AC 2011-103: A LONGITUDINAL STUDY ON STUDENTS' DEVELOPMENT AND TRANSFER OF THE CONCEPT OF INTEGRATION

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A longitudinal study on students' development and transfer of the concept of integration

Abstract: We present results from the first two years of a project investigating how engineering students develop problem solving skills through their academic career. The project consists of a longitudinal study as well as cross-sectional studies in multiple courses in mathematics, physics and engineering. In this article, we focus on the transfer of knowledge from mathematics to physics courses. We track how students' understanding of function and integration evolves as they progress through the Calculus courses using individual semi-structured interviews. Most students develop a solid understanding of the mathematical concepts of function and integration. However, they are unable to access different representations (for example, use geometric reasoning) when solving a similar problem in the physics interviews. Transfer of knowledge depends upon students recognizing a connection between the different subjects.

Introduction

Engineering students take mathematics courses to acquire the ability to use mathematics as a language for solving problems in physics and engineering. The first immediate role of a math course is to teach students a set of tools they can use to solve specific abstract problems (finding solutions to equations, determining the minimum and maximum of a function, computing a certain integral, etc.); but behind this first role lies the idea that the knowledge acquired could be applied in different contexts. The central goal of mathematics education is to help students develop the mathematical sophistication in order to use this language to solve problems beyond the original learning setting. In conjunction with colleagues in Physics and Electrical Engineering at Kansas State University, we have been carrying out a longitudinal study tracking how engineering students' conceptual understanding and problem solving abilities evolve throughout their undergraduate career. As part of this study, the physics department asked selected students questions about finding work done when they were given a formula for the force as a function of distance and also when they were given a graph showing force as a function of distance. The physicists reported to the group that students did not seem to have learned the notion of integral as an area under the curve, because students did better on the algebraic variations than on the geometric variations of these questions, despite the fact that the functions in the algebraic variations were quadratic while the ones in the geometric variations were just linear. At the very least, students were not successfully transferring their geometric knowledge to the context of physics.

This article is a report on the mathematical portion of that study. We will address whether and how students' conceptual understanding grows across the calculus sequence. We will then discuss issues with transfer and what factors support or hinder mathematical problem solving in applied courses. The goal is to identify when and how different students develop their problem solving skills so that we can better support this process.

Background

For this study we needed a framework to measure conceptual understanding. This is important both for tracking conceptual growth over time and also for analyzing transfer of knowledge between contexts. In order to measure whether knowledge can be transferred, it is important to check whether the knowledge has been properly learned in the first place, and whether such knowledge is mere procedural skill or a more complete conceptual understanding. A standard framework for measuring conceptual understanding in mathematics education is provided by the APOS theory introduced by Dubinsky [4]. APOS is an acronym for Action-Process-Object-Schema. Initially, students focus on carrying out specific Actions to solve specific problems. As they practice these actions, they recognize that the repeated actions provide a Process for determining specific quantities or properties. Over time, this process is reified into an Object, a noun rather than an adjective. At an elementary level, students move from carrying out the process of addition to understanding the operation of addition which has properties such as associativity and commutativity. Once this reification has been carried out, the student is ready to assimilate the object into their mental Schema where they understand the relationships between different concepts. The phrase "APOS scale" in this article refers to rating a student's conceptual understanding by whether they appear to be working with Actions, Processes, Objects, or Schemas in applying concepts to problems. This cycle is a process of abstraction of course, and it seems quite likely that students will be more successful in transferring knowledge that is understood at higher levels of abstraction than they will be knowledge that they only comprehend at the more context-bound level of Action.

The act of applying what one has learned in one situation to different situations is defined as "transfer" in the educational literature [7][9]. The theory of transfer of learning started with the theory of identical elements introduced by Thorndike and Woodworth [10]. In a typical study, the researcher provides or explains the solution of a problem involving a selected concept to a student then asks the interviewee to solve a problem differing from the first only in surface features. According to Thorndike and Woodworth, the likelihood of transfer is proportional to the number of "identical elements" the two problems share. Studies show that this type of transfer is difficult and rare [3].

Indeed, features that are surface differences for experts are generally not for students. In a response to this critique, Lobato promotes an "actor-oriented" perspective [5]. The conceptualization of transfer shifts away from the expert's viewpoint to an actor's or learner's viewpoint. In this approach, the goal is to understand the "relations of similarities created" by the learner and how they are supported by the environment. The focus is not on whether the right type of transfer is obtained but rather on determining what kind of similarities the students see.

Another modern approach to transfer has been proposed by Bransford & Schwartz [1]. Transfer studies in their view have relied too much on "sequestered problem-solving", in which a student is explained a problem then asked "cold" to solve a similar problem thus giving negative results. They promote an approach using "preparation for future learning" which focuses on students' ability to learn new skills. A good assessment of learning would take into account the speed of learning the concepts in a new underlying material rather than the early attempts of students in applying a concept to the new context [6].

In this study, we have decided to apply Lobato's actor-oriented perspective in analyzing transfer. The sequestered problem-solving approach of Thorndike and Woodworth has not proved easy to measure in practice and relies on an oversimplified view of how students approach problems.

While the preparation for learning approach of Bransford & Schwartz is a better fit for our current understanding of how students learn, it requires measurements of the speed of learning over time that are not suited to our study. Lobato's focus on "relations of similarity" provides a better fit for the notion of conceptual understanding using the APOS scale and allows us to address what is transferred even in situations where students were unable to solve selected problems.

Research questions

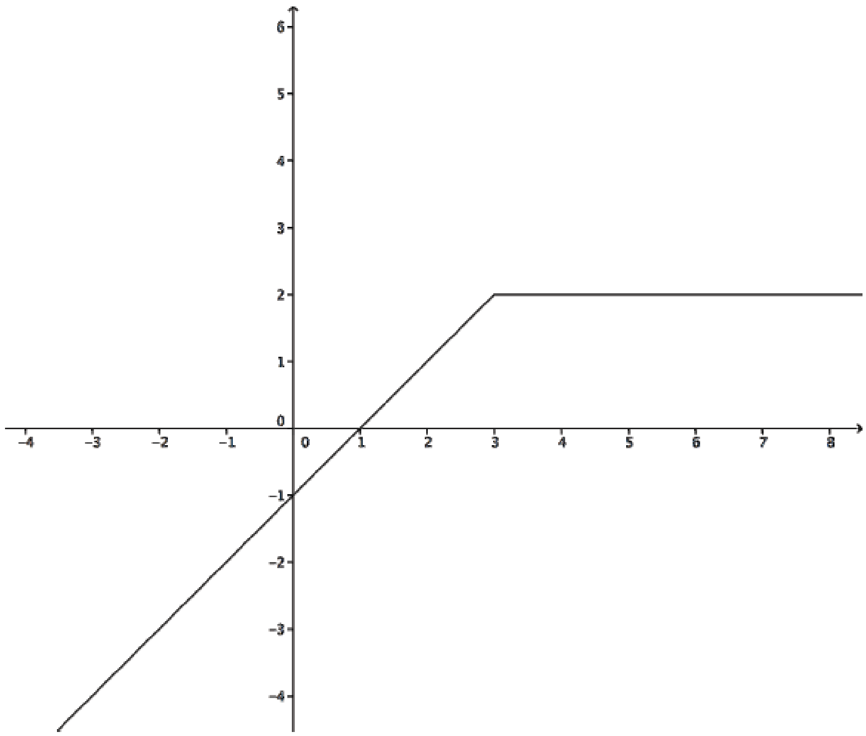
The general research questions of our study ask how engineering students develop their understanding of quantitative topics and what factors support or inhibit them transferring their knowledge from mathematics to courses in science and engineering. In this paper we will focus on the concepts of function and accumulation (integration). These are primary topics in the calculus sequence and also play a major role throughout applications. In view of the reports from the physicists that students find it difficult to use geometric notions of integration in physics classes, we will also consider whether this specific topic has been learned, and if so, why there are difficulties with transfer. So the specific research questions for this paper are

1. How does students' conceptual understanding of the concepts of function and integration grow over time along the APOS scale?
2. What factors support or hinder students from transferring their knowledge of integration from mathematics to physics?

Methodology

This study was carried out at a large public university in the Midwest. An initial pilot study testing our ability to measure conceptual understanding on the APOS scale was carried out with 18 students in Fall of 2005 and Spring of 2006. Organizing and designing the longitudinal survey took time, but a group of volunteers was chosen from the Calculus I class during the Fall of 2009. This group consisted of nineteen engineering students most of whom were in their first semester of college. This group of students was then tracked as much as possible as they progressed through the Calculus sequence. Of the original nineteen Calculus I students, eight agreed to another interview in February of 2010 for Calculus II; four students were added who had not been previously interviewed. These twelve Calculus II students were asked to volunteer again in April after they had progressed a little further in their Calculus II class. Eleven of the students agreed to a second interview. Finally, of the twelve students interviewed at least once in Calculus II, nine agreed to an interview in the Fall of 2010 while taking Calculus III.

Students' knowledge and ability were assessed using semi-structured interviews. Calculus interviews consisted of both scripted questions as well as unscripted follow-up questions designed to better indicate the students' level of understanding. Questions were posed addressing students ability to work algebraically, geometrically, and verbally. A sample question from the first Calculus II interview is given below.



Given the graph of *g*(*x*) above, can you draw the graph of $G(x) = \int_0^x g(t)dt$ **?**

After the interviews, the students were rated with a modified APOS framework to evaluate their conceptual understanding of each of the topics: function, derivative, and accumulation. The normal Action, Process, Object, Schema framework of APOS theory was modified by adding in intermediate levels: Action/Process, Process/Object, etc. This was done to afford a finer level of evaluation as well as to mitigate the effects of the fuzzy borders between the APOS levels; e.g., if students are on the border between Action and Process, instead of deciding definitively – and possibly arbitrarily – whether they were Action or Process, we can instead say they are Action/Process.

For example, the first few levels were operationalized as follows:

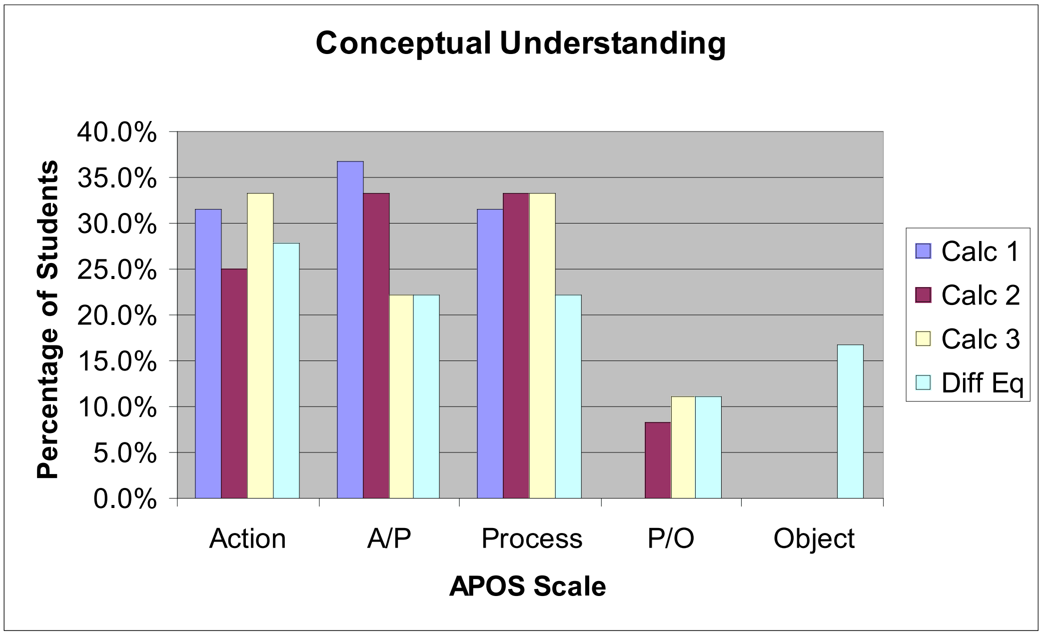
Action: Students can "do the math" to arrive at an answer, but their only strategy is to follow algorithms they have been taught without any vision of what lies ahead or what the steps mean. If students do not know how to do a problem, they will run through their catalog of procedures until either they find one that works or they give up. Functions, integrals, etc. are static formulas that are evaluated by means of an algorithm.

Action/Process: Students still follow prescribed steps but they show some understanding and adaptability. They can recognize mistakes and contradictions, but they may not be able to explain them or figure out what went wrong. Alternatively they may skip back and forth between Action and Process.

Process: Students can reason through a problem, recognize the reason for steps, connect different representations, etc. The object in question, though, is still an intangible process rather than an object with properties that exhibit themselves as that process.

Results

The results are given in the graph below. Note that this graph cheats slightly in that we have included results from the differential equation students in the pilot project alongside the results from the continuing cohort of students in the longitudinal study (who will hit differential equations this semester). The data shows that a group of about 25-30% of the students starts at the Action level and stays there throughout the calculus sequence. These students learn the procedures so that they can pass the classes, but do not grow conceptually. However, there is evidence of growth among the other students, as the percentages move up along the scale with each class. In answer to our first research question, we conclude that most students do gain in conceptual understanding, but that a significant fraction mainly learns procedurally.



In contrast to the experience reported by the physicists, in the calculus interviews students did equally well on problems where information was provided either graphically or algebraically. Not only did students do as well on problems requiring a geometric understanding, almost all students naturally used geometric reasoning in problems where it was appropriate, such as the example given above. Students definitely did learn the geometric notion of integral as area under a curve. Why they were unsuccessful in transferring this knowledge to apply toward problem solving in physics is considered in the next section.

Discussion

Our second research question addressed what factors supported or hindered students in transferring the knowledge they demonstrated in the mathematics interviews to solving problems in the physics interviews. The physicists had observed that students were more successful in transferring in an algebraic context than in a geometric context. There are several possible explanations for why students transfer knowledge better using algebraic representations than geometric representations. The most straightforward explanation may be simply that the algebraic representations are naturally more abstract, and therefore students are less likely to be confused by the context. If this is the case, then as students progress along the APOS scale, they ought to become more effective at transferring their knowledge, since movement along the APOS scale measures abstraction of understanding.

Another explanation is that the issue we are seeing is not with transfer but with problem solving. In considering whether students can transfer knowledge to a new problem solving situation, one must consider whether failure in the new situation is caused by a lack of transfer or by a lack of problem solving ability. Schoenfeld defined four different aspects of mathematical problem solving[8]. First, students need the basic Knowledge to solve the problem. Second they need to be able to use Heuristic reasoning to recognize which knowledge and techniques apply to the problem at hand. Third, they must possess metacognitive Control to be successful in carrying out a multistep problem, with the ability to shift between different techniques as needed without losing the underlying thread of the problem. Finally, students are sometimes defeated by their Belief systems that lead them not to consider ideas and techniques they know. For example, students may compartmentalize theoretical knowledge as irrelevant to “practical” problems, and thus not recognize that existence proofs often contain insight into constructing the quantities in question.

In this framework, troubles with Control or Belief may cause students to be unable to recognize the need to use the geometric concept of integration in the problem at hand. Students who believe that geometric reasoning is part of the “theory” of integration in mathematics may not consider using geometric techniques when doing “practical” problems. Alternatively, students may start working with formulas as they relate kinetic energy, work, and force, and lack the control needed to shift gears from the formulas of physics to apply the geometric tools they mastered in calculus.

We are continuing our longitudinal study to see how students develop as they complete the calculus sequence and proceed through their engineering classes. Based on our initial findings, we are looking to see what percentage of students achieve higher levels of understanding and will follow-up to see how higher levels of understanding support transfer (or not). We are also adding questions on beliefs about the roles of mathematics and physics to our protocols to see if we can identify whether issues with problem solving are hindering transfer, at least in geometric representations. As long as we are still collecting and analyzing our results, the implications for effective instruction will remain uncertain. It appears that students are learning the calculus they need to be successful later. The issue is helping them recognize when to use the tools they have learned.

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