AC 2011-2317: A MULTIDISCIPLINARY INVESTIGATION INTO VARIOUS POSSIBLE GEOMETRIES OF IMPERIAL ROMAN ARTILLERY: A CASE STUDY

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Introduction

Multidisciplinary projects provide unique opportunities to foster critical thinking in undergraduate engineering students and to help students develop an understanding of the research process. In addition, multidisciplinary projects which combine engineering analysis and a study of technological history are an interesting way to increase student interest in the engineering design process. In this paper, the authors will present a case study of one such interdisciplinary project, conducted by an undergraduate mechanical engineering student, in which the student investigated and compared three different geometric configurations of a Roman siege weapon, known as a ballista, which is rather like a large crossbow that uses torsion springs to shoot an arrow or a stone.

The Ballista

The ballista was a siege weapon of the Greek and Roman period which was made up of two vertical springs which were composed of frames containing highly tensioned cords made of sinew or hair. Figure 1 shows a 1/12 scale model of a 1 talent (58 lb stone) ballista built by one of the authors. The full size machine would top out at about 18 feet high.

Figure 1 The form of the ballista that is described by Heron and Philon.

In this configuration wooden arms were inserted perpendicularly through the cord bundles about half way up the spring cylinders which were made up of the cords. The two outside ends of the arms were attached to a bowstring which was drawn back by a windlass and held in place by a trigger device. Then the projectile, either an arrow or a stone sphere, was placed at the mid-point of the bowstring. When the trigger was released, the arms sprang outwards under the opposing torsional couples exerted by the springs. There is little dispute about the form of the Greek and early Roman ballistae since they are described by the ancient writers[1], illustrated in diagrams included in manuscripts[2], and included in a couple of extant sculptures[3].

Fairly new archeological evidence, from France[4], the region around the Danube[5], and the near east[6], suggests a previously unknown or unrecognized geometric configuration for the ballista of the 2nd -4th centuries A.D. This configuration, shown in Figure 2, has a frame which is wider and lower than had been described by the ancient sources. A full sized cheiroballistra, built by one of the authors, is shown in Figure 3.

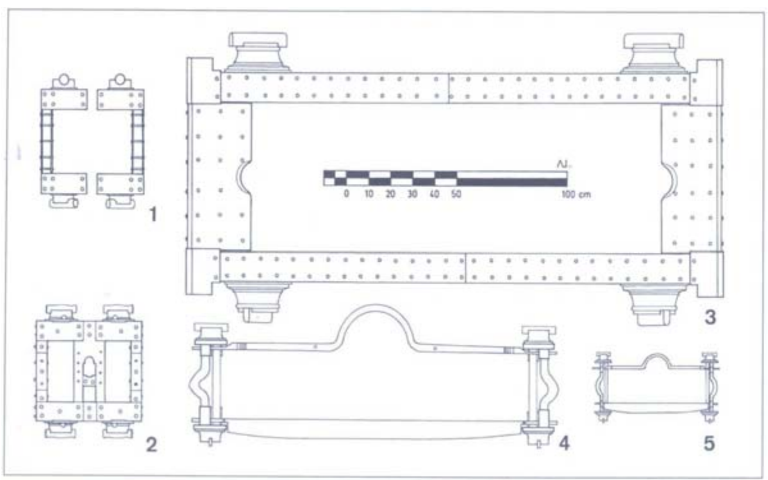

Figure 2 Torsion artillery frames viewed from the front:
1. Ampurius; 2. La Cardad; 3. Hatra; 4. Orşova: 5. Heron's cheiroballista.

Figure 3 The Cheiroballista; a hand held one span arrow projecting weapon.

Some of these findings, and the description given by Heron for his cheiroballistra, point to a change in operational geometry since, for this configuration, the arms point forward toward the target, swinging inward and not out the sides as had been previously used for the Greek and Roman ballista. These two distinctly different ballista configurations can be seen in Figure 4.

An advantage of this cheiroballistra-type configuration is that the arms could be drawn back through an angle of at least 90 degrees whereas with outward directed arms of the Greek/Roman ballista the arms cannot be drawn back through an angle of more than 45-55 degrees. However, a disadvantage for these outward swinging arms is that the angle of the bowstring becomes more parallel to the bow arm when attempting to go past a 55 degree angle causing the arm to be pulled out of the cord bundles which results in a decreased torque in the bow arm.

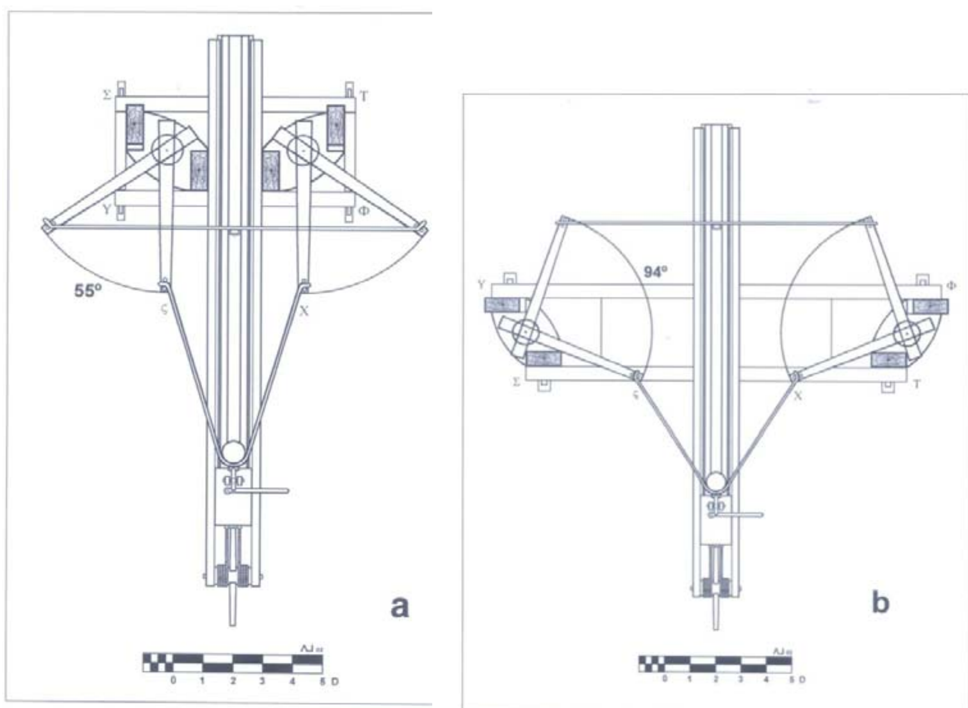

Figure 4 The two principal ballista configurations: the out swinging (a) and the in swinging forms (b).

Student Project Description

For this project, the student began by reviewing the historical background literature to gain an understanding of the technical and historical aspects of the project. With the help of the authors, the student then re-derived the mechanics equations (discussed later) that were used in both the analytical and experimental portions of the project. The configuration shown in Figures 1 and 4a, having the out-swinging arms, had been constructed previously. Therefore, the student designed, based on a reverse-engineering analysis of existing records, illustrations, and extant sculptures, and fabricated, using the existing torsion spring in the ballista shown in Figure 1, the ballista with the in-swinging arms as shown in Figure 4b. The student also designed and fabricated the intermediate form, with arm geometries between the out-swinging and the in-swinging arm configurations, which uses the wider spaced springs, but retains the outward swinging arms. The student then shot a projectile from each ballista configuration and determined the arm torque and projectile velocity. For each configuration, the student also obtained moment and angle data which he plotted. From this plot he obtained an equation for the moment as a function of angle. He then used the moment equation as an input into a computer program developed by one of the authors to calculate the average velocity of the projectile for each configuration. He then compared these calculated and experimental velocities for each configuration.

Shown in Figures 5, 6 and 7 are the three actual ballista configurations analyzed by the student for this project. The three designs use the same torsion springs but different positions and orientations. The orientation and position of the two arms necessitated different lengths of bowstring for each of the three cases. The student shot a projectile from each configuration and measured the arm torque and the projectile velocity.

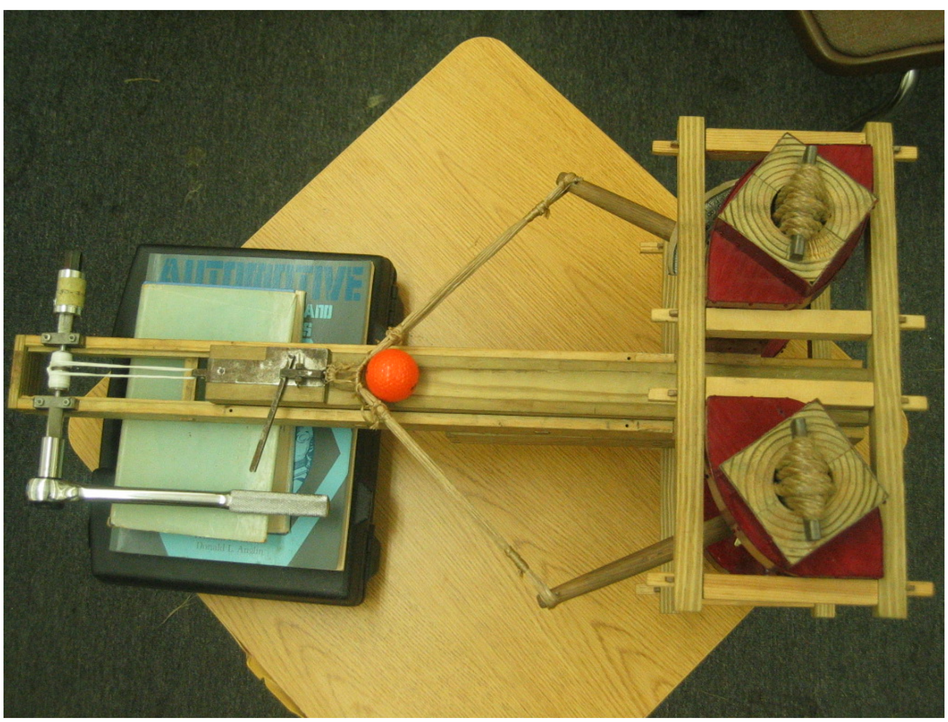

Figure 5 First Configuration (Old form)

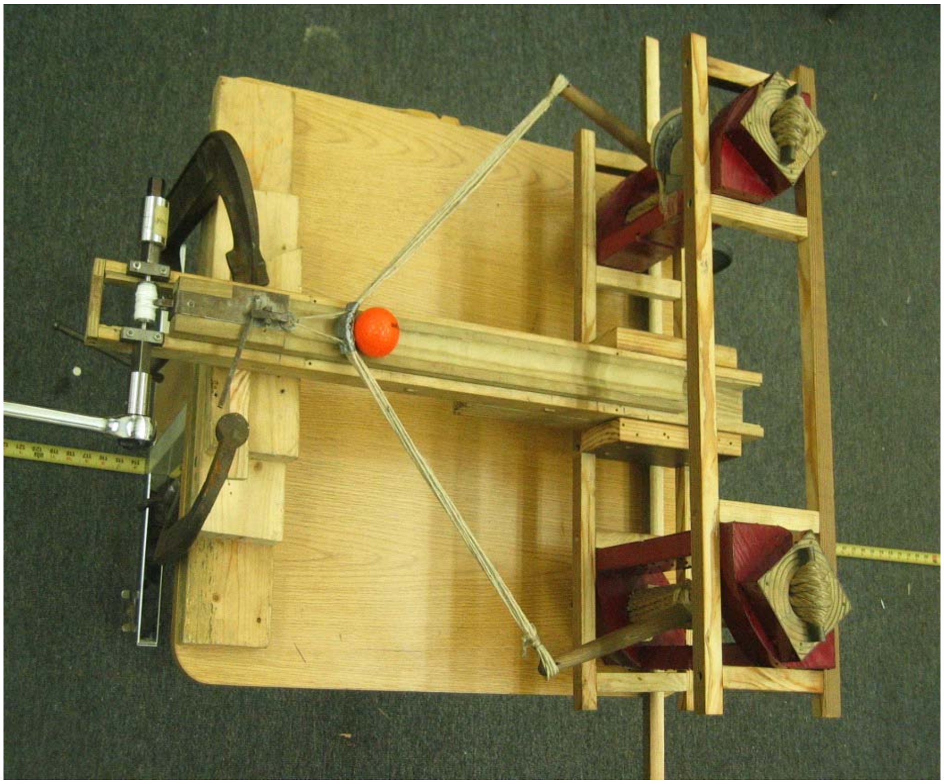

Figure 6   Second Configuration (intermediate form)

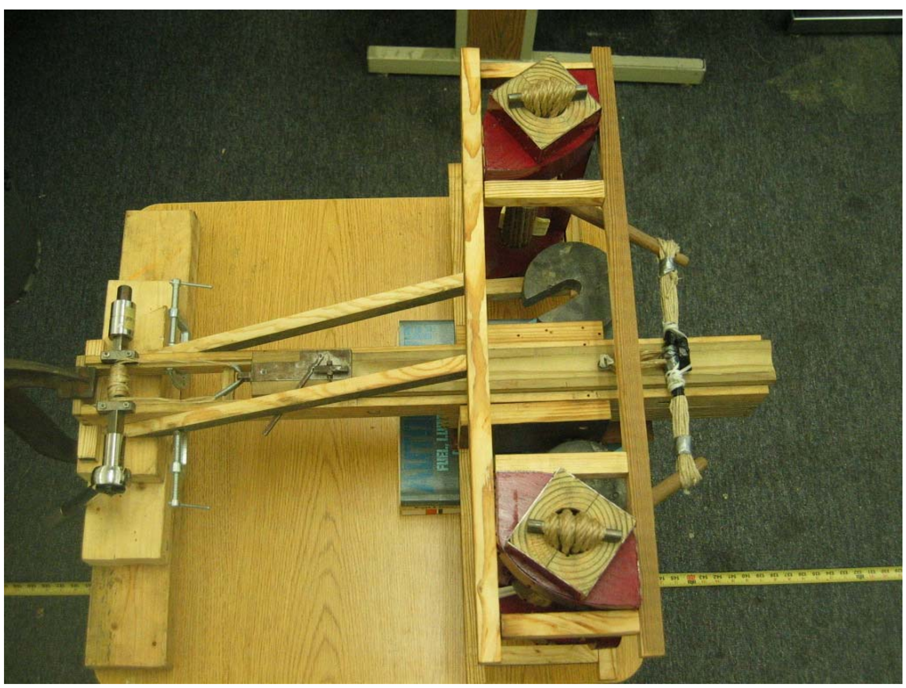

Figure 7   Third configuration (new form)

Equations of Motion

The general configuration used for the mathematical modeling of the ballista is shown in Figure 8. The following equations of motion used for the mathematical model were derived based on this figure and the assumption was made that the ballista was fired in the horizontal position with the horizontal velocity of the projectile being calculated. The subscript b_1 refers to the right hand side and b_2 the left.

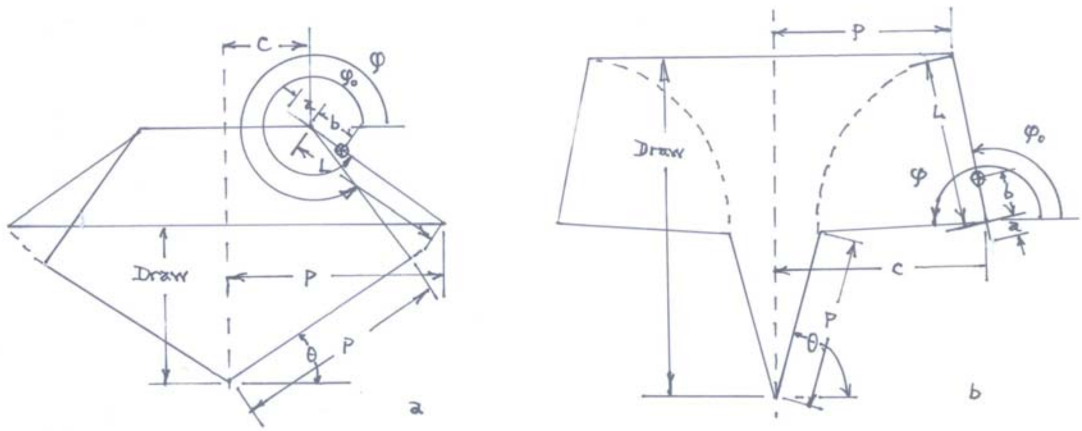

Figure 8 Geometry used in the analytic analysis (a) for the first and second configuration and (b) used for third configuration.

As can be seen in Figure 8, 2c is the distance between the torsion bundles, 2p is the length of the bow string, L is the length of the arm from the torsion bundle to the point of connection of the bow string, L + a is the total length of the arm which is through the torsion bundle, and b is the distance of the center of mass of the arm from the torsion bundle. The angle of the arm is φ with φ_o being the initial angle of the arm. The angle of the bowstring is θ.

Position of centers of mass of each of the arms:

$$x_{b1} = c + b\cos\varphi$$
$$x_{b2} = -x_{b1}$$
$$y_{b1} = y_{b2} = b\sin\varphi$$

Position of the center of mass of the projectile:

$$x_p = c + L\cos\varphi - p\cos\theta = 0$$
$$y_p = L\sin\varphi - p\sin\theta$$

Velocity of the center of mass of each of the arms:

$$\dot{x}_{b1} = -b\dot{\varphi}\sin\varphi$$
$$\dot{x}_{b2} = -\dot{x}_{b1}$$
$$\dot{y}_{b1} = \dot{y}_{b2} = b\dot{\varphi}\cos\varphi$$

Velocity of the center of mass of the projectile:

$$\dot{x}_p = -L\dot{\varphi}\sin\varphi + p\dot{\theta}\sin\theta = 0$$
$$\dot{y}_p = L\dot{\varphi}\cos\varphi - p\dot{\theta}\cos\theta$$

Acceleration of the center of mass of each of the arms:

$$\ddot{x}_{b1} = -b\dot{\varphi}^2\cos\varphi - b\ddot{\varphi}\sin\varphi$$
$$\ddot{x}_{b2} = -\ddot{x}_{b2}$$
$$\ddot{y}_{b1} = \ddot{y}_{b2} = -b\dot{\varphi}^2\sin\varphi + b\ddot{\varphi}\cos\varphi$$

Acceleration of the center of mass of the projectile:

$$\ddot{x}_p = -L\sin\varphi\ddot{\varphi} - L\cos\varphi\dot{\varphi}^2 + p\sin\theta\ddot{\theta} + p\cos\theta\dot{\theta}^2 = 0$$
$$\ddot{y}_p = L\cos\varphi\ddot{\varphi} - L\sin\varphi\dot{\varphi}^2 - p\cos\theta\ddot{\theta} + p\sin\theta\dot{\theta}^2$$

Because of symmetry only the right side of the ballista need be considered:

$$\sum M = M - bA_y\cos\varphi + b\sin\varphi + (L-b)T\sin(\varphi-\theta) = I\ddot{\varphi}$$

$$\sum F_x = A_x - T\cos\theta = m_b\ddot{x}_b$$

$$\sum F_y = A_y - T\sin\theta = m_b\ddot{y}_b$$

Here M is the moment due to the highly tensioned cord bundle and is a function of φ, T is the tension in the bow string, A_x and A_y are the forces of the arm on the cord bundle, and I is the mass moment of inertia of the arm about the torsion bundle. Since the tension in the bowstring is assumed the same on both sides of the projectile then $\sum F_x = 0$.

For the static case $\sum F_y = 2T\sin\theta - P = 0$, which yields $P = 2T\sin\theta$. Summing the moments about the center of the torque bundle gives $T = \dfrac{M}{L\sin(\theta-\varphi)}$, and, therefore, the force P to pull back both arms is $P = \frac{2M\sin\theta}{L\sin(\theta-\varphi)}$.

For the dynamic case P goes to zero giving $\sum F_y = 2T\sin\theta - D_y = m_p\ddot{y}_p$, where $D_y = \mu m_p g$ for the friction of the projectile on the runway and μ *is* the coefficient of kinetic friction. This changes the torque to $T = \frac{m_p(\ddot{y}_p + \mu g)}{2\sin\theta}$. Substituting the force equations and the kinematic equations yields the following equation

$$\ddot{\varphi}(-I_{\oplus} - m_b b^2 - m_p L^2\cos\varphi\sin(\theta-\varphi)) + \ddot{\theta}\left(\frac{m_p pL\cos\theta\sin(\theta-\varphi)}{2\sin\theta}\right) +$$
$$\dot{\varphi}^2\left(\frac{m_p L^2\sin\varphi\sin(\theta-\varphi)}{2\sin\theta}\right) + \dot{\theta}^2\left(\frac{-m_p pL\sin(\theta-\varphi)}{2}\right) + M - \frac{D_y L\sin(\theta-\varphi)}{2\sin\theta} = 0.$$

From the kinematic acceleration of the center of mass of the projectile ($\ddot{x}_p$) a second equation was derived

$$\ddot{\varphi}(-L\sin\varphi) + \ddot{\theta}(p\sin\theta) + \dot{\varphi}^2(-L\cos\varphi) + \dot{\theta}^2(p\cos\theta) = 0.$$

These two equations can be solved numerically to yield all of the positions, velocities, accelerations, forces and moments.

Project Procedures and Results

First configuration (Old form)

This original configuration has historical evidence. Therefore this configuration was built according to the historical directions given by Heron and Philon[7]. The distance between the two torsion springs is 7.25 inches, with the arms projecting outward.

A force (*P*) was applied to the bowstring to draw it back, and the initial angle (φ_0) of the right arm was measured. This was done several times and the angle (θ) of the bowstring with respect to the main axis was measured each time. The force *P* was measured directly using a force meter several times and averaged. Then the data was applied to the moment equation

$$M = \frac{PL\sin(\theta-\varphi)}{2\sin\theta}$$

where M (ft-lb) is the moment applied by the torsion springs, P (lb) is the force applied to the bowstring, θ is the angle of bowstring with respect to main axis, L is the length of the arm, which is 0.54167 ft, and φ is 360^o-$\varphi_{0.}$

The student then created a "Moment vs. φ" graph, shown in Figure 9, and the following equation was computer fitted (solid line) to the averaged data (line with data points) which came from indirect measurements

$$M(\varphi) = -0.001\varphi^3 + 0.9739\varphi^2 - 309.37\varphi + 32799(ft - lb)$$

where φ is in degrees.

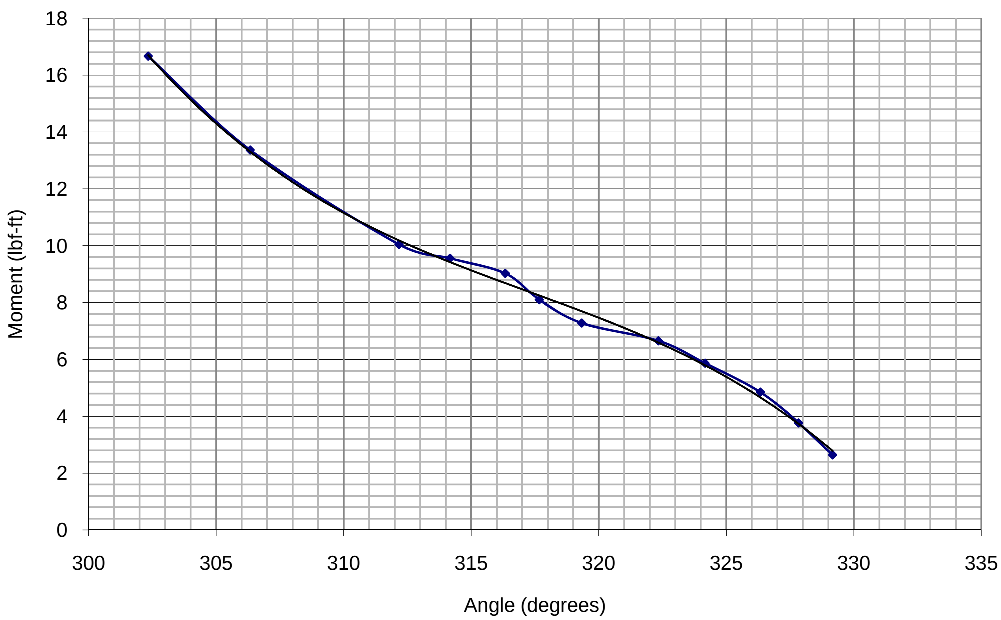

Figure. 9 Moment of a torsion bundle vs. Angle of twist for the first configuration

The fitted equation, derived from the experiment data using EXCEL, was then entered into a computer program, developed by the one of the authors[8], to calculate the averaged velocity. From the computer model, the averaged velocity of projection was calculated to be 60.8 feet/s. The experimental velocity was determined by the student to be 58.8 feet/s.

Second configuration (Intermediate)

The student took apart the first configuration and reconfigured the two torsion springs so that they were spaced further apart. The arms were still projecting out to the side, but the torsion springs were separated a distance 16.5 inches. This configuration was compared by the student to the first configuration to see if a longer bowstring would produce more energy and therefore a higher projectile velocity and a greater range.

The student then created a "Moment vs. φ" graph, shown in Figure 10, and the following equation was computer fitted (solid line) to the averaged data (line with data points) which came from indirect measurements

$$M(\varphi) = 0.008\varphi^3 - 0.7311\varphi^2 + 228.25\varphi - 23678 \text{ (ft-lb)}$$

where φ is in degrees.

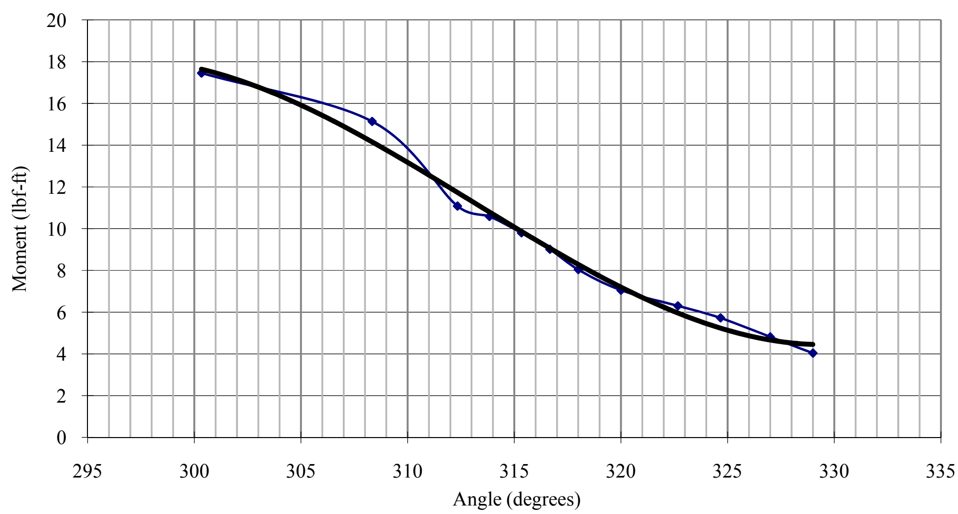

Figure 10 Moment of a torsion bundle vs. Angle for the second configuration

From the computer model, the averaged velocity of projection was calculated to be 81.7 feet/s. The experimental velocity was determined by the student to be 74.8 feet/s.

The student found that this intermediate form did provide more energy than the first configuration, which means this configuration could shoot a projectile slightly further than the first configuration.

Third configuration (New form)

The written record says very little about this configuration. This configuration is principally suggested from the archeological finds[9], and controversial interpretations of some sculptures[10]. Based on reverse-engineering, the student designed and fabricated this configuration so that the torsion springs were separated apart as in the second configuration, but the arms were facing forward this time.

The student then created a "Moment vs. φ" graph, shown in Figure 11, and the following equation was computer fitted (solid line) to the averaged data (line with data points) which came from indirect measurements

$$M(\varphi) = -0.0001\varphi^3 + 0.0432\varphi^2 - 5.8297\varphi + 258.23 \text{ (ft-lb)}$$

where φ is in degrees.

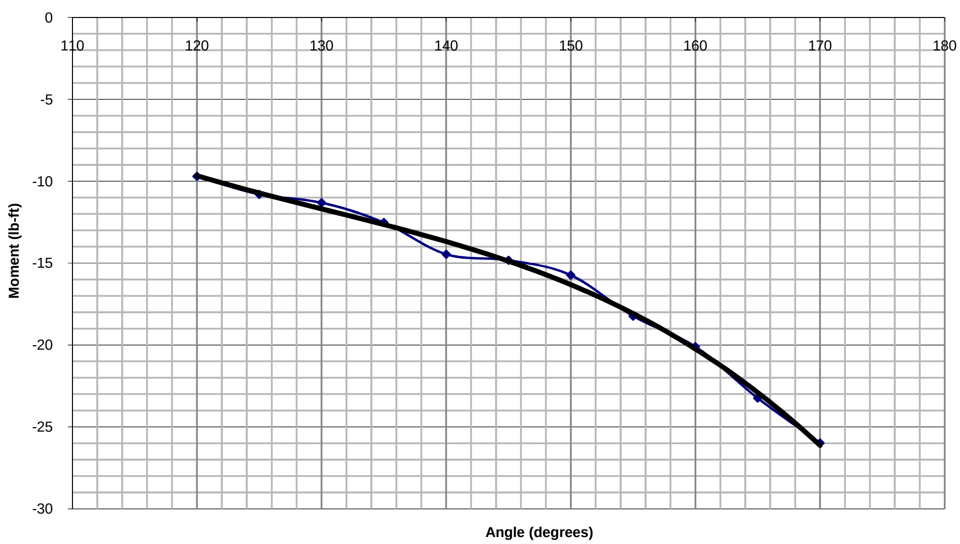

Figure 11 Moment of the torsion bundle vs. Angle for the third configuration

From the computer model, the averaged velocity of projection was calculated to be 125.2 feet/s. The experimental velocity was determined by the student to be 130.2 feet/s.

From the "Moment vs. Angle" graphs (Figures 9, 10, and 11), the arms of the new configuration can be pulled back 50 degrees, 29.7 degrees for the intermediate form, and only 27 degrees for the original configuration. Also the maximum moment for the new configuration is 26 ft-lb, 17.46 ft-lb for the second configuration and 16.7 ft-lb for the first configuration.

The three ballistae were shot level from a horizontal position in order to measure the drop from horizontal. By knowing the distance of the drop, the velocity could be calculated by

$$V = R\sqrt{\frac{g}{2D}}$$

R (ft) is the range, g (32.2 ft/s^2) is the gravity in English units, D (ft) is the drop, and V (ft/s) is the velocity of the projectile.

Tables 1, 2 and 3 below show the experimental and analytically-based computer results the student obtained for each of the three ballista configurations.

Table 1 Results for the first configuration

		Maximum Pull (lb)	Velocity (ft/s)
Old	Measured	40	58.8
	Computer	41.59	60.8

Table 2 Results for the second configuration

		Maximum Pull (lb)	Velocity (ft/s)
Intermediate	Measured	40	74.8
	Computer	35.95	81.7

Table 3 Results for the third configuration

		Maximum Pull (lb)	Velocity (ft/s)
New	Measured	88.53	130.2
	Computer	85.17	125.2

Comparison of the results for the first configuration and the second configuration shows that the wider of the torsion springs provide slightly more energy. Comparison of the results of the first configuration and the third configuration shows that the different orientation of the arms affects the shooting performance. In this case, with the arms projecting, inward more energy is provided than with the arms projecting outward.

Conclusion

In this case study, three different historical configurations of ballistae were tested, analyzed, and compared by the student. Two of these configurations were designed and fabricated by the student. After the student analyzed each of these three configurations, he conducted tests using scale-model ballistae, to ascertain which configuration had the greatest initial projectile velocity. He then compared these experimental test results to those of an analytic model. Through his work with this multidisciplinary project, the student gained critical thinking skills since much of the actual design of these ballistae had to be reverse-engineered based on his analysis of the functionality due to the lack of detailed historical accounts. The student also gained a better understanding of the research process by having to identify and use various analytical tools and to develop appropriate tests. However, perhaps as importantly, through this multidisciplinary project which combined historical analysis, engineering mechanics and design, and experimental design and testing, the student gained a greater interest in engineering design and its applications.

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