A HANDY TOOL FOR CONVENIENT ERROR PROPAGATION ANALYSIS: A USER FORM FOR ERROR INFLUENCE COEFFICIENTS

Sheldon M. Jeter

Georgia Institute of Technology

INTRODUCTION

Complete uncertainty analysis in experimental engineering requires two distinct and complementary calculations. Statistical analysis of repeated measurements is needed to compute the Uncertainty A, which is the uncertainty due to random variation. Complementary physical analysis of the measurement system is also needed to evaluate the Uncertainty B or the range in possible bias or built in error. The more interesting and important applications of Uncertainty B analysis are encountered when considering an indirect measurement. An indirect measurement is merely a value calculated from a set of direct measurements. Error Propagation Analysis (EPA) is usually necessary to estimate the Uncertainty B for indirect measurements.

This paper first reviews the basic principles of experimental uncertainty. It then reviews the principles and pertinent details of EPA. It next presents an example that illustrates the calculations of Uncertainty A and Uncertainty B. The latter calculation requires EPA, so the paper presents and explains an Excel User Form to facilitate this task. The example demonstrates that this form makes even relatively complex EPA simple and quick.

TYPES OF UNCERTAINTY

Common experience and a little scrutiny reveal that two types of experimental uncertainty exist, random and systematic. Conventional practice and consensus standards (ISO, 1995) also recognize these two types. Formally, the random uncertainty is called Uncertainty A. It is operationally defined as the uncertainty that can be evaluated by statistical analysis of the experimental data. The measure of Uncertainty A is an error limit based on observed random variation in the data. Conventionally, Uncertainty A has been called imprecision. In contrast, the systematic uncertainty known as Uncertainty B must be evaluated by physical analysis of the entire measurement system. Uncertainty B is explicitly not a measure of random variation. Instead, it is the estimated possible range

of systematic error that might be built into a measurement system of the design and construction actually used. Conventionally, Uncertainty B has been called inaccuracy.

Uncertainty analysis results in some sort of error band for a measurement. The Expanded Uncertainty, U, is the half-with of this error band. Conventionally, the 95 % error band is reported. After statistical analysis, the Expanded Uncertainty, U_A, is typically computed using the appropriate coverage factor, k_c, and the pertinent summary statistic called the Standard Uncertainty, u, as

$$U_A = k_c\, u \qquad\qquad (1)$$

For example, consider a so-called single sample experiment. In this case, the experimental conditions are kept fixed and basically similar measurements are repeated. The expected value of the measured quantity is the average of the data, and the Standard Uncertainty is frequently either well-known Sample Standard Deviation (SSD) or some simple function of the SSD. The coverage factor is generally computed using the t-distribution, since that distribution properly addresses the necessarily finite, and sometimes quite small, experimental samples. More complicated cases such as double or multiple sample experiments wherein one or more independent variables are varied require slightly more sophisticated statistical analysis such as regression analysis. In any case, evaluation of Uncertainty A is relatively straightforward. The evaluation of Uncertainty B is also relatively straightforward, but as shown in the next section the analysis requires error propagation analysis not statistics.

REVIEW OF ERROR PROPAGATION ANALYSIS

The most elementary concept in error propagation analysis (EPA) is the division of all measurements into two distinct categories, direct measurements and indirect measurements. Direct measurements are read from an instrument without further processing or modification. They are the raw material of experimental engineering. In practice, the more meaningful measurements are indirect measurements. An indirect measurement is calculated from one or more direct measurements. Examples can be as simple as a calibration correction or as complex as the calculation of the efficiency of an energy conversion system. In a typical calibration correction a single direct measurement, such as a thermometer scale reading, is used to compute the indirect measurement, the corrected temperature. In a typical efficiency calculation several direct measurements, such as flow rates, power, and temperatures, are used to compute the efficiency.

To illustrate and define how direct and indirect measurements are related, assume that m independent direct measurements, identified as the set of x_i data, contribute to an indirect measurement, y. The measurement formula is then merely the calculation formula,

$$y = y(x_1, x_2, \cdots x_m) \qquad\qquad (2)$$

The operational mathematics of EPA is essentially incorporated in two equations. The first of these two basic equations shows how uncertainty in the dependent variable or indirect measurement, y, is caused by the uncertainty in some independent or directly measured variable, x_j. Call this uncertainty $U_{y,j}$, where the j indicates the influence of the jth direct measurement. Note that uncertainties are assumed to be relatively small; therefore, the classical Euler formula for a small perturbation can be used to calculate it. The resulting uncertainty in the calculated y (which is the previously identified $U_{y,j}$) is related to the uncertainty in the measured x_j, which is identified as U_j, as follows,

$$U_{y,j} = \frac{\partial y}{\partial x_j} U_j \qquad (3)$$

Note that it seems to be more convenient to use the Expanded Uncertainty rather than the Standard Uncertainty in EPA calculations to avoid introducing any ambiguity about the appropriate coverage factor.

The second basic operational formula shows how independent uncertainties are combined when several direct measurements are used to calculate an indirect measurement. It can usually be assumed that the governing probability distributions are approximately symmetric. As a result, analysis shows that the squares of the contributing uncertainties sum to give the squared combined uncertainty, or

$$U_y^2 = U_{y,1}^2 + U_{y,2}^2 + \cdots + U_{y,i}^2 + \cdots = \left(\frac{\partial y}{\partial x_1} U_1 \right)^2 + \left(\frac{\partial y}{\partial x_2} U_2 \right)^2 + \cdots + \left(\frac{\partial y}{\partial x_i} U_i \right)^2 + \cdots \quad (4)$$

The preceding results are near corollaries of the basic principles of uncertainty theory or experimental statistics. Nevertheless, systematic treatment of uncertainty is relatively recent in the engineering literature, and in the mechanical engineering literature these findings are usually attributed to Kline and McClintock (1953). Note in particular the significant role of the partial derivatives, which are recognized to be the influence coefficients that show how each x_i influences the y. In practice, evaluating these partial derivatives is likely to be the only even slightly troublesome task in EPA. To make this task easy, an Excel utility called a User Form was programmed to automate the numerical calculation of partial derivatives for use in EPA, and this form is presented in this paper.

In principle, EPA could be used to evaluate the Uncertainty A of an indirect measurement from the observed uncertainties of the direct measurements, but this calculation would be superfluous since it is easier just to evaluate the uncertainty of an indirect measurement from statistical analysis of its observed variation. In contrast, EPA is invaluable and irreplaceable for evaluating the Uncertainty B of an indirect measurement since this uncertainty cannot usually be estimated directly. Adequate specific or at least generic information about the uncertainties of the contributing direct measurements should always be available in experimental engineering; consequently, EPA can always be used to estimate the otherwise unknown uncertainty of the indirect measurement.

COMBINING UNCERTAINTY

The final result needed to completely quantify the error limits of an experimental measurement is the Combined Uncertainty, which represents the overall effects of randomness and bias. To compute the overall Combined Uncertainty, one first computes the Expanded Uncertainty A using statistics. One then computes the Expanded Uncertainty B by applying EPA. The Uncertainty A, which is due to random variation, and the Uncertainty B, which quantifies the possible bias, are obviously independent sources of error; therefore, one can safely compute the Combined Uncertainty using the usual combining rule, as

$$U_C^2 = U_A^2 + U_B^2 \tag{5}$$

The overall uncertainty problem thus amounts to statistical evaluation of Uncertainty A and EPA evaluation of Uncertainty B. A representative practical application will be presented next and used to illustrate a convenient means of conducting and presenting these calculations.

PRACTICAL APPLICATION

The representative practical application selected to illustrate a convenient system for calculating and presenting uncertainty data is the experimental measurement of the overall conductance of a counterflow heat exchanger operated under fixed conditions. The experimental setup is shown in Figure 1. This experiment is mundane, but it is complicated enough to be a representative of the more challenging uncertainty calculations that are encountered in practical energy systems engineering.
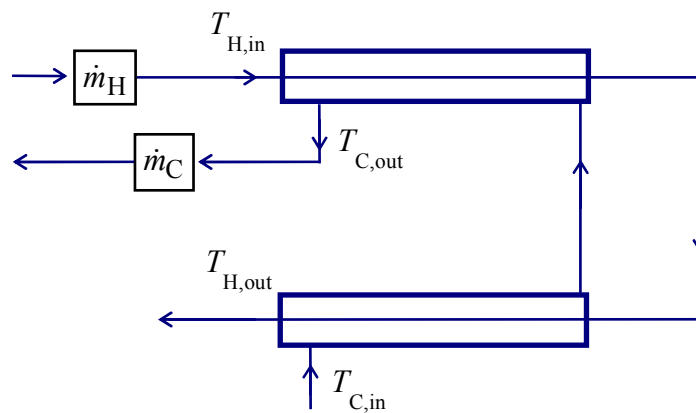

Figure 1. Schematic of Heat Exchanger Experiment

As most readers will recognize, calculation of the conductance involves the six direct measurements shown schematically in Figure 1. In the actual experiment, a vibrating U-tube Coriolis flowmeter was used for the hot water flow rate, and a variable area flowmeter or rotameter was used for the cold water. Type T thermocouples read by a digital thermometer with electronic correction for the cold junction temperature were used for all of the temperature measurements.

In this particular application, the heat exchanger conductance is to be calculated from measured data using the log-mean temperature difference. Usually the overall conductance is represented as the hypothetical product of a unit conductance U and a reference area A. In the context of this paper, the symbol UA would be confusing, so the unambiguous symbol C_{HX} will be used. In principle the conductance is calculated by

$$C_{HX} = \frac{\dot{Q}_{ave}}{F_G \cdot LMTD} \tag{6}$$

Any heat transfer textbook will recite this formula where
$\quad \dot{Q}_{ave}$ is the average heat transfer rate,
$\quad F_G$ is the geometrical correction factor, and
$\quad LMTD$ is the log mean temperature difference.
The preceding measurement function is deceptively simple in appearance. In fact as shown below, both the heat rate and the $LMTD$ are rather complicated.

The best estimate of the heat rate is the average between the rate that heat is adsorbed by the cooler stream and the rate that heat is given off by the warmer stream, or

$$\dot{Q}_{ave} = (\dot{Q}_{hot} + \dot{Q}_{cold})/2 \tag{7}$$

The two heat rates are computed from the six direct measurements. First for the hot stream,

$$\dot{Q}_{hot} = (\dot{m}_H\, c_{P,H})(T_{H,in} - T_{H,out}) \tag{8}$$

Here for the warmer or hot stream (H),
$\quad \dot{m}_H$ is the mass flow rate,
$\quad c_{P,H}$ is the heat capacity, and
$\quad T_{H,in}$ and $T_{H,out}$ are the inlet and outlet temperatures.
Then for the cold stream,

$$\dot{Q}_{cold} = (\dot{m}_C\, c_{p,C})(T_{C,out} - T_{C,in}) \tag{9}$$

where the corresponding terms have the analogous definitions.

Combining the subsidiary calculations described above with the familiar general formula for the *LMTD*, gives the following formula for the heat exchanger conductance:

$$C_{HX} = \frac{\left(\left(\dot{m}_H\, c_{P,H}\right)\left(T_{H,in} - T_{H,out}\right) + \left(\dot{m}_C\, c_{p,C}\right)\left(T_{C,out} - T_{C,in}\right)\right)/2}{F_G \cdot \dfrac{\left(T_{H,in} - T_{C,out}\right) - \left(T_{H,out} - T_{C,in}\right)}{\ln\left\{\dfrac{T_{H,in} - T_{C,out}}{T_{H,out} - T_{C,in}}\right\}}} \tag{10}$$

This detailed formula above is quite complex, especially by experimental engineering standards. In fact, it could be even more complex than it appears since the temperature dependence of the heat capacities should be considered if the temperature changes are significant. In this particular case of counterflow, the geometrical correction factor F_G is, thankfully, unity. Otherwise, the calculation formula would be even worse. Such complexity makes purely analytical EPA calculations awkward.

The practical example will now continue with actual computation of the conductance data and the accompanying Uncertainty A statistics.

EXAMPLE UNCERTAINTY A CALCULATIONS

The heat exchanger application will proceed with the routine thermodynamic calculations and the associated statistical calculations that result with the Uncertainty A. Representative measured and calculated experimental data are in Table 1 below. This table is meant to represent work in progress, so all legible digits from the direct measurements are reported and a generous number of digits are retained in the intermediate calculations to prevent round-off errors from propagating. If the table were being prepared for formal presentation, only significant digits identified by the (yet incomplete) uncertainty analysis would be retained.

Six sets of experimental data are presented in the table along with the needed intermediate calculations and the final calculations of the heat exchanger conductance. A seventh column contains the averaged values of the direct measurements and the corresponding intermediate and final calculations. These averaged data are the most representative data; consequently, they were used as the most appropriate baselines for the uncertainty calculations. Since the measured results are very consistent, any set of data could have been used as the baseline for the EPA in this experiment.

All of the calculations in the table follow from Equations (7) through (10) along with regression formulas for the heat capacities that are trivial and are not shown.

In this paper, the Uncertainty A calculations are presented first only to facilitate a more logical presentation. In practice, the investigator is well advised to conduct at least

preliminary Uncertainty B analysis ahead of time using generic or even estimated data. Otherwise a poor experimental design could result in excessive Uncertainty B that cannot be easily corrected later. Indeed a complete redesign may be necessary.

Table 1. Experimental Data for Heat Exchanger Experiment

	units	1	2	3	4	5	6	AVG
$\dot{m}_H$	kg/s	0.051	0.052	0.052	0.052	0.053	0.052	0.052
$\dot{m}_C$	kg/s	0.083	0.083	0.083	0.083	0.083	0.083	0.083
$T_{H,in}$	°C	65.3	64.4	64.0	64.6	65.5	66.2	65.0
$T_{H,out}$	°C	47.7	47.4	47.4	47.6	48.0	48.6	47.8
$T_{C,in}$	°C	11.2	11.1	11.1	11.1	11.1	11.1	11.1
$T_{C,out}$	°C	22.1	22.2	22.0	21.9	22.2	22.4	22.2
$T_{H,avg}$	°C	56.5	55.9	55.7	56.1	56.8	57.4	56.4
$T_{C,avg}$	°C	16.7	16.7	16.6	16.5	16.6	16.7	16.6
$c_{P,H}$	J/kg-K	4182	4182	4182	4182	4183	4183	4186
$c_{P,C}$	J/kg-K	4185	4185	4185	4185	4185	4185	4180
$\dot{Q}_{hot}$	W	3736.7	3697.6	3636.0	3722.5	3856.4	3788.6	3743.4
$\dot{Q}_{cold}$	W	3758.0	3826.0	3758.1	3724.1	3860.0	3928.0	3804.6
$\dot{Q}_{avg}$	W	3747.3	3761.8	3697.0	3723.3	3858.2	3858.3	3774.0
LMTD	K	39.746	39.175	39.081	39.512	40.052	40.597	39.694
C_{HX}	W/K	94.283	96.027	94.600	94.232	96.329	95.038	95.077

Average C_{HX} =   95.085
SSD C_{HX} =   0.899
Standard u_A of average C_{HX} =   0.367
U_A of the average C_{HX} =   0.94

The Uncertainty A calculations are, as usual, quite straightforward. In this case of a single sample experiment, the average value of the conductance is the ultimate measured variable. Since the measurement average is calculated from six (6) data, the Standard Uncertainty is the familiar standard deviation of an average given by the formula,

$$u_{AVG} = SSD/\sqrt{N} = 0.367\,\text{W/K} \tag{11}$$

Where N is the number of data in the data set, which is 6 in this case. The Expanded Uncertainty is computed using Equation 1. The coverage factor is computed using the number of statistical degrees of freedom, 5 in this case, so

$$U_A = k_c \, u_{AVG} = 2.57(0.367 \, \text{W/K}) = 0.94 \, \text{W/K} \tag{12}$$

This calculation completes the analysis of Uncertainty A, the uncertainty due to random variation evaluated by statistical analysis of repeated measurements. In the final calculation, only two digits are retained since two is the maximum reasonable number of significant digits in any uncertainty.

The presentation will now digress to present the background and design of an Excel User Form developed for convenient calculation of the influence coefficients needed in Uncertainty B calculations. Then the Uncertainty B calculations will be presented and the uncertainty analysis will be completed.

A USER FORM FOR INFLUENCE COEFFICIENTS

A glance at Equation (10) is convincing that in this, as in any modestly challenging situation, analytical calculation of the influence coefficients in practical applications can be difficult. Indeed, the derivatives are sometimes so difficult or time-consuming that harried experimenters may be tempted to avoid computing them and thereby neglect completing the uncertainty analysis. This circumstance is both undesirable and unnecessary because adequate numerical derivatives can always be computed. Assuming that the investigator is using a professional spreadsheet such as Excel for the calculations, three alternatives to analytical derivatives come to mind.

The first alternative is essentially manual calculation of the numerical derivative using a simple first finite difference formula such as

$$\frac{\partial y}{\partial x_i} = \frac{y_2 - y_2}{\Delta x_i} \tag{13}$$

To compute this partial derivative, the experimenter would need to individually increment each individual direct measurement and then calculate the influence coefficient using the preceding formula. Note that great accuracy is not necessary since only two significant digits at most are required for an uncertainty. The result would then need to be copied as a "value" (i.e., an explicit numerical constant) into a spreadsheet block used for EPA. This tactic is simple but somewhat tedious, especially since the scrupulous investigator would want to investigate the accuracy of the derivative, which would require repeated calculations.

The next alternative is using a mathematical program with a built in utility for partial derivatives. An especially interesting choice is the popular EES program (Klein, 1992-2002), which includes an excellent EPA utility. Unfortunately, not all investigators or students will have access to that program, and in any event EES and similar programs are less popular for experimental analysis than the spreadsheet programs such as Excel.

The third alternative is introduced in this paper. It is to add a partial derivative utility to Excel. This has been done by creating a User Form called Num_Der. The Visual Basic for Applications (VBA) code for the form is shown in Appendix A and the accompanying dialog box is shown in Appendix B below.

Note that the dialog box includes text boxes for the user to identify the cells that contain (1) the dependent variable cell, which must contain a formula, where the indirect measurement, y, is ultimately calculated, (2) the independent variable cell, which must contain a value, that contains the pertinent direct measurement, (3) the cell where the value of the derivative is to be inserted, and (4) a cell for an optional user-defined value of the increment of the direct measurement.

The basic algorithm is organized as follows:

(1) The code first checks to ensure that the indirect measurement cell actually contains a formula and that the direct measurement cell actually contains a value.

(2) The code next establishes a first estimate of an appropriate increment in the direct measurement. Note that an effort is made to avoid numerical noise. The choices are:
     (a) The user-defined increment is used if specified.
     (b) .0001 times the measurement value is used unless that value is small.
     (c) The value of x itself is used if it is less than 1.0×10^{-9}.

(3) The code next evaluates the numerical derivative using the following standard second forward difference formula,

$$\frac{\partial y}{\partial x} = \frac{-3y(x) + 4y(x + \Delta x) - y(x + 2\Delta x)}{2\Delta x}$$

(14)

(4) The code next evaluates the derivative using an increment .01 times smaller than the original and compares the values. If the values agree, the average of the two values is inserted in the output cell. If the values disagree, user intervention is suggested.

A variety of test functions and practical problems have been addressed with this routine and no practical or hypothetical problems have been discovered. The step in which an appropriate increment is identified could probably be made more sophisticated and potentially more effective; however, in the absence of practical problems, no attempt has been made to refine it. Note that this algorithm uses the same data entry cell as in used in the actual calculations. This arrangement virtually ensures that the units will be consistent throughout the uncertainty calculations.

Next the User Form will be used to facilitate the needed Uncertainty B calculations.

UNCERTAINTY B CALCULATIONS

Table 2 presented below exemplifies a recommended format for calculating and presenting the Uncertainty B data and for presenting the accompanying Uncertainty A and Combined Uncertainty data.

As mentioned before with regard to this experiment, a vibrating U-tube Coriolis flowmeter was used for the hot water flow rate, and a variable area flowmeter or rotameter was used for the cold water. Type T thermocouples read by a digital thermometer with electronic correction for the cold junction temperature were used for all of the temperature measurements. In actuality, the uncertainty data for all of these direct measurements were obtained by calibration. In this case, the thermocouples and the digital thermometer were calibrated as a system by comparison with a field standard platinum resistance temperature detector in a thermostatic calibration block, and the flow meters were calibrated using a weighing tank.

Uncertainty analysis was conducted for all of these calibrations, and the uncertainties from these calibrations are entered into column two of the table. Not surprisingly, the four uncertainties for the very similar temperature calibrations are almost identical. Note that the Combined Uncertainty of each calibration must be used as the Uncertainty B of the corresponding measurement. This is because any residual Uncertainty A from the calibration is now essentially built in to the instrument by way of the calibration formula, and further repeated measurements when the instrument is used cannot reduce this error. In practice, generic data supplied by the manufacturer or obtained from reliable technical literature can be used to replace or complement calibrations.

Using the Excel User Form, the Uncertainty B calculations are almost as simple and easy as the Uncertainty A calculations shown above. The uncertainty data are merely entered into the appropriate cells. Then using the baseline data, the influence coefficients are calculated with the form. Note that in its present configuration, the form by design requires that the input data be present as fixed numerical values. If, as in this actual case, the averages are computed with cell formulas, it will be necessary to convert the formulas to values or to copy the formula results into new cells as fixed values. The individual contributing uncertainties are computed using Equation (3) and are combined according to Equation (4). Ultimately, the Uncertainty B is calculated to be 6.9 W/K as shown in the footnotes to Table 2. The Combined Uncertainty in this case is also 6.9 W/K because the Uncertainty A is so small. Note that the process of summing the squares nearly eliminates the contribution of the smaller uncertainty when they differ as much as an order of magnitude. Also note that stopping with the statistical Uncertainty A analysis and thereby neglecting the Uncertainty B calculations greatly underestimates the overall true uncertainty in this very representative case.

The footnotes to the table include the Uncertainty A statistic and the calculation of the Combined Uncertainty. Recall that only a few repeated measurements and routine statistical calculations were necessary to compute the Uncertainty A. In this case, only

the Sample Standard Deviation and the coverage factor from the t-distribution are necessary. The Combined Uncertainty is computed using Equation (5) completing the analysis.

In many cases, experimenters lamentably stop their error analysis after computing only the statistical Uncertainty A. This is unfortunate both because the resulting underestimate of the uncertainty is misleading and because the missing EPA calculations are almost invariably trivial in concept and easy to compute in practice, as shown in this example. Indeed, inspection of the example table below shows that all that is needed in typical Uncertainty B analysis is a tabulation of the uncertainties of the instrumentation used for the direct measurements, calculation of the influence coefficient from the assuredly known measurement formula, and a little arithmetic. Indeed the only challenging task would be the analytical or numerical calculation of the influence coefficients, but even this task is nearly automatic with the User Form presented herein.

Table 1. Uncertainty Estimation by Error Propagation Analysis

Measurement	U_j	$\dfrac{\partial C_{HX}}{\partial x_j}$	$\left(\dfrac{\partial C_{HX}}{\partial x_j}U_j\right)^2$ $(\text{W/K})^2$	Basis	Source
$T_{H,in}$ (C)	0.2 K	1.58 (W/K)/K	0.100	calibration	(1)
$T_{H,out}$ (C)	0.2 K	-3.94 (W/K)/K	0.621	calibration	(1)
$T_{C,in}$ (C)	0.2 K	-3.04 (W/K)/K	0.369	calibration	(1)
$T_{C,out}$ (C)	0.2 K	5.40 (W/K)/K	1.165	calibration	(1)
$\dot{m}_H$ (kg/min)	0.40 kg/min	15.84 (W/K)/ (kg/min)	40.149	calibration	(2)
$\dot{m}_C$ (kg/min)	0.24 kg/min	9.05 (W/K)/ (kg/min)	4.722	calibration	(2)

Sources: (1) reference to temperature calibration report,
(2) reference to weighing tank calibration report

(summed) $(U_{y,j})^2$ = 47.1 $(\text{W/K})^2$
Expanded Uncertainty B, U_B = 6.9 W/K
Expanded Uncertainty A, U_A = 0.94 W/K
Combined Uncertainty, U_C = 6.9 W/K

CONCLUSION

Complete uncertainty analysis resulting in the Combined Uncertainty should never be avoided by experimenters and should always be demanded by users of experimental data. Uncertainty A information is quickly obtained by standard statistical analysis of repeated measurements and is usually quite straightforward. Uncertainty B information is obtained by physical analysis of the measurement system usually involving error propagation analysis (EPA). EPA requires merely the collection and management of the uncertainty data relating to the direct measurements and some simple calculations. The only practical obstacle to quick and efficient EPA is calculation of the relevant influence coefficients. The Excel User Form presented in this paper is demonstrated to be a reliable and efficient means of numerically calculating these coefficients; therefore complete uncertainty analysis is nearly automatic and should never be avoided.

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BIOGRAPHY

SHELDON M. JETER is Associate Professor of Mechanical Engineering at the George W. Woodruff School of Mechanical Engineering at Georgia Tech. He has degrees from Clemson University, the University of Florida, and Georgia Tech. He has been on the academic faculty at Georgia Tech since 1979. His research interests are thermodynamics, experimental engineering, heat and mass transfer, and energy systems.

Appendix A: VBA Code of the User Form for Partial Derivatives

```
Private Sub CommandButton1_Click()
'User Form for Numerical derivatives
'Under development, 16 April 2004; completed and tested 20 April 2004
'This user form uses the following standard formula to approximate the derivative
'
'              -3*y(x) + 4*y(x+h) - y(x+2h)
'
'   y '(x) = ----------------------------
'
'                       2*h
'
Actsh = ActiveSheet.Name
Y_value = Worksheets(Actsh).Range(Y_cell.Value).Value
X_value = Worksheets(Actsh).Range(X_cell.Value).Value

If Worksheets(Actsh).Range(Y_cell.Value).HasFormula = False Then
   MsgBox "The selected y-cell does not contain a formula. User form will terminate."
   GoTo 999
End If

If Worksheets(Actsh).Range(X_cell.Value).HasFormula = True Then
   MsgBox "The selected x-cell contains a formula not a value. User form will terminate."
   GoTo 999
End If

If (Delta_X_cell.Value = "") Then
   If (Abs(X_value) > 0.000000001) Then
      Delta_x = 0.0001 * X_value
   Else
      Delta_x = 0.000000001
   End If
Else
   Delta_x = Worksheets(Actsh).Range(Delta_X_cell.Value).Value
End If

X_2 = X_value + Delta_x
Worksheets(Actsh).Range(X_cell.Value).Value = X_2
Y_2 = Worksheets(Actsh).Range(Y_cell.Value).Value
X_3 = X_value + 2 * Delta_x
Worksheets(Actsh).Range(X_cell.Value).Value = X_3
Y_3 = Worksheets(Actsh).Range(Y_cell.Value).Value

'evaluate the derivative
dydx1 = (-3 * Y_value + 4 * Y_2 - Y_3) / (2 * Delta_x)

'repeat with smaller interval
Delta_x = 0.01 * Delta_x
X_2 = X_value + Delta_x
Worksheets(Actsh).Range(X_cell.Value).Value = X_2
Y_2 = Worksheets(Actsh).Range(Y_cell.Value).Value
X_3 = X_value + 2 * Delta_x
Worksheets(Actsh).Range(X_cell.Value).Value = X_3
Y_3 = Worksheets(Actsh).Range(Y_cell.Value).Value
```

```
'evaluate the derivative
dydx2 = (-3 * Y_value + 4 * Y_2 - Y_3) / (2 * Delta_x)

'compare the two values
DerError = 2 * Abs((dydx2 - dydx1) / (dydx1 + dydx2))
If (DerError > 0.01) Then
    MsgBox "The derivative does not appear to converge. User intervention suggested"'
End If

DYDX = 0.5 * (dydx1 + dydx2)

Worksheets(Actsh).Range(X_cell.Value).Value = X_value
Worksheets(Actsh).Range(Der_cell.Value).Value = DYDX

Sheets(Actsh).Select
999 Unload Me
End Sub
```

Appendix B: Screen Shot of the Dialog Box for the User Form for Partial Derivatives