement. Of the twelve responses in part (a), seven indicated a perceived increase in problem solving ability by the student. Two of the comments also mentioned the relationship between the assignments and knowledge gained in previous courses. Of the nine responses in part b, three indicated that the workload was too heavy, three suggested that the lectures should be more closely aligned with the analysis problems, and one suggested more guidance on problem validation. Of the seven responses in part c, three were positive or neutral, one suggested assigning less difficult problems, on asked for more examples of validation, and two suggested a closer tie between lectures and assignments.

Conclusions

One of the goals of ME-4511 Engineering Analysis - as reflected in course Outcomes 1, 3, and 4 - is to allow students to apply the problem solving skills introduced in previous courses to a wide range of problems. Upon entering the fourth year of study, the undergraduate engineering student will have been exposed to a number of techniques for solving engineering problems. However, the approach to solving each problem is often obvious from the context of the course; indeed, one of the significant challenges of the engineering educator is to overcome the habit of the student to “solve by example” - that is, to find an example in the textbook or the lecture notes

that corresponds to the problem to be solved, and then make whatever slight modifications are necessary (see for example the review of research into physics problem solving found in reference 4). One of the reasons for the development of ME-4511 Engineering Analysis was to take problems out of this limiting context, so that students would be required to approach the problem methodically rather than simply applying a template solution. Ultimately it will be the ability to solve a wide variety of problems, rather than topical knowledge, that will be of long-term value to the engineering graduate. However, implementing a class to achieve this goal is not an easy task, since developing problems that do not fit within the traditional topical boundaries, and yet are not frustratingly difficult to solve, is extremely difficult.

After offering the course for two consecutive years, it is clear that one of the most significant obstacles to be overcome by the students is a lack of confidence, rather than a lack of knowledge or ability. When students seek help outside of class, it usually turns out that they already know how to approach the problem, but are unwilling to proceed unless they can clearly see how their approach will lead to a solution. A willingness to proceed step-by-step, rather than trying to see the solution as a whole, is often the distinguishing feature of those teams that solve a problem most efficiently. This course does seem to enhance this ability in the majority of students, as observed by the instructor and also as reflected in the fairly confident survey responses shown in Table 3.

Several steps will be taken to improve the course in the future. The most obvious improvement is to continue to develop new analysis problems to better achieve course outcomes 1, 3, and 4. The second course outcome, which refers specifically to numerical methods, is a remnant of a previous course which no longer exists. This outcome is somewhat misaligned with the overall objectives of the course, and will probably be removed in the future - while still retaining some lectures on the numerical solution of differential equations. Unfortunately, this tends to lead students in the solution of some of the analysis problems, but there is currently no other place for this topic in the curriculum. An outcome may also be added in the future to emphasize the importance of validation and communication. Both issues are included in the course repeatedly, so inclusion in the course outcomes would simply formalize their importance.

References

1. "Criteria for Accrediting Engineering Programs," (2013 - 2014), ABET Engineering Accreditation Commission, <http://www.abet.org/DisplayTemplates/DocsHandbook.aspx?id=3149>
2. David Jonassen, Johannes Strobel, and Chwee Beng Lee, "Everyday Problem Solving in Engineering: Lessons for Engineering Educators," *Journal of Engineering Education*, April 2006.
3. David Sawyers, "Pipe Network Analysis: An Exercise in Computer-Aided Analysis," *Proceedings of the ASEE North-Central Section Conference*, Columbus, Ohio, 2013.
4. Tolga Gok, "Development of Problem Solving Strategy Steps Scale: Study of Validation and Reliability," *The Asia-Pacific Education Researcher*, v20, 2011.

Appendix 1 - Analysis Problem Statements

Problem 1: Acceleration of a falling chain.

A 15m-long uniform chain is wound around a 0.5-m diameter drum which rotates about a fixed horizontal axle. A segment of the chain of length $x_0 = 1.5$ m is initially unwound from the drum. The chain has a mass per unit length of $\rho = 1$ kg/m, and is initially at rest. If the weight of the overhanging section of chain is sufficient to overcome friction, it causes the remainder of the chain to smoothly unwind from the drum

The axle diameter is 10 cm, and the width of the drum is 0.75 m. The gap between the drum and axle is 2 mm wide, and is filled with a lubricant that has a viscosity of 0.5 kg/m-s.

1. Develop the governing differential equation for $V(x)$, where V is the speed of the falling chain, and x is the length of chain that has unwound from the drum at any instant.
2. Obtain an analytical solution for $V(x)$ by assuming that friction between the shaft and the drum is negligible.
3. Obtain a numerical solution for $V(x)$ when $x_0 < x < 10$ m, including the effects of shaft friction.
4. Validate your solution to part 3 by solving numerically with the assumption of negligible friction.
5. Plot $V(x)$ from each of your solutions to parts 2-4 on the same graph.

You may work in teams of two to complete this assignment. Your solution should be provided in memo report format. Particular care must be taken to address the following:

- Your report should clearly explain the problem (in greater detail than the outline provided above).
- Present and discuss the equations and solution procedure used to obtain the results, including derivation from basic principles or commonly-known equations. Be sure to justify your decisions - which may include assumptions, simplifications, and choice of solution technique.
- You must provide references to reliable sources for all additional information needed to solve the problem.
- Your report should contain a meaningful validation section.

Problem 2: Aerodynamic loading of a tapered radio mast

A radio tower is a tapered circular cylinder with a base diameter of 0.2m, tip diameter of 0.1m, and height of 30m. The windspeed V (in m/s) varies with height such that $V = 5 \tanh(y/10)$ where y is the height above ground level in meters. Calculate the moment about the base of the tower caused by the aerodynamic drag force.

You may work in teams of two to complete this assignment. Your solution should be provided in memo report format. Particular care must be taken to address the following:

- Your report should clearly explain the problem (in greater detail than the outline provided above).
- Present and discuss the equations and solution procedure used to obtain the results, including derivation from basic principles or commonly-known equations. Be sure to justify your decisions - which may include assumptions, simplifications, and choice of solution technique.
- You must provide references to reliable sources for all additional information needed to solve the problem.
- Your report should contain a meaningful validation section. Your validation and evaluation of the effects of uncertainty should be quantitative, not merely qualitative.

Problem 3: Viability of small-scale pumped storage facility

The Village of Ada is considering the construction of a small-scale pumped-storage facility. When the demand for electricity is low, this system would pump water into a storage tank similar to the one currently seen to the east of campus. When electricity demand increases, the water would be used to generate electricity. Is this system feasible? In other words, should the village pursue this project?

You may work in teams of two to complete this assignment. You will submit your answer in the form of a memo report. Your report should include the following:

- How did you define “feasible”? Are there other ways to define this term for the proposed project and if so, why is your definition most appropriate?
- What are the major questions that must be answered, or issues to be considered, in order to determine feasibility?
- What assumptions did you make? You should justify any significant assumptions or simplifications.
- How did you solve the problem? Discuss significant calculations, as well as supporting information. Remember to cite references appropriately.
- If you determine that the project **is not feasible**, under what conditions might it become more appealing?
- If you determine that the project **is feasible**, what are the major risks (conditions under which the project might become less appealing)?

Problem 4: Optimization of a vehicle-portable water tank.

You are to specify the dimensions (Length and nominal diameter) that minimize the cost of a horizontal cylindrical water tank to be installed in the bed of a 2013 F-150 pickup. You only need to specify the dimensions of the tank (you do not need to consider any support framework or mounting brackets).

The tank must contain 300 gal of water, must not exceed the weight limit of the truck, and must fit between the wheelhouses and inside the closed tailgate. The truck to be used is a Regular cab 4x2 with a 6 ½' styleside bed, 5.0L engine and Heavy-Duty Payload package. The tank will be made from rectangular sheets of 11 gauge T304 stainless steel, with each circular endcap welded to the ends of the cylinder. For this exercise you may assume the sheets can be obtained in an infinite range of sizes.

You may assume that the variation in cutting and forming costs with size is negligible, and that welding is performed at a cost of \$10/in. Therefore the total cost of the tank is the sum of the material cost and the welding cost.

Submit your solution as a memo report, clearly explaining the problem definition and solution procedure. You should validate your solution, and provide reference citations as needed.

Problem 5: Pipe diameter optimization for a fluid network

A pipe network consists of a 10 ft vertical pipe which draws water from a well that is open to the atmosphere. This vertical pipe is connected to a pump, which is then connected to a 500 ft long horizontal pipe, which is connected at the other end to two horizontal outlet pipes with lengths of 50ft and 20 ft. Both outlet pipes connect to open-topped tanks. All the pipe sections are smooth and have the same diameter, which can be specified in ¼” increments from ½” to 2”. The pipe costs $\$0.50 \times (LD)$, where L is the pipe length and D is the diameter. An adjustable valve is located at the exit of the shorter pipe length to ensure that both flowrates are the same.

A pump is located between the vertical section and the first horizontal section of pipe. The pump curve is $h_p = \alpha(100 - 0.01Q^2)$, where h_p is the pump head in feet, Q is the flowrate in gpm, and α is a dimensionless parameter that specifies the size of pump/motor combination chosen (α can have any value between 0 and 10). The cost of the pump/motor is $\$700\alpha$.

If the flowrate at each exit is 10 gpm, determine the pipe diameter that minimizes the total system cost. Also calculate the pump inlet pressure (in psig), and the power required (in hp) if the pump/motor has a combined efficiency of 70%.

This problem can be solved using EES, but there are several numerical issues that must be considered.

- Since EES is an iterative solver, it may or may not converge. In addition, when solving a nonlinear problem it may converge to an answer that is mathematically correct but is not appropriate for the physical problem being solved. For an iterative solver, it is sometimes necessary to begin with reasonable initial guesses in order to obtain a good converged result. This is especially true of variables with values that are typically several orders of magnitude different from the default initial guess.
- As the numerical values change during the iteration process terms may temporarily become negative, even if the converged value is positive. It may be necessary to formulate the equations in a way that allows you to force certain terms to always be positive.
- Since you only have a limited number of options for diameter, it is probably best to set up a table for your calculations.
- Finally, it is best not to rely on the unit conversions in EES. You should enter your values and equations using a consistent set of units.

You may work in teams of two to complete this assignment. Your solution should be provided in memo report format. Particular care must be taken to address the following:

- Your report should clearly explain the mathematical problem to be solved, including derivation from basic principles or commonly-known equations. Be sure to justify your decisions - which may include assumptions, simplifications, and choice of solution technique.
- You must provide references to reliable sources for all additional information needed to solve the problem.
- Discuss how you solved the mathematical problem, including any significant problems encountered and how these problems were overcome.
- Your report should contain a meaningful validation section.

Problem 6: Analysis of a two-mass spring-mass-damper system.

1. Develop the governing differential equations for the two-degree-of-freedom spring-mass-damper system shown in Figure 1, where the forcing functions have the form $F_i(t) = \phi_i \sin(\omega_i t)$.

2. Develop a Matlab program to do the following:

- Read the system parameters ($m_1, m_2, c_1, c_2, k_1, k_2, \phi_1, \phi_2, \omega_1, \omega_2$) from a text file, in the order listed with each parameter on a separate line of the file.
- Solve to find x_1 and x_2 as functions of time.
- Plot the solution $x(t)$ for each mass.
- Plot the phase space $v(x)$ for each mass. Use the subplot command to show all four plots in the same figure; include properly labeled axes and titles.

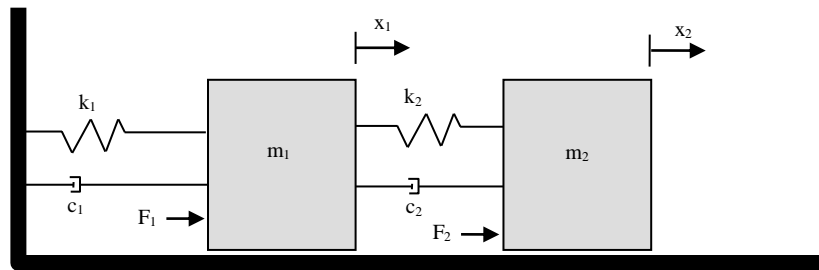


Figure 1: Specification of a 2-DOF vibrating system.

3. Use your program to solve the problem for the following cases:

Case	m_1 (kg)	m_2 (kg)	c_1 (kg-s)	c_2 (kg-s)	k_1 (N/m)	k_2 (N/m)	ϕ_1 (N)	ϕ_2 (N)	ω_1 (rad/s)	ω_2 (rad/s)
a	2	0	2.5	0	2	0	0	0	0	0
b	2	0	2.5	0	2	0	1	0	2	0
c	2	1	2.5	0.5	2	1	1	1	2	2

In all cases, the following initial conditions should be used. These will not be included in the data file, and can be “hard-wired” into the program.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.0 \\ 0.1 \end{bmatrix} m \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} m/s$$

Deliverables

You may work in teams of two to complete this assignment. You will not submit a written report - instead, provide the following:

- Your neatly handwritten derivation of the governing equations.
- A printout of the plots for each case a-c.
- A printed copy of your Matlab file(s).
- A one-page typed discuss of how you have validated your program and solutions.