AC 2009-808: A MULTICOURSE EFFORT FOR INSTILLING SYSTEMATIC ENGINEERING PROBLEM-SOLVING SKILLS THROUGH THE USE OF A MATHEMATIC COMPUTER-AIDED ENVIRONMENT

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A Multicourse Effort for Instilling Systematic Engineering Problem Solving Skills Through the Use of a Mathematic Computer Aided Environment.

Abstract

This paper describes a coordinated, multicourse effort at the Mechanical Engineering (ME) Department at Mississippi State University (MSU) to inculcate disciplined/systematic engineering problem solving skills through the use of mathematical worksheets such as those found in Mathcad. It is important to understand that the worksheets addressed here are not forms that are pre-designed for students to fill in the blanks. Students are required to solve problems in much the same manner they traditionally do with pencil and paper. These mathematical CAD programs include: a) word processing features that allow students to present their work in a professional manner, b) automatic recalculation and sophisticated graphing capabilities, which allow students to quickly fix their errors and perform parametric studies, c) symbolic and numerical processing which allows students to tackle more complex and realistic problems, and d) a large number of mathematical tools that greatly facilitate the analysis of engineering problems. Fully exploiting the above features require careful guidance so students can learn a systematic methodology that allows for a better understanding on how to approach and solve engineering problems. For ME students at MSU, these worksheets are required in three courses: Engineering Analysis, System Dynamics, and Energy Systems Design. A significant advantage to sequencing courses in this manner is that students spend more time (and effort) in engineering functions (formulation, verification, and validations) than in the arithmetic function (primarily accomplished by Mathcad). Details, examples, and assessment of effectiveness are discussed in the paper.

Introduction

One important objective in engineering education is to inculcate in the students a systematic and practical approach to problem solving. Traditionally, this has been largely accomplished through the use of homework following guidelines for problem formulation and solution using engineering paper and handheld calculators. However, the widespread availability of laptops and user-friendly mathematical computer solvers is displacing the use of calculators in favor of worksheets from mathematical CAD programs such as Mathcad, Maple, etc. The features available in these programs include automatic recalculation, copy, pasting, and line editing of equations and text, symbolic processing, automatic handling of engineering units, and instantaneous graphic capabilities. Students can spend more time and effort in engineering functions such as problem formulation, solution, verification, and validation rather than recalculating results and plots each time a mistake is found or a parameter is perturbed.

On the other hand, without careful guidance, the use of these worksheets can quickly degenerate into careless and pointless use of equation solvers, displaying the solutions using a mismatch set of units, generating the wrong algebraic solutions, etc. Also, often, the software itself is not fully debugged and display erroneous results. Thus, the students must be carefully coached on the correct use of these mathematical worksheets. This is achieved by raising the expectations on

the problem formulation/solving documentation procedure plus an added emphasis on verification and validation (V&V) of the results. A benefit of V&V is that students spend more time rethinking the entire solution process in terms of the physical meaning of the results and the mathematical well-posedness. The genesis of the evolution of the authors' realization of the pedagogical impacts of mathematical computer solvers on engineering education is documented in Hodge and Taylor (1998), Hodge (2005), and Hodge and Luck (2006).

At the ME department at MSU, these worksheets are used in three courses: Engineering Analysis (EA), System Dynamics (SD), and Energy Systems Design (ESD). Disciplined, systematic problem solving skills are inculcated through the use of strictly enforced homework format in EA and SD. The homework format consists of clearly formulated Given, Find, Solution, and Verification/Validation sections. Pictures and sketches are introduced into the worksheets by copying and pasting from computer generated sketches or drawings, by browsing the internet, or by scanning. Students are required to carefully read the problem statement and summarize this information in the Given section of the homework. The variables to be found or the design requirements are to be listed in the Find section. It is interesting that the Given and Find sections of the homework format are in congruence with the Given and Find formulations in Mathcad for solutions of algebraic and differential equations. This helps reinforce the importance of describing the problems in a well-posed manner. The Verification/Validation section is considered of equal importance as the Solution section and is addressed and graded accordingly. Students are required to show that their answers are reasonable by checking the units are consistent, the magnitudes are reasonable, and the models/equations behave as expected. The physical units processing as well as the 2D and 3D plotting features of Mathcad are heavily used in these sections. Finally, the ESD course, takes the previously acquired problem solving skills one step further into engineering design scenarios.

The sections below will describe the philosophy behind the courses and details and examples of typical homework.

Philosophy behind the Engineering Analysis (EA) course:

A main objective behind the EA course is to consolidate the mathematical skills acquired during the freshman and sophomore years while emphasizing their use in posing and solving engineering problems. This is partly accomplished through the process of learning how to use Mathcad. Students are introduced to Mathcad functionality by demonstrating how to define functions, perform differentiation and integration, simplify systems of algebraic and trigonometric expressions, perform series analysis, find optimal solutions, fit equations to data, and solve differential equations. They are asked to verify and interpret the results using units, magnitudes, and graphical comparisons. This process of verification and interpretation allows them to focus on the definition and meaning of the mathematical expressions used to describe engineering problems.

Example 1

Figure 1 below illustrates an instructor provided solution for a problem in a first homework set. Starting from a blank worksheet, students are expected to generate their own solution but keeping the same format of providing sections labeled as Given, Find, Solution, and Verification. This particular homework set is designed to review basic concepts of integration and differentiation, and, more importantly, to introduce the process of verification. Fresh from calculus courses, students often lack practice and good understanding on the meaning and practical aspects of basic algebra and calculus skills. In this homework students not only learn how to graph a function, use the trace and zoom functions, provide markers and set appropriate limits to the plots but also learn how to use all these ideas within the context of the graphical meaning of derivatives and integrals. This simple philosophy of teaching students the features in the software by requiring them to provide clear, intuitive, graphical interpretation of concepts learned in calculus allows for a very effective way to reinforce their knowledge while preparing them for typical interpretations expected in engineering practice. The same philosophy of verifying the results in order to review key mathematical concepts is used in several homework sets covering topics such as Taylor Series and truncation errors, root finding, maximization/minimization, polynomial fitting, and differential equation solvers.

Example 2

Figure 2 illustrates an after-the-fact, instructor provided solution to a homework problem from the course textbook where students practice how to setup a problem in matrix form and verify the solution. Again, it is important to understand that students are responsible to generate their own solutions starting from a completely blank worksheet. This homework is an example of a series of homework sets where students practice basic matrix notation, solving a system of linear algebraic equations, and provide a conceptual explanation on the meaning of the results. Again, student review all these skills while learning how to create and invert matrices using the software. Note how they are forced to interpret the meaning of the terms in the inverse of the matrix in part (b) of the problem. In this problem student are given air flowrates in and out of a restaurant and carbon dioxide concentrations in the incoming air as well as carbon dioxide generation by smokers and by the kitchen grill. Students are then asked to find the equilibrium concentrations of carbon dioxide in each room in the restaurant. This time, the verification is based on an important engineering concept: conservation of mass. The idea here is to use basic understanding of equilibrium, compatibility, and energy and mass conservation equations to verify the solutions. The objective is for students to be able to relate the latter engineering concepts to a simple system of equations, linear in this problem.

Note how students are introduced to the use of units in this assignment. The consistency of the units of the results of the calculations is verified automatically by the software. Mathcad will not proceed with a calculation unless all units are consistent. This forces the students to carefully retrace their calculations and correct any errors in the consistency of the units in the equations they define.

Example 3

Figure 3 illustrates a homework problem where students practice solving systems of first order differential equations using the built-in differential equation solvers from Mathcad and verify the solutions using steady state information as well as conservation of energy. Again, the figure displayed is not the actual work of the students but rather the solution provided by the instructor as an example of the approach and the overall format required from their homework. Students are required to start from a blank worksheet. The only requirement for each worksheet is that Given, Find, Solution, and Verification headings be provided as well as meaningful content under each heading. Students are given ample of guidance during the lecture sessions on how to understand the basic equilibrium processes, solve the resulting differential equations and verify similar type of problems.

This homework example requires a more mature set of analytical engineering skills from the students. Note the careful use of units throughout the entire solution. Requiring using units forces the students to carefully consider the meaning of each additive term in the differential equations. Inconsistent units will result in the software refusing to run. Note the graphical interpretation of the results during the verification process. Also note the use of the word "Slopes" in the definition of the first order differential equations. This approach of using simpler words such as "slopes" while discussing differential equations makes the students much more comfortable with the subject. For instance, higher order differential equations are typically solved numerically as a system of first order differential equations. That is, a system of equations describing slopes. Students are very receptive to the simpler concept stating that a slope is just the ratio of "rise" over "run" and the system of first order differential equations is merely a process of simultaneously finding all the slopes so that the next incremental changes in the dependent variables can be identified.

Finally, note the subtle use of the Mathcad "Given" and "Find" solver commands in the Verification section. These commands are covered with much detail during the lectures. Students learn that, at steady state, differential equations become simple algebraic or even transcendental equations that are readily solved through root solving algorithms. Finally, it can be observed how the students are reminded in the verification process that simple equilibrium equations such as conservation of energy, or mass, or just summation of forces can be very effective tools in double checking the consistency of the results and hence, allowing for a higher level of confidence on the results.

Problem 1.

Given :   The function $f(x) := e^{\sin(x)} \cdot \cos(x) + 1.$

Find :   Use Mathcad to

　　a) Calculate the derivative of f(x) when $x := \dfrac{\pi}{4}$

　　b) Calculate the integral of f(x) in the interval $0 \le x \le \pi$

Use mathcad graphic capabilities to verify the solutions.

Solution :

Answer a)　　　　Answer b)

$$\dfrac{d}{dx} f(x) = -0.42$$　　　$$\int_0^\pi f(x)\,dx = 3.14$$

Verification :

The validation consists in approximating the derivatives by the approximate slopes and the areas (integrals) by the areas of a box of height equal to the mean value in the interval of interest.

a)
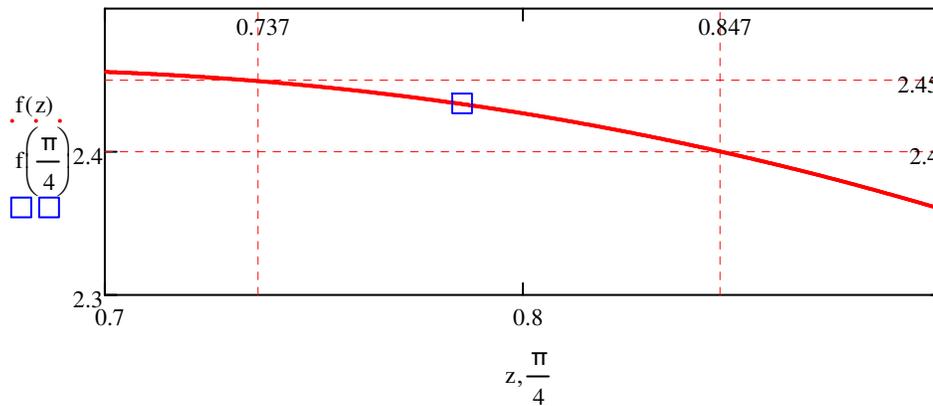
The trace function was used to collect the following data from the plot on the left:

point 1　　　x1 := 0.737   y1 := 2.45

point 2:　　　x2 := 0.847   y2 := 2.40

slope at $x = \dfrac{\pi}{4}$　　Slope $:= \dfrac{y2 - y1}{x2 - x1}$　　　Slope = −0.45

Thus the slope left of -0.45 is quite close to the exact value of -0.42

Figure 1. Worksheet for Example 1

b)

$$\text{sqr} := \begin{pmatrix} 0 & 0 \\ \pi & 0 \\ \pi & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \text{area1} := \begin{pmatrix} 2.2 \\ .7 \end{pmatrix} \qquad \text{area2} := \begin{pmatrix} -.2 \\ 2.5 \end{pmatrix}$$

$$\text{f(z)}$$
$$\text{sqr}^{\langle 1 \rangle}$$
$$\text{area1}_0$$
$$\text{area2}_0$$

$$z, \text{sqr}^{\langle 0 \rangle}, \text{area1}_1, \text{area2}_1$$
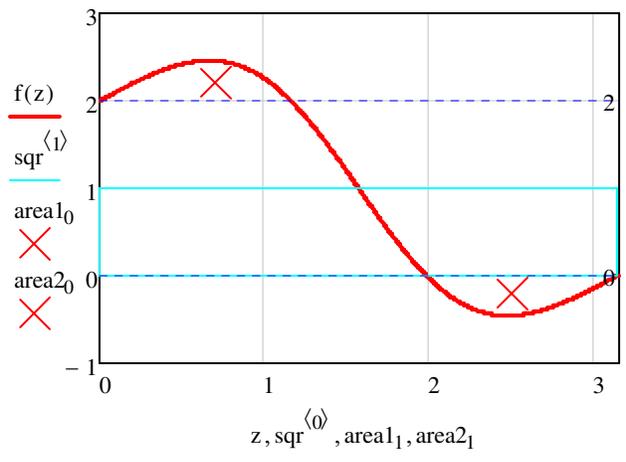
Note that the area (X) bounded on the top by the red curve and on the bottom by y= 2 cancels out the area (X) bounded on the top by y=0 and the bottom by the red curve.

The area of the blue box is roughly equal to the area f(z).

$$\text{ApproxArea} := 1 \cdot \pi \qquad \text{ApproxArea} = 3.142$$

Thus, the answer appears reasonable as it is quite close to the graphical approximation.

Figure 1. Concluded.

Given: The following schematic for Carbon Monoxide Distribution in a Restaurant
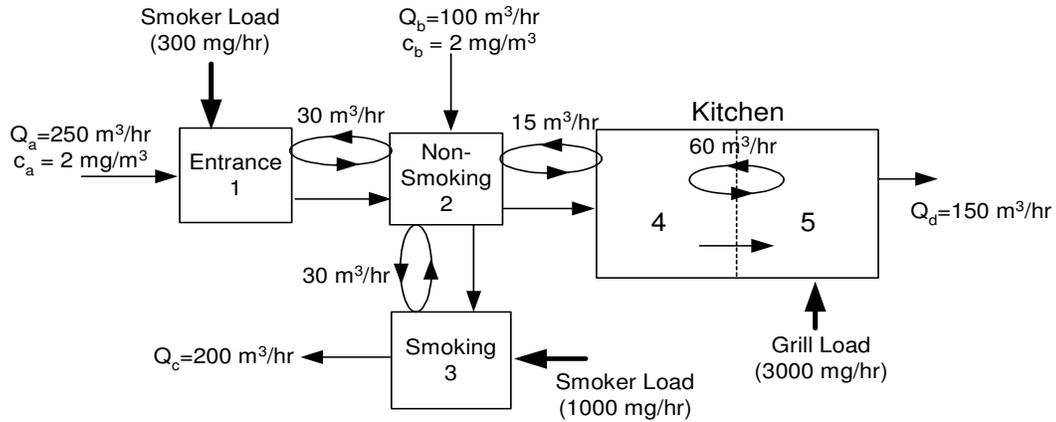
Monoxide Gas Distribution in a Restaurant

**Find:** a) steady state carbon monoxide concentration in each room

b) percent contribution in section 4 from smokers, grill, and outside air

Solution:

Labelling the volumetric flowrates and the concentrations of carbon monoxide.

$$E_1 := 30 \frac{m^3}{hr} \qquad Q_a := 250 \frac{m^3}{hr} \qquad Q_b := 100 \frac{m^3}{hr} \qquad Q_c := 200 \frac{m^3}{hr} \qquad Q_d := 150 \frac{m^3}{hr}$$

$$E_2 := 15 \frac{m^3}{hr} \qquad C_a := 2 \frac{mg}{m^3} \qquad C_b := 2 \frac{mg}{m^3}$$

$$E_3 := 30 \frac{m^3}{hr} \qquad S_1 := 300 \frac{mg}{hr} \qquad S_3 := 1000 \frac{mg}{hr} \qquad S_5 := 3000 \frac{mg}{hr} \qquad E_4 := 60 \frac{m^3}{hr}$$

The following equations represent the balance of the carbon dioxide flow in each room.

$$Q_a \cdot C_a + S_1 + E_1 \cdot (C_2 - C_1) - Q_a \cdot C_1 = 0$$

$$Q_b \cdot C_b + Q_a \cdot C_1 + E_1(C_1 - C_2) + E_3(C_3 - C_2) - Q_c \cdot C_2 - Q_d \cdot C_2 + E_2(C_4 - C_2) = 0$$

$$Q_c \cdot C_2 + E_3(C_2 - C_3) - Q_c \cdot C_3 + S_3 = 0$$

$$Q_d \cdot C_2 + E_2 \cdot (C_2 - C_4) - Q_d \cdot C_4 + E_4 \cdot (C_5 - C_4) = 0$$

$$Q_d \cdot C_4 + E_4 \cdot (C_4 - C_5) + S_5 - Q_d \cdot C_5 = 0$$

Figure 2. Worksheet for Example 2.

Solution using the inverse matrix method:

$$F := \begin{pmatrix} -E_1 - Q_a & E_1 & 0 & 0 & 0 \\ Q_a + E_1 & -E_1 - E_3 - Q_c - Q_d - E_2 & E_3 & E_2 & 0 \\ 0 & Q_c + E_3 & -E_3 - Q_c & 0 & 0 \\ 0 & Q_d + E_2 & 0 & -E_2 - Q_d - E_4 & E_4 \\ 0 & 0 & 0 & Q_d + E_4 & -E_4 - Q_d \end{pmatrix} \quad R := \begin{pmatrix} -Q_a \cdot C_a - S_1 \\ -Q_b \cdot C_b \\ -S_3 \\ 0 \\ -S_5 \end{pmatrix}$$

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{pmatrix} := F^{-1} \cdot R$$

Answer a)

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{pmatrix} = \begin{pmatrix} 3.227 \\ 3.452 \\ 7.8 \\ 8.647 \\ 22.933 \end{pmatrix} \cdot \frac{mg}{m^3}$$

b) percentage contributions in Room #4

$$coeffsinverse := F^{-1}$$

Answer b)

$$smoker := \frac{coeffsinverse_{3,0} \cdot -S_1 + coeffsinverse_{3,2} \cdot -S_3}{C_4}$$

$$smoker = 14.222 \cdot \%$$

$$grill := \frac{coeffsinverse_{3,4} \cdot -S_5}{C_4}$$

$$grill = 62.649 \cdot \%$$

$$intake := \frac{coeffsinverse_{3,0} \cdot (-Q_a \cdot C_a) + coeffsinverse_{3,1} \cdot (-Q_b \cdot C_b)}{C_4}$$

$$intake = 23.129 \cdot \%$$

Verification:

Conservation of mass means that the total amount of carbon monoxide entering the building must equal the amount leaving. Because we are writting this equation independently of the previous derivations, these answers seem to be reasonable as shown below.

$$Q_a \cdot C_a + S_1 + Q_b \cdot C_b + S_3 + S_5 - Q_d \cdot C_5 - Q_c \cdot C_3 = 0 \frac{kg}{s}$$

The carbon monoxide contributions from each area are reasonable in that they add up to 100 %.

$$smoker + grill + intake = 100 \cdot \%$$

Figure 2. Concluded.

Problem 1 .

<u>Given</u> : Consider the water heater problem shown below. The thermostat setting for turning the heaters off is 130 F. Assume a water flowrate demand of 2gpm starting 100 minutes after turning on the heaters and the same thermostat settings for both water heaters.
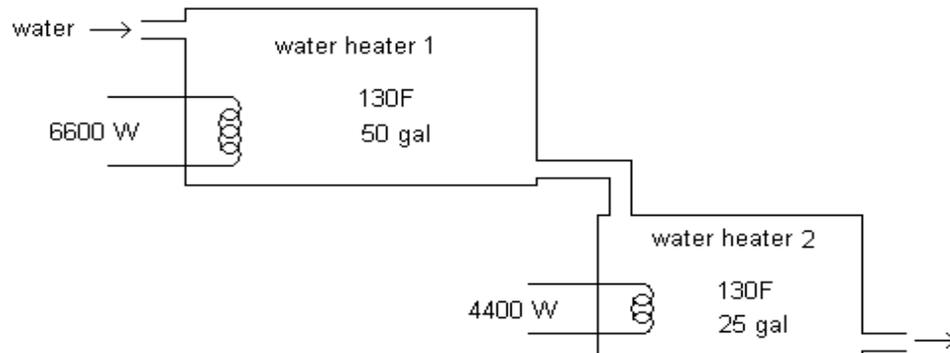
<u>Find</u> : plot the temperature of the water leaving each water heater until steady state temperatures are reached.

<u>Solution :</u>

Example of a household water heater :

$$\text{KfromF}(T) := (T - 32) \cdot \frac{5}{9} + 273 \qquad (\text{ Convertion function from degrees F to K })$$

$$\text{FfromK}(T) := (T - 273) \cdot \frac{9}{5} + 32 \qquad (\text{ Convertion function from degrees K to F })$$

$$kJ := 1000 \cdot J$$

$$C_p := 1 \cdot \frac{cal}{gm \cdot K} \qquad (\text{ Cp of water })$$

$$\rho := 1000 \, \frac{kg}{m^3} \qquad (\text{ Density of water })$$

$$V1 := 50 \cdot gal \qquad\qquad V2 := 25 gal \qquad (\text{ Volume of water tank 1 \& 2 })$$

$$T_{in} := \text{KfromF}(50) \cdot K \qquad\qquad T_{in} = 283 \, K \qquad (\text{ Inlet water temperature })$$

$$T_a := \text{KfromF}(65) \cdot K \qquad\qquad T_a = 291.333 \, K \qquad (\text{ Ambient temperature })$$

Figure 3. Worksheet for Example 3.

$$Q(t) := \begin{cases} 2 \cdot \dfrac{gal}{min} & \text{if } t > 100 min \\ 0 & \text{otherwise} \end{cases}$$

(Flowrate : Start the flow after 100 min)

$$Qgen1(T) := \begin{cases} 6600\,W & \text{if } T < KfromF(130) \cdot K \\ 0 & \text{otherwise} \end{cases}$$

(Heater ON if the temp. is below 130F)

$$Qgen2(T) := \begin{cases} 4400\,W & \text{if } T < KfromF(130) \cdot K \\ 0 & \text{otherwise} \end{cases}$$

Using an energy balance to solve for the time rate of change (Slopes) of the temperatures :

$$Slopes(t, T) := \begin{bmatrix} \dfrac{1}{\rho \cdot C_p \cdot V1} \cdot \left[\rho \cdot C_p \cdot (T_{in} - T_a) \cdot Q(t \cdot s) - \rho \cdot C_p \cdot (T_0 \cdot K - T_a) \cdot Q(t \cdot s) + Qgen1(T_0 \cdot K) \right] \cdot \dfrac{s}{K} \\ \dfrac{1}{\rho \cdot C_p \cdot V2} \cdot \left[\rho \cdot C_p \cdot (T_0 \cdot K - T_a) \cdot Q(t \cdot s) - \rho \cdot C_p \cdot (T_1 \cdot K - T_a) \cdot Q(t \cdot s) + Qgen2(T_1 \cdot K) \right] \cdot \dfrac{s}{K} \end{bmatrix}$$

$$Tinit := \begin{pmatrix} KfromF(50) \\ KfromF(50) \end{pmatrix} \cdot K \qquad tinit := 0 \cdot s \qquad tfin := 3600 \cdot 4 \cdot s \qquad N := 1000$$

$$Sol := rkfixed\left(\dfrac{Tinit}{K}, \dfrac{tinit}{s}, \dfrac{tfin}{s}, N, Slopes \right) \qquad T_1 := Sol^{\langle 1 \rangle} \qquad T_2 := Sol^{\langle 2 \rangle} \qquad t := Sol^{\langle 0 \rangle}$$

Answer :

Figure 3. Continued.

Figure 3. Concluded.

Note the use of the Given and Find (Mathcad) commands above in order to find the steady state solution of the differential equation.

Philosophy behind the System Dynamics course:

In the System Dynamics course students learn to model and obtain the transient and frequency response of mechanical, hydraulic, electric, and thermal systems and interpret the results based on the system parameters. They learn to pose and solve state variable equations using analytical methods as well as numerical solvers and how to combine the state variable equations into higher order differential equation. Verification of the homework solutions is based on the process learned in the Engineering Analysis course.

Example 4

Figure 4 illustrates a homework set in the System Dynamics course that makes extensive use of Mathcad symbolic commands to simplify the algebra and obtain analytical solutions for differential equations. Given the step response of a spring-damper system students are asked to

find the time constant of the system and use this information along with the steady state information to determine the spring and damping constants for the system.

Problem 3:

Given:

The unit step response of a first order system system is shown below (this is just a spring connected in parallel to a damper through a massless plate and a force pulling on the plate). The units for the y-axis and x-axis are mm and seconds, respectively.
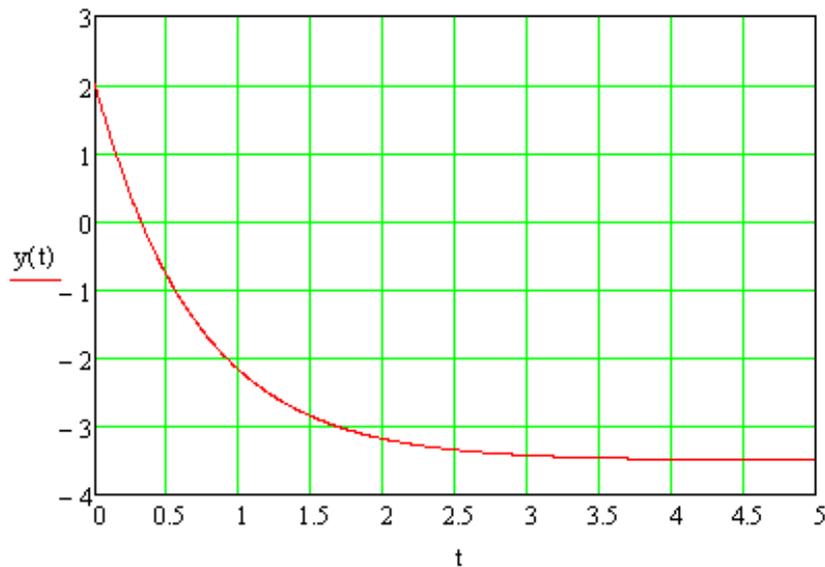
Find:

a) Write down the solution for the STEP response of the system in terms of the time constant, the initial value and the steady state value. Use only symbols, not numbers for this part.

b) Use the equation for part (a) and the value of time when y = -1mm to find the time constant. Repeat the same process when y= -3.3 mm and find the time constant. Explain which of the two choices of y is less sensitive to errors when reading the graph? Which of the two values of time constant is more reliable? Clearly explain your answer.

c) Take the derivative of the equation from part (a) and show how to find the time constant based on the initial slope and the steady state value.

d) Write down the differential equation of the system and use this equation along with the steady state information from the plot below to find the spring constant assuming that the step force is of magnitude 5.

e) Find the value of the damping b.

Figure 4. Worksheet for Example 4

Solution:

a) The differential equation of the a 1st order system subject to a step input is:

$$\tau \cdot \frac{dx(t)}{dt} + x(t) = x_{ss} \cdot \Phi(t)$$

The Laplace transform of this equation is:

$$\tau \cdot (s \cdot X(s) - x(0)) + X(s) = \frac{x_{ss}}{s}$$

rearranging,

$$\tau \cdot s^2 \cdot X(s) + s \cdot X(s) = \tau \cdot s \cdot x(0) + x_{ss}$$

Solving for X(s) and applying partial fraction expansion:

$$X(s) = \frac{\tau \cdot s \cdot x(0) + x_{ss}}{s(\tau \cdot s + 1)} = \frac{A}{s} + \frac{B}{\tau \cdot s + 1}$$

Solving for A and B:

$$A = \frac{\left(\tau \cdot s \cdot x(0) + x_{ss} \right)}{s \cdot (\tau \cdot s + 1)} \cdot s \quad \text{substitute} \ , s = 0 \ \rightarrow A = x_{ss}$$

$$B = \frac{\left(\tau \cdot s \cdot x(0) + x_{ss} \right)}{s \cdot (\tau \cdot s + 1)} \cdot (\tau \cdot s + 1) \ \text{substitute} \ , s = -\frac{1}{\tau} \ \rightarrow B = \tau \cdot x(0) - \tau \cdot x_{ss}$$

So $\quad X(s) = \dfrac{x_{ss}}{s} + \dfrac{\tau \left(x(0) - x_{ss} \right)}{\tau \cdot s + 1} = \dfrac{x_{ss}}{s} + \dfrac{x(0) - x_{ss}}{s + \dfrac{1}{\tau}}$

Applying the inverse Laplace Transform:
$$\boxed{x(t) = x_{ss} + \left(x(0) - x_{ss} \right) \cdot e^{\frac{-t}{\tau}}}$$

Figure 4. Continued.

b) Using the Mathcad Trace tool, we find t=0.555s for y= -1.011mm, x$_{ss}$= -3.5mm, and x(0)=2mm.

Substituting these values in the equation of part a) gives:

$$\tau_1 := -1.011 \cdot mm = -3.5 \cdot mm + (2 + 3.5) \cdot mm \cdot e^{\dfrac{-0.56 \cdot s}{\tau_1}} \begin{vmatrix} solve, \tau_1 \\ float, 3 \end{vmatrix} \rightarrow 0.706 \cdot s$$

$$\boxed{\tau_1 = 0.706\,s}$$

Using the Mathcad Trace tool, we find t=2.355s for y= -3.3098mm.

Putting these values in the equation of part a) gives:

$$\tau_2 := -3.3098 \cdot mm = -3.5 \cdot mm + (2 + 3.5) \cdot mm \cdot e^{\dfrac{-2.36\,s}{\tau_2}} \begin{vmatrix} solve, \tau_2 \\ float, 3 \end{vmatrix} \rightarrow 0.701 \cdot s \qquad \boxed{\tau_2 = 0.701\,s}$$

The first choice using the value of time when y=-1mm is the less sensitive to errors when reading the graph because around y= -1mm the slope of the curve is higher so it is easier to determine the corresponding time value. Thus it is the first value of the time constant who is the more reliable since it is calculated with the values of t and y that are the less sensitive to errors.

c) The derivative of the equation from part a) is:

$$\frac{dx(t)}{dt} = \frac{-\left(x(0) - x_{ss}\right)}{\tau} \cdot e^{\dfrac{-t}{\tau}}$$

substituting t=0:

$$\frac{dx(0)}{dt} = \frac{x_{ss} - x(0)}{\tau} \cdot e^{0} = \frac{x_{ss} - x(0)}{\tau}$$

solving fot τ

$$\boxed{\tau = \frac{x_{ss} - x(0)}{\dfrac{dx(0)}{dt}}}$$

Figure 4. Continued.

d) The differential equation of the system, assuming that the step force is magnitude 5N, is:

$$b \cdot \frac{dx(t)}{dt} + k \cdot x(t) = f(t) = -5N$$

At steady state $x = x_{ss}$ and the derivative term $dx(t)/dt$ is zero. This yields

$$x_{ss} := -3.5mm \qquad k \cdot x_{ss} = -5N \qquad \text{or} \qquad k := \frac{-5N}{x_{ss}} \qquad \boxed{k = 1.429 \frac{N}{mm}}$$

e) Since $\quad \tau = \dfrac{b}{k} \quad$ the damping coefficient b is: $\qquad b := \tau_1 \cdot k \qquad \boxed{b = 1.009 \dfrac{N}{\dfrac{mm}{s}}}$

Verification:

The plot below shows that the results from parts a) through e) are consistent because they reproduce the plot given.

$$x_0 := 2 \cdot mm \qquad \tau := \frac{b}{k}$$

$$x(t) := x_{ss} + \left(x_0 - x_{ss}\right) \cdot e^{\dfrac{-t \cdot s}{\tau_1}} \qquad x'(t) := \frac{d}{dt} x(t) \qquad x'(0) = -7.79 \times 10^{-3} \, m$$

Figure 4. Continued.

The blue straight line intersects the steady state line at a time equal to τ. This verifies the equation from part (c) which can be written as:

$$x_{ss} = \frac{dx(0)}{dt} \cdot \tau + x(0) \qquad \text{or} \qquad x(t) = x'(0) \cdot t + x_0 \qquad \text{at time} \qquad t = \tau$$
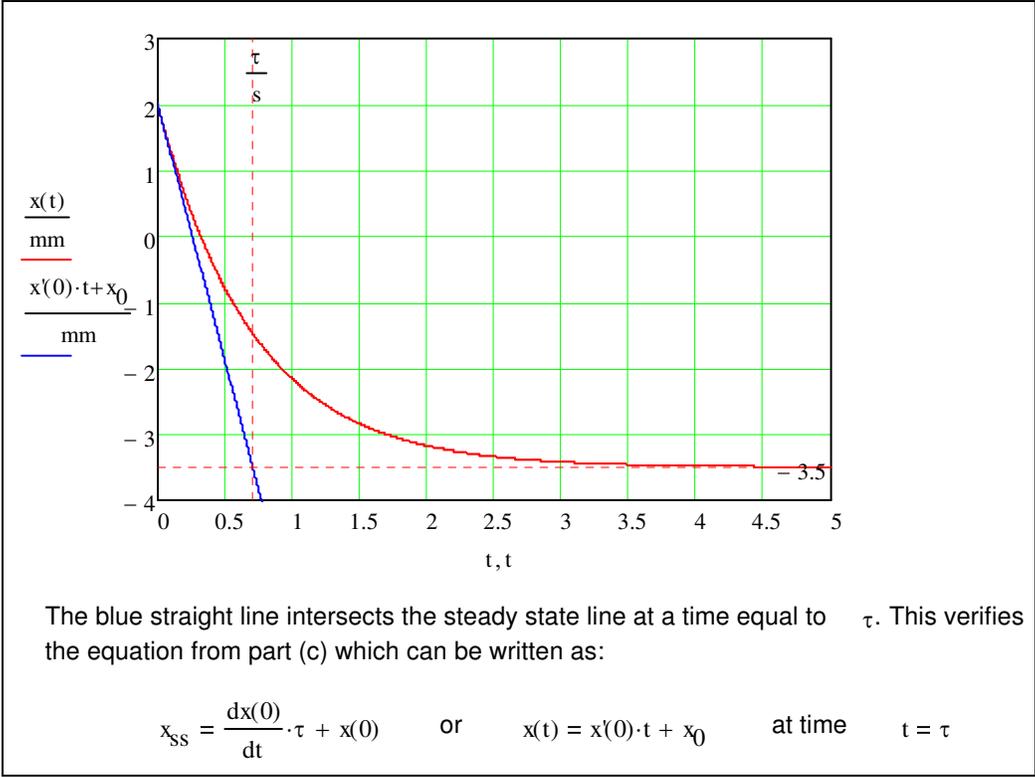
Figure 4. Concluded.

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Philosophy behind the Energy Systems Design course

In the Energy Systems Design course students learn to analyze and design energy systems components (series and parallel piping systems, piping networks, and heat exchangers), to select and confirm the appropriateness of pumps, and to model and understand the operation of energy systems. The evolution of the Energy Systems Design course was delineated in Hodge (1998). Hodge and Taylor (1999) is a textbook based on the materials covered in the course. Example 4 was taken from a typical homework assignment in this course.

Example 5

A system to pump oil between two reservoirs is illustrated in Figure 5. The pipe, which is 300 ft long, is made of commercial steel and is to handle 0.4 cfs. The oil has a density of 56.1lbm/ft^3 and a viscosity of 0.00576 lbm/ft-sec. The pump/motor efficiency is 67 percent. Electricity costs $0.05 per kWh, and the demand charge is $10.00 per kW per month. As a function of diameter, determine the power required to pump the oil from the lower reservoir to the upper reservoir. Select a diameter, and defend your diameter selection.

A schematic of the system is presented in Figure 5.
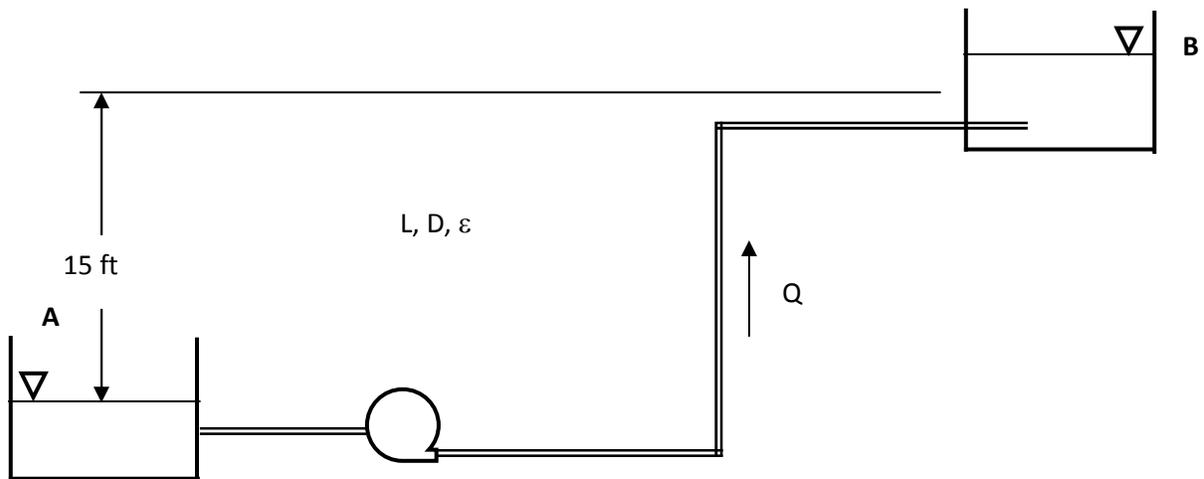
Figure 5. Oil Pumping System Schematic.

The energy equation for the system can be written as

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + f\frac{L}{D}\frac{V^2}{2g} + (K_{ent} + 2K_{elbow} + K_{exit})\frac{V^2}{2g} - W_s\frac{g_c}{g} \quad (1)$$

The minor losses are the entrance, the elbows, and the exit. Since A and B are located at free surfaces of reservoirs open to the atmosphere, $P_A = P_B$ and $V_A = V_B = 0$. The energy equation thus reduces to the form:

$$W_s = Z_B - Z_A + (f\frac{L}{D} + K_{ent} + 2K_{elbow} + K_{exit})\frac{V^2}{2g} \qquad (2)$$

The Mathcad worksheet containing the problem solution is included as Figure 6. The power input to the pump is required as a function of the pipe diameter. The pipe diameter will be varied, and the pump increase in head and the power required calculated for each diameter. Diameters from 1.5 to 9 inches, in increments of 0.5 inches, are prescribed by $D_i = (1 + 0.5 \cdot i)$, where i is the Mathcad range variable.

The calculation of the increase in head of the pump, W_s, required for a given pipe diameter is a Category I problem. The pump increase in head, W_s, cab be calculated directly for each pipe diameter. The power required by the pump is

$$PowerR = \frac{W_s Q \rho}{\eta} \qquad (3)$$

Graphical representations of variables of interest are provided in the worksheet. For the relatively small diameter range considered, the power required by the pump changes by two orders of magnitude (from 406 hp to 5.9 hp). The figures provide graphic evidence of the almost D^5 dependence that power required and pressure drop have on diameter when major losses are dominant in piping systems. An examination of the power required graph provides some insight into diameter selection. The power required has a diminishing returns relationship as the diameter is increased. At about D = 3 inches (the "knee" in the curve) further increases in diameter yield little decreases in either the pump power required or the pump increase in head. Thus, a cursory examination eliminates diameters near 1 inch (too much power) or near 6 inches (too large a pipe) and indicates a diameter of 3 inches to be a rational choice.

Economic metrics, such as the minimum present worth, are often used to determine the desired configuration of a system. The present worth depends on the system operating costs as well as the initial system costs. Since the hours per week of use is not provided, a two-shift operation (80 hours/week) is assumed. Additionally, the expected lifetime of the system is taken as 10 years and an interest rate of 7 percent is reasonable. The total system operating costs consist of the energy cost and the demand cost. The total system operating costs per year as a function of pipe diameter is depicted in the worksheet. For very large pipe diameters, the yearly operating costs asymptotically approach a small number ($250), depending on the pipe diameter.

The initial system costs are composed of the pipe purchase/installation cost and the pump cost. Pipe and pump costs are taken from Hodge and Taylor (1999). The total initial cost of the system is the sum of the pipe and pump costs. The initial costs of the system are dependent only on the pipe diameter and not on the hours per week of operation. The initial system cost as a function of pipe diameter is presented in the worksheet. The system initial costs are dependent mostly on the pipe diameter and length since the pump cost is very small compared to the pipe cost except at the smaller diameters.

With the operating costs per year and the total initial costs known, the present worth for a n-year operating period is

$$\mathrm{Pr}\,esentW = CostTot \cdot \frac{(1+\text{int})^n - 1}{\text{int} \cdot (1+\text{int})^n} + CostInit \qquad (4)$$

where int is the interest rate (7 percent) and n (10 years) is the number of years considered. The present worth for the system as functions of pipe diameter is shown in the worksheet. A minimum present worth exists since operating costs decrease as diameter increases and total initial costs increase as diameter increases. The minimum present worth corresponds to a pipe diameter of 3.5 inches. Hence, the cursory indication of about 3 inches from power required is amazingly accurate.

The recommended pipe diameter is 3 .5inches for this system. The results of both the simple analysis examining only the behavior of the increase in power (or pump increase in pressure) and the more complex present worth economic analysis are essentially the same.

Verification of the solution is provided by observing that the value of the increase in head of the pump, W_s, asymptotically approaches the elevation difference of 15 feet. This is appropriate as frictional losses decrease as D^5 and for large pipe diameters become very small. Consider the fluid velocity as a function of pipe diameter. At a diameter of 3 inches, the fluid velocity is about 8 ft/sec, and at a pipe diameter of 3.5 inches, the fluid velocity is near 6 ft/sec. A general rule of thumb in piping systems is that economic constraints dictate that the velocity should not exceed 10 ft/sec. This solution is very congruent with the rule of thumb thus providing additional verification for the solution.

ORIGIN ≡ 1   Set origin for counters to 1 from the default value of 0.

Input the pipe geometry:

Diameter in feet   Length in feet   Roughness in feet:

$i := 1 .. 16$ $L := 300 \cdot ft$ $\varepsilon := 0.00015 \, ft$

$D_i := (1 + 0.5 \cdot i) \cdot in$

Input the system boundary (initial and end) conditions:

Pressures in psi: Elevations in feet:

$$\begin{pmatrix} P_a \\ P_b \end{pmatrix} := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \frac{lbf}{in^2} \qquad \begin{pmatrix} Z_a \\ Z_b \end{pmatrix} := \begin{pmatrix} 0 \\ 15.0 \end{pmatrix} \cdot ft$$

Input the loss coefficients:

K factor   Equivalent length

$K := 1.5$ $C := 60$

Input the fluid properties:

Density in lbm/ft^3 Viscosity in lbm/ft-s

$$\rho := 56.1 \cdot \frac{lb}{ft^3} \qquad \mu := 0.00576 \frac{lb}{ft \cdot sec}$$

Input the flow rate in cfs:

$$Q := 0.4 \cdot \frac{ft^3}{sec}$$

Define constants and adjust units for consistency: $g := 32.174 \dfrac{ft}{sec^2}$

$$g_c := 32.174 \frac{ft \cdot lb}{lbf \cdot sec^2}$$

Define the functions for Reynolds number, fully-rough friction factor, and friction factor:

$$Re(q, D) := \frac{4 \cdot \rho \cdot q}{\pi \cdot D \cdot \mu} \qquad f(q, D, \varepsilon) := \begin{vmatrix} \dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{Re(q, D)} + \left(\dfrac{\varepsilon}{3.7 \cdot D} \right)^{1.11} \right] \right]^2} & \text{if } Re(q, D) > 2300 \\ \dfrac{64}{Re(q, D)} & \text{otherwise} \end{vmatrix}$$

$$f_T(D, \varepsilon) := \frac{0.3086}{\left[\log\left[\left(\dfrac{\varepsilon}{3.7 \cdot D} \right)^{1.11} \right] \right]^2}$$

The generalized energy equation is:

$$W_{s_i} := \frac{P_b - P_a}{\rho} + (Z_b - Z_a) \cdot \frac{g}{g_c} + \frac{8}{\pi^2} \cdot \frac{Q^2}{g_c \cdot (D_i)^4} \cdot \left(f(Q, D_i, \varepsilon) \cdot \frac{L}{D_i} + K + C \cdot f_T(D_i, \varepsilon) \right)$$

Power imparted to fluid: $Power := W_s \cdot Q \cdot \rho$ Pump mechanical efficiency. $\eta := 0.67$

Figure 6. Worksheet for Example 5.
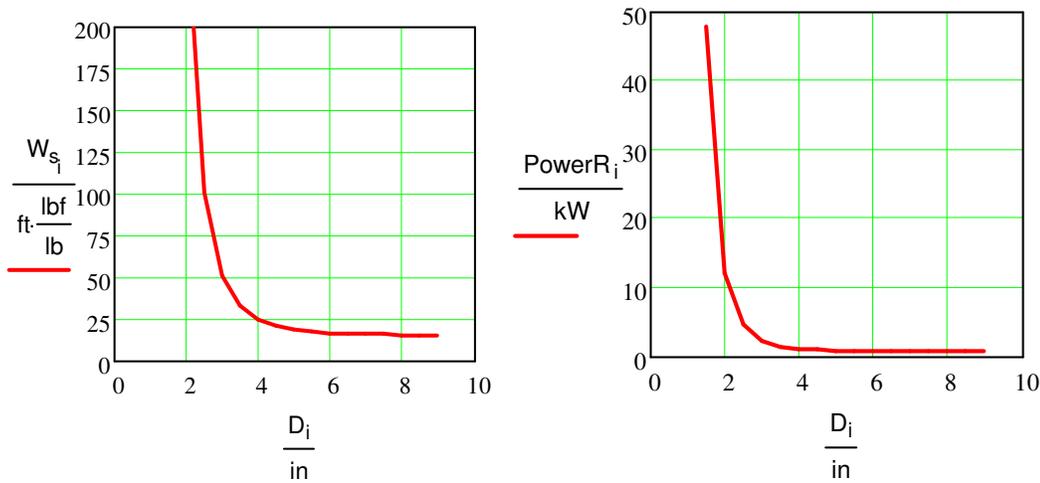
Consider the cost of running the pump for 80 hours per week (two shifts).

$$wk := 7 \cdot day \qquad month := 4.33 \cdot wk \qquad HW := 80 \cdot \frac{hr}{wk}$$

Energy Costs:

The total electricity cost is composed of an energy cost (kWh) and a demand cost (kW).

The energy cost is the product of the kWh used per year and the kWh rate:

$$CostE := PowerR \cdot HW \cdot 52 \cdot wk \cdot \frac{0.05}{kW \cdot hr}$$

The demand cost is the product of the demand (kW) per billing period times the number of billing periods in a year (usually 12) times the demand rate per billing period.

$$CostD := PowerR \cdot 12 \cdot month \cdot \frac{10.0}{kW \cdot month}$$

The total electricity cost is the sum.

$$CTot := CostE + CostD$$
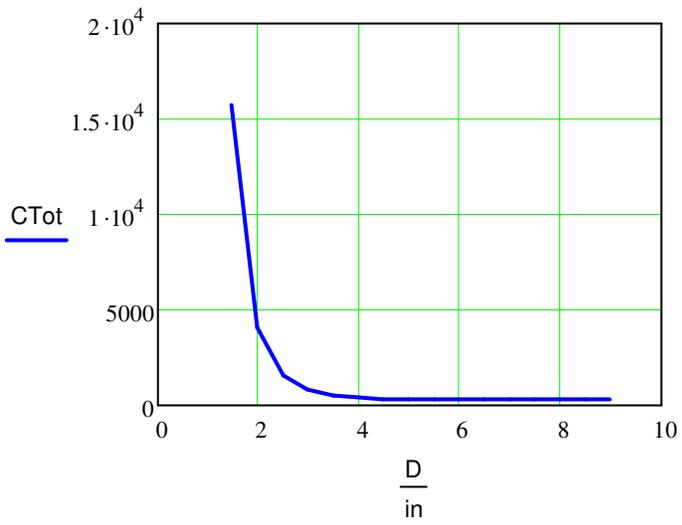
Figure 6. Continued.

Initial Costs:
The initial costs are composed of the pipe cost (purchase and installation) and the pump cost (purchase and installation).
Pipe pumpcosts (estimated using information in Hodge and Taylor, 1999).

$$CostP := \frac{L}{ft} \cdot \left[0.4 \cdot \left(\frac{D}{in} \right)^2 + 1.831 + 8.178 \cdot \frac{D}{in} \right] \qquad CostPump := 6000 \cdot \left(\frac{Power}{130 \cdot hp} \right)^{0.5}$$

The total cost is the sum of the pipe and pump costs.

$$CostInit := CostP + CostPump$$
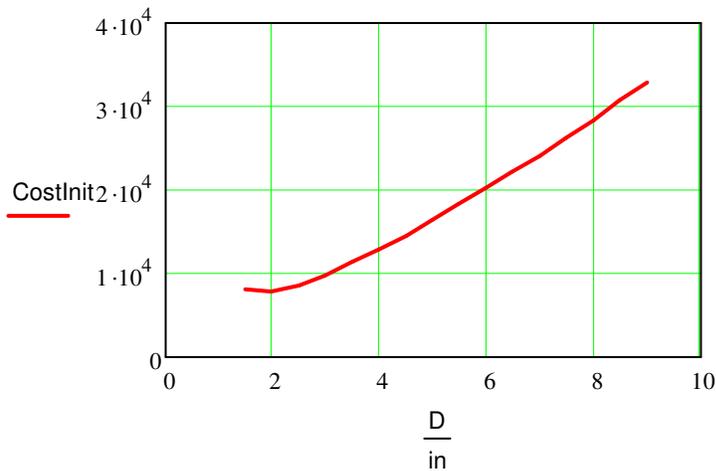
Consider the present worth for 10-years of operation at 7 percent.

$n := 10$ Number of years considered $int := 0.07$ Interest rate

$$PresW := CTot \cdot \frac{(1 + int)^n - 1}{int \cdot (1 + int)^n} + CostInit \quad \text{Present worth for n years at int interest rate}$$
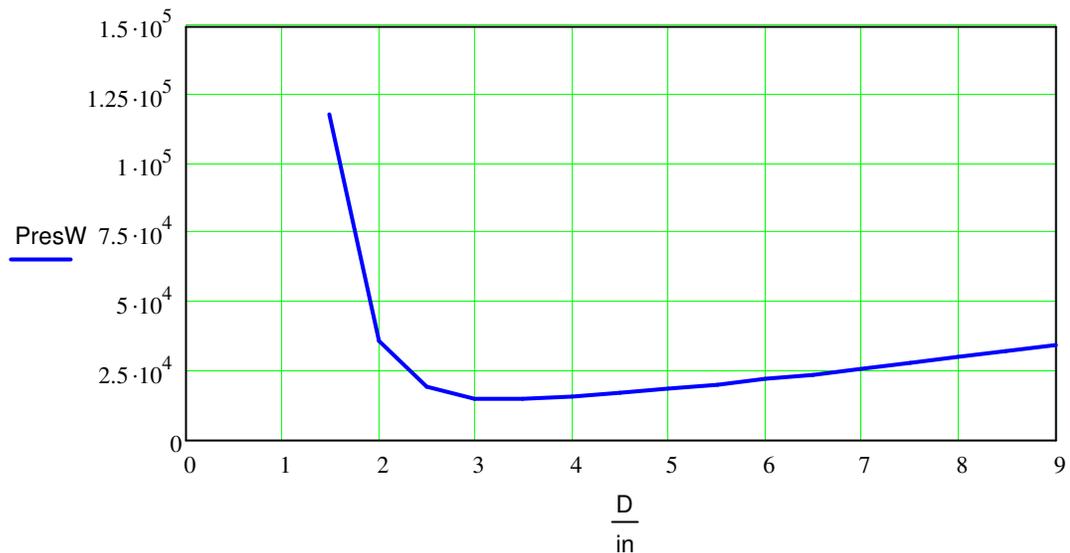
Figure 6. Continued.

A more useful representation of the present worth is achieved if the plot is confined to the region of interest.
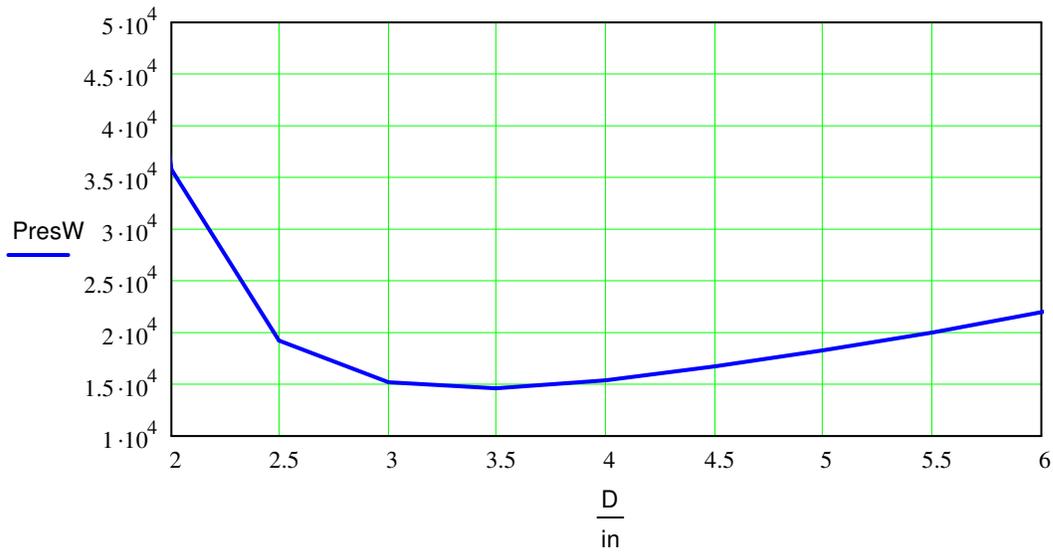
Fluid velocity calculation.

$$\text{Area} := 0.25 \cdot \pi \cdot D^2 \qquad V := \frac{Q}{\text{Area}}$$
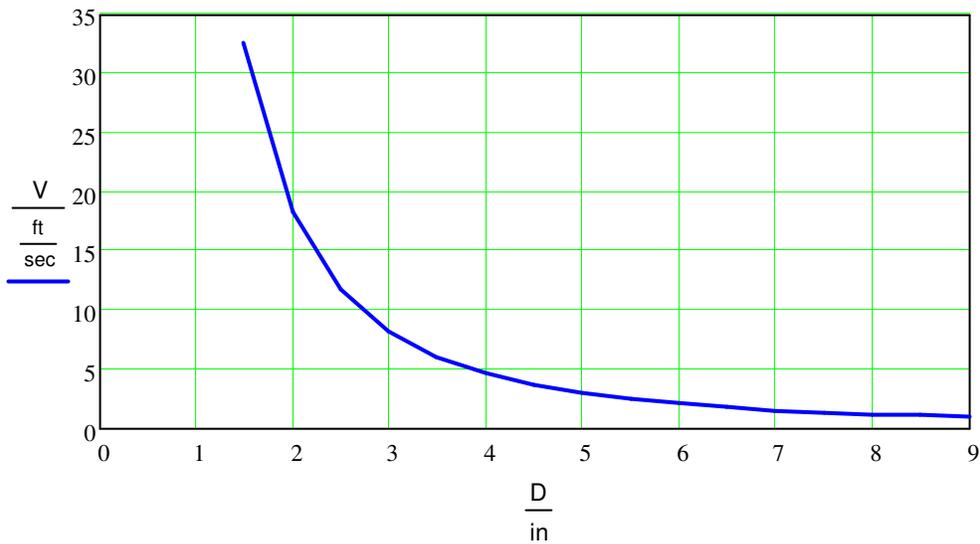

Figure 6. Concluded.

Assessment and Conclusions

The purpose of this paper is to discuss a process for inculcating a systematic and practical problem solving approach in engineering students. In all the examples explored in this paper, the same process is used. The treatments of all the example problems are identical and emphasize the three steps: (1) formulate a well-posed system of equations, (2) utilize user-friendly mathematical computer solvers to do the "arithmetic," and (3) verify the results. In this paper, the arithmetic has been accomplished by using Mathcad. Other computational software systems (Mathematics, Matlab,....) offer the same capability, albeit in different formats, but with the same results.

Anecdotally, students appreciate the attention to problem solution using the three-step unified approach. The use of Mathcad relieves the student from assimilating different numerical techniques ("procedures") to solve a system of equations. The net result is that more involved and more realistic problems can be assigned. With less time spent on arithmetic, more time is available for students to engage is higher-level synthesis and understanding.

Examples illustrating a unified approach to solutions of engineering problems in three requied courses in ME at MSU have been presented and discussed. The approach offers advantages in providing students with capability to solve more "real world" problems while concentrating on the engineering aspects of the problems.

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