A Pathway Towards STEM Integration: Embodiment, Mathematization, and Mechanistic Reasoning

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Dr. Paul J. Weinberg is an Associate Professor of Mathematics and STEM Education at Oakland University (Rochester, MI), where he teaches methods courses for pre- and in-service secondary mathematics teachers. In addition, he teaches mathematics content courses, in the Department of Mathematics and Statistics, for elementary education majors. Dr. Weinberg's research focuses on students' reasoning within STEM disciplines, in the context of schooling; this focus has principally been in the field of engineering. He is interested in ways of characterizing and developing disciplinary practices (e.g., mechanistic reasoning) in K-12 classrooms in order to promote and support disciplined inquiry. He has published his research in the Journal of Pre-College Engineering Education Research (J-PEER), Cognition and Instruction, and ZDM: The International Journal on Mathematics Education. In addition, Dr. Weinberg has coauthored a book, The First-Year Urban High School Teacher, focusing on the challenges of supporting teaching and learning in the nation's highest poverty schools and districts. He has recently begun a research study that will supports mechanistic reasoning through mathematical description in a 3rd grade after-school engineering program. Dr. Weinberg received a doctorate, with a focus on Mathematics and Science Education, from Peabody's College of Education and Human Development at Vanderbilt University in 2012.

Abstract

A challenge facing STEM education is integration, advancing student conceptual development and disciplinary practices within and across STEM domains. The study described here was supported through a focus on mechanistic reasoning about systems of levers. The literature suggests that children (and adults) have difficulties recognizing an output link's rotation and causally ascribing that rotation to the constraint of the fixed pivot. To foster mechanistic reasoning, fifteen third-grade students in an urban elementary school were engaged in the design of kinetic toys composed of systems of levers within an after-school STEM program. To make the operations of these systems salient, students participated in an embodied activity highlighting properties of these mechanisms. Students re-described and inscribed these embodied experiences in order to support reasoning about mathematical (the geometry of circles) and physical systems (systems of levers). The coding of student talk and gesture during the first two days of this engineering program as well as pre- and post-assessments showed students made gains on measures of mechanistic reasoning, mathematical reasoning, and engineering practices.

STEM Integration

A challenge facing STEM education is integration, advancing student conceptual development within and across Science, Technology, Engineering, and Mathematics (STEM) domains (National Research Council [NRC], 2007). The National Academy of Engineering (NAE) (Honey, Pearson, & Schweingruber, 2014) has argued that integrated STEM education should bring together concepts from more than one discipline (e.g., mathematics and science; science, technology, and engineering); it may connect a concept from one domain to a practice of another, such as applying properties of geometric shapes (mathematics) to engineering design (Weinberg, 2017a; 2017b; 2019); or it may combine two practices, such as scientific inquiry (which includes modeling, argument, etc.) and engineering design (in which blueprints can be developed).

The NAE (2014) has also argued that STEM education is a potential vehicle for concept development in mathematics through the mathematical description of natural and designed systems in science and engineering. However, in spite of the potential for mathematical learning through STEM integration, studies have shown that most STEM curricula (designed to target mathematics content) do not reach this potential (Tran & Nathan, 2010; Prevost, Nathan, Stein, Tran, & Phelps, 2009). These studies indicate that the vast majority of such curricula address few mathematics content standards (e.g., NCTM, 2000) and those standards that are addressed are done so in a shallow, disconnected manner. Moreover, such treatment of mathematics ignores the fundamental role of mathematical description in the other STEM fields (i.e., other than mathematics).

Mechanistic Reasoning

Reasoning about mechanism is foundational to disciplined inquiry in science and engineering; thus, it should be one of the foundations of a Science, Technology, Engineering, and Mathematics (STEM) education (Bolger, Kobiela, Weinberg, & Lehrer, 2012; National Research Council, 2011; Russ, Scherr, Hammer, & Mikeska, 2008; Weinberg, 2017a; 2017b; 2019). Individuals who engage in mechanistic explanations focus on the processes that undergird causal relationships and consider how the elements (and the relations between those elements) of system components affect one another. These explanations may describe natural (e.g., biological) or designed (e.g., mechanical) systems. Machamer, Darden, and Carver (2000) note that "[c]omplete descriptions of mechanisms exhibit productive continuity without gaps from the set up to terminal conditions" (p. 3). They provide the following example of a mechanistic explanation of the workings of neurotransmitters: "the mechanism of chemical neurotransmission, a presynaptic neuron transmits a signal to a post-synaptic neuron by releasing neurotransmitter molecules that diffuse across the synaptic cleft, bind to receptors, and so depolarize the post-synaptic cell." (p. 3)

Supporting students to engage in mechanistic reasoning requires opportunities for them to consider components, structure, and mechanisms of many and diverse systems. The National Research Council (NRC) (2009) indicated the import of forging a tight connection between engineering principles, disciplinary knowledge, and disciplinary practices (e.g., mechanistic reasoning). Moreover, they stated that K–12 engineering education should be supported by the engineering design process. The NRC indicated that the design process should be "open to the idea that a problem may have many possible solutions … [and provide] a meaningful context for learning scientific, mathematical, and technological concepts." (p. 4) In addition, the engineering design process should further support the development of mechanistic reasoning within the

learning of scientific phenomena. Accordingly, the Next Generation Science Standards Engineering Concepts and Practices (NGSS Lead States, 2013) include mechanistic reasoning as one of their core competencies. This research engages students in authentic STEM integration, where mechanistic reasoning can support and be supported by mathematics.

The Difficulty of Supporting Mechanistic Reasoning

The literature suggests that mechanistic reasoning about even simple systems is not trivial for many children (or adults) (Lehrer & Schauble, 1998; Bolger et al., 2012; Weinberg, 2017a; 2017b; 2019). In Bolger et al.'s study, children predicted and explained the motion of pegboard linkages (Figure 1). Lehrer and Schauble interviewed second- and fifth-grade students, within engineering tasks, to assess their reasoning about the mechanics of gears. In both of these studies, the majority of participants did not engage in mechanistic explanations.

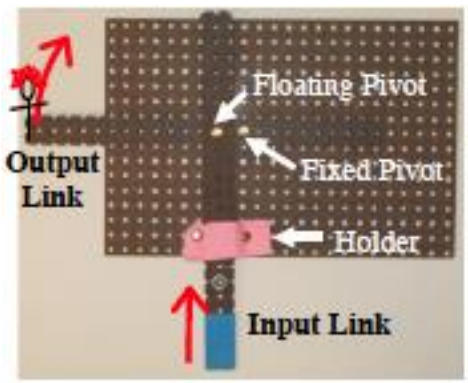


Figure 1. Example of a system of pegboard linkages.

In Weinberg (2017a; 2017b; 2019), participants predicted and explained the motion of pegboard linkages represented on an assessment. Most children's mechanistic reasoning was fragmented, displaying few of the mechanistic elements necessary to describe lever motion. First, most did not seem to see the rotary motion of linkages. For example, children's gestures often indicated straight paths or impossible motions that were consistent with the shape of the systems. Talk or gesture to indicate rotation of links was not common in most children's

explanations, even after children moved a machine. Second, most children rarely if ever suggested that fixing a link to the pegboard would constrain its motion. Bolger et al. (2012) noted that out of 72 total pegboard machines explained, there were only 6 instances in which a child recognized both constraint of the fixed pivot and rotation, indicating the difficulty of coordinating these two ideas.

Weinberg (2017a; 2017b; 2019) developed the AMRP assessment with Item Response Theory (IRT) modeling. He showed that there were mean differences in difficulty between participants who did not have the propensity to diagnose the rotation of a system of levers (M = - 0.67) and those who did (M = - 0.26 logits, $p < 0.1$, one-tailed t-test). In addition, there was also a difference between those who had the propensity to diagnose rotation (M = - 0. 26) and those who also had the propensity to diagnose the relationship between the constraint caused by the fixed pivot and subsequent lever motion (M = 0.52 logits; $p < 0.05$, one-tailed t-test). Moreover, even if participants showed the propensity to notice rotation and this fixed pivot constraint, it was significantly more challlenging to causally connect these and other mechanistic elements (M = 1.80 logits; $p < 0.0001$, one-tailed t-test). Accordingly, the present study was designed to support students to recognize rotation, constraint via the fixed pivot as well as causally connect these mechanistic elements.

Mechanistic Reasoning and Mathematical Description

Kobiela, Bolger, Weinberg, Rouse, and Lehrer (2011), Bolger et al. (2012), and Weinberg (2017a) have shown that when students engage in the mathematical description of systems of levers (re-describing these systems with mathematical analogues) it makes their mechanisms more salient. Kobiela and colleagues and Weinberg characterized the relationship between mathematical description and mechanistic reasoning about systems of levers within engineering design. These

studies investigated how mathematical description mediates mechanistic reasoning as well as the tracing of these systems from input to output. Bolger and colleagues (2012) showed that by mathematically describing systems of levers, participants supported both mathematical and mechanistic reasoning. In Weinberg (2012), when working with systems of levers, 76% of those participants who could causally trace all mechanistic elements through systems of levers, from input to output, on at least one item made a reference to the mathematics of circles. In addition, those participants who spontaneously referenced mathematics to explain machine motion, on at least one item, had higher mechanistic reasoning ability scores on the assessment than those who did not reference mathematics in their explanations.

Theoretical Framework

In order to support the growth of mechanistic reasoning (focusing on rotation and constraint via the fixed pivot), we engaged elementary students in the design and construction of kinetic toys. The embodiment in this study was developed in order to support student reasoning about elements of mechanistic reasoning that have proven to be difficult in previous studies (e.g., Bolger et al., 2012; Weinberg, 2017a; 2017b; 2019). In these studies, students had had difficulty with seeing rotary paths and reasoning about fixed pivot constraints (Figure 2).

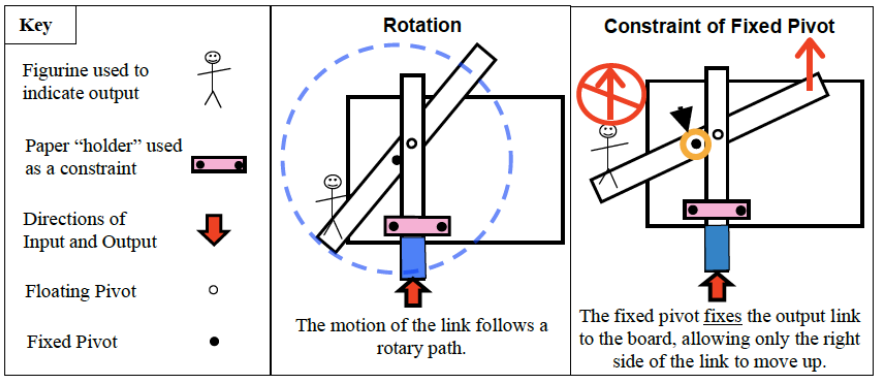


Figure 2. Example of rotation and fixed pivot constraint.

Our approach to learning by design included efforts to embody (Lakoff & Nunez, 2001; Barsalou, 2008) and mathematize (Freudenthal, 1973; Kline, 1980) a system of levers (Figure 1), highlighting how a fulcrum disrupts the straight path of a lever. Abrahamson and Sánchez-Gárcia (2016) described a theory of action-based mathematics learning that informed the instruction in this study. The approach was anchored in a body-syntonic (Papert, 1980) wherein initial mathematical description emerged from bodily activity, which was then re-expressed with the operation of the physical system of levers.

This study's embodiment was called the Rope Walk (Figure 3). In the Rope Walk, one student held one end of the rope and remained in place (the "Holder") while the other held the other end of the rope and attempted to walk in a straight path perpendicular to the rope's orientation (the "Walker"). Due to the constraint of the rope, the Walker's attempted straight path was disrupted. Instead, the constraint produced a circular (i.e., rotary) path, maintaining the Walker at a constant distance from the Holder as the Walker continued to walk at about the same rate. The task was performed twice with different lengths of rope: The Long Hold and the Short Hold; then the Holder and the Walker switched jobs (Figures 3a and 3b).

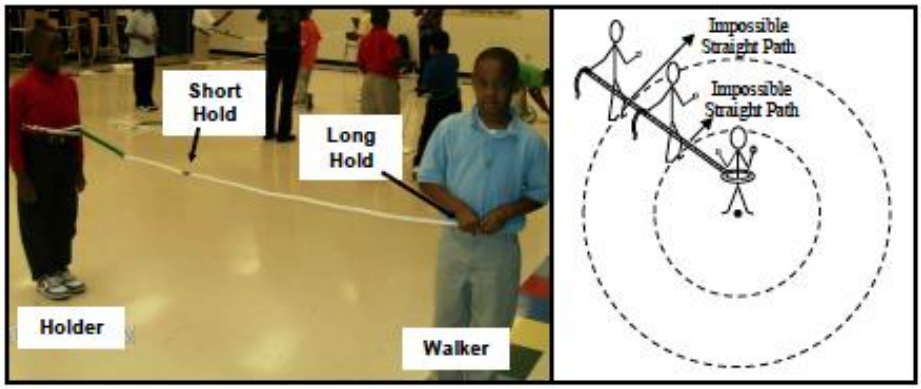


Figure 3. Image and diagram that show the Rope Walk.

Students re-described and inscribed these embodied experiences in order to support reasoning about mathematical (the geometry of circles) and physical systems (systems of levers).

The embodiment activity supported conceptual development through the learner's coordination of sensorimotor activity with mathematical and mechanical systems (Piaget, 1968). Through the Rope Walk embodiment the students constructed an artifact (Vygotsky, 1978). Artifacts are human constructions that include both material and symbolic forms (Saxe, 2002). The Rope Walk served as an artifact that had cultural and historical significance to the students as they mathematically re-described this artifact within the geometry of circles and simple systems of levers. The relationship between embodiment, mathematical system, and physical system is represented (Figures 4 and 5).

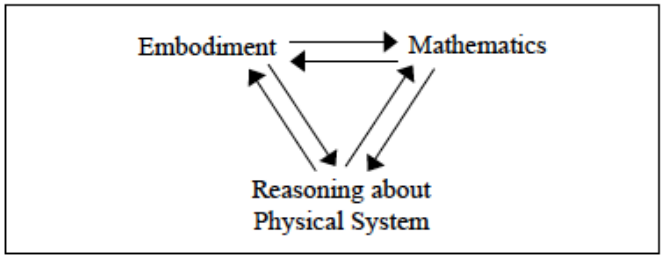


Figure 4. Model indicating the relationship between embodiment, mathematical description, and reasoning about the physical system.

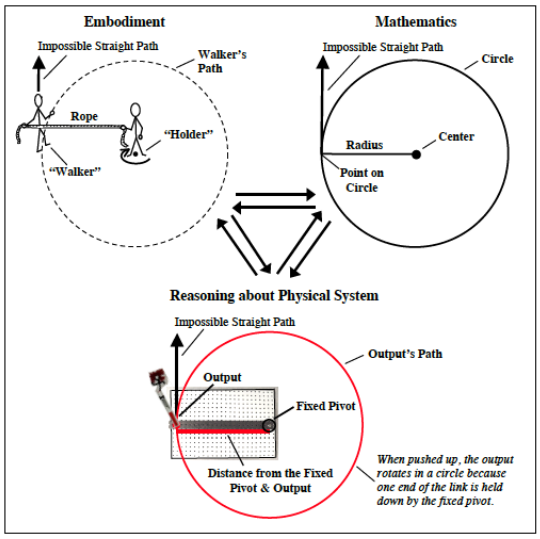


Figure 5. The Ropewalk disrupted straight paths. Through the curriculum, this embodied experience, mathematical description, and work with the physical system supported mechanistic reasoning.

Sfard (2007) notes that a relationship between subject (student participant(s)) and object (the Rope Walk embodiment, the geometry of circles, and the physical system of levers) cannot be conceptualized as the "acquisition" of entities such as ideas or concepts. Accordingly, she describes a *commognitive* theory of learning. This theory describes learning as taking place through the development of disciplinary discourse. Accordingly, this study traces the students' and the classes' learning according to changes in student talk and gesture during whole group discussions during the first two days of instruction.

Research Question

The research question addressed in this study is: Are embodiment and mathematical description viable resources for supporting reasoning about the mechanisms of systems of levers. Specifically, how do 3rd grade students appropriate these resources and forms of reasoning through discourse (i.e., talk and gesture; Gee, 2004). In addition, how will these 3rd grade students perform on assessments of mechanistic reasoning, mathematics, and engineering practices?

Method

Participants

Students (n = 15, 9 male) in the target classroom attended an urban elementary school in the midwestern United States. Most students (75%) qualified for free or reduced lunch. Three students had Individualized Education Programs (IEPs). In addition, post-interview assessment scores from the Assessment of Mechanistic Reasoning Project (AMRP) were compared with those from a group of 112 participants (Table 1) from a previous study. This comparison group

was used to measure those scores from participants in this study against a diverse group who had not engaged in the curriculum.

Table 1.
Comparison Group

Respondents	Number included in analysis
Elementary school students	28 (female = 17)
Middle school students	25 (female = 16)
High school students	20 (female = 4)
University undergraduates (non-science majors)	16 (female = 13)
University undergraduates (engineering majors)	13 (female = 5)
Adults (without college education)	10 (female = 8)

Procedure

Classroom instruction. Two researchers (ES and PW) served as the classroom instructors for this 9-week after-school engineering class (1.5 hours/week). The engineering topics were informed by the *Common Core State Standards* (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), *Standards for K-12 Engineering Education* (NRC, 2010), and the MechAnimations Curriculum (Bolger, Kobiela, & Lehrer, 2013). Each class session was videotaped and rendered for further analysis.

MechAnimations curriculum. The motion of systems of levers relies upon the rotary paths of individual levers around fixed and floating pivots (Figure 1). Bolger, Kobiela, Weinberg, and Lehrer (2012) as well as Weinberg (2017a; 2017b; 2019) showed that the understanding of these systems of levers is not trivial, children and adults had difficulty seeing and reasoning about the relationships between the fixed-ness of pivots and the rotary paths of levers. In the curriculum, these concepts were taught through the Rope Walk, a physical embodiment of lever motion that highlighted how a fulcrum disrupts the straight path of a lever.

In the Rope Walk (Figure 4), one student (the Holder) held one end of a rope and remained in place while another student (the Walker) held the other end of the rope and attempted to walk in a straight path perpendicular to the rope. The constraint imparted by the

Holder disrupted the straight path and produced a circular path, similar to the path produced by a link fixed to pegboard by a pivot.

After the Rope Walk embodiment, the curriculum focused on its mathematical description in order to support student mechanistic reasoning about systems of levers (Figure 4). Mathematically, the Holder served as the center of the circle and the radius stood in for the rope. The points on the circle were traced out by the Walker, traversing the constrained path caused by the fixed length of the rope. In this sense, the mathematical features of the experience also map directly onto the physical system: the Holder becomes the fixed pivot, the Walker becomes an output on a link traveling in a circular path. Table 2 elaborates the corresponding elements between the task's embodied experience, mathematical system, and the physical system of levers.

Table 2

Embodied Experiences, Mathematics and the Physical System Related to the Rope Walk.

Embodied Experience	Mathematics	Physical System
• A person holds one end of a rope. Another person holds the other end.	• A point (center) is drawn with a line (radius) (r_1) extending out from it. Another point is drawn at the other end.	• A link is attached to a pegboard at the end with a fixed pivot. The other end is marked as the output (with a figurine).
• One person tries to walk straight while the other rotates in place. His straight path is quickly disrupted by the constraining force of the rope held by the other person.	• One end of the radius cannot be swept in a straight path without moving both ends of the line.	• The output cannot be moved in a straight line without removing the fixed pivot.

• The person walking is forced by the rope onto a path going around the person rotating in place. The walker is always the same distance from the center person.	• A circle is created by sweeping the radius (constant length) around the center point.	• The output rotates around the fixed pivot. At any point in the rotation, the output always remains the same distance from the fixed pivot.
• The rope is now exchanged for a shorter rope and the person walking again attempts to walk straight.	• The new, shorter, radius (r_2) is created. Again, the radius cannot be swept in a straight path without moving both ends of the line.	• The output (figurine) is now moved in closer to the fixed pivot. Again, the output cannot be moved in a straight line without removing the fixed pivot.
• The walker is again forced onto a path going around the person rotating in place. This time, the path is shorter and the walker goes around faster.	• A new smaller circle (with smaller circumference) is created when the shorter radius is swept around the center point.	• The output again rotates around the fixed pivot. This time, the path looks smaller.

Through the curriculum, MechAnimations were constructed during design challenges. MechAnimations (Figure 6a) are systems of levers made of card stock and decorated; their components are hidden from view. Within these design challenges, students modelled the working and appearance of these MechAnimations using pegboard (Figure 6b) and paper (Figure 6) systems of linkages.

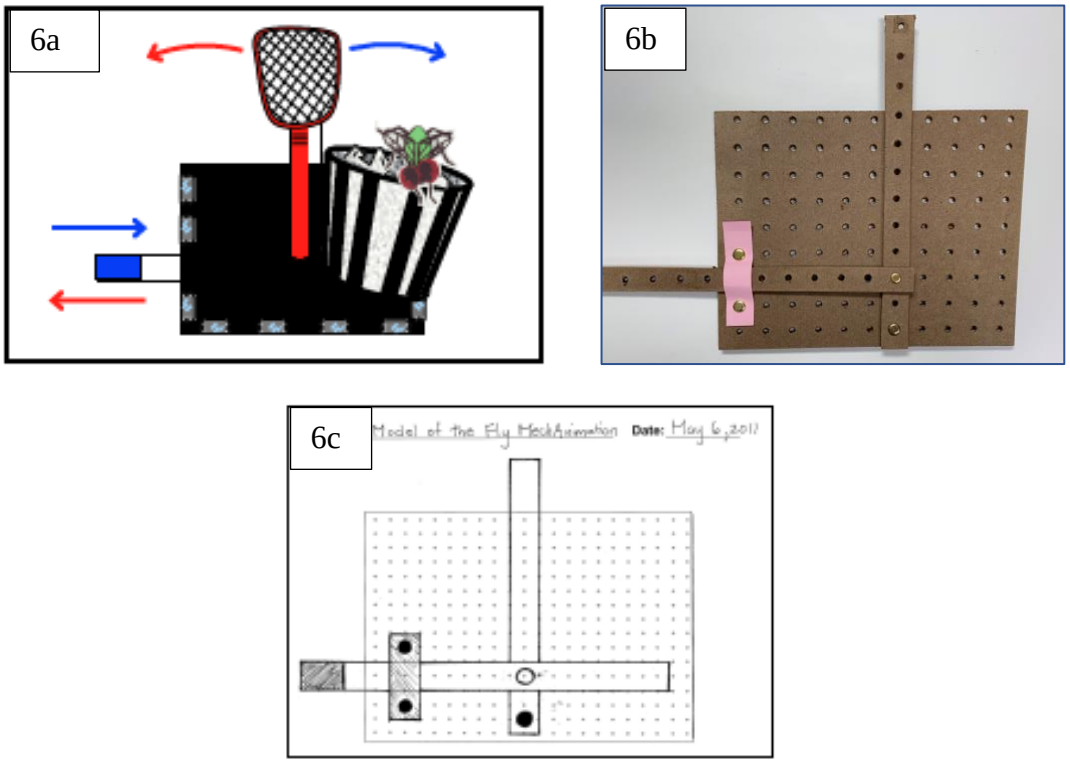


Figure 6. Fly MechAnimation (6a), pegboard model (6b), and paper model (6c).

Pre- and post-interviews. Before and after instruction, participants responded to a paper-and-pencil assessment in addition to a radius-circumference item and a blueprint item.

Instruments

Radius-circumference item. This item (Figure 7) assessed student understanding of the relationship between a circle's radius and its circumference (mathematical system) within the pegboard system. Within the pegboard system, the analogue to the radius and circumference is the distance from a point on an output link and the distance traveled by that point. For this task, the researcher presented the student with a pegboard mechanism, with three pieces of tape on the output link (blue, green, and red). The participants were asked the following questions, "If I pushed up on the input, which color would move the most?" "Why?" This was intended to

determine if participants could see that the red, green, and blue tape would travel in a circular path and that the red tape would travel in the largest path.

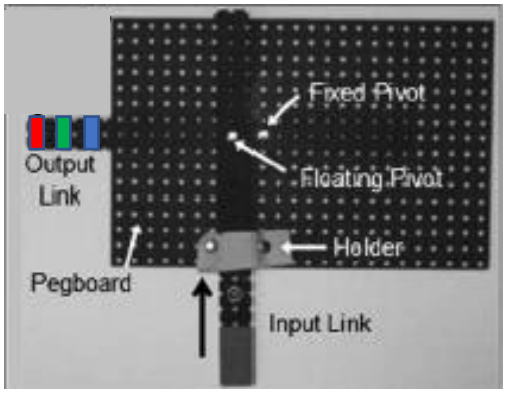


Figure 7. The Radius-circumference item.

Blueprint item. The blueprint item (Figure 8) assessed the engineering practice of designing a blueprint. The students were presented with an unfamiliar MechAnimation (Figure 8a), with all parts hidden from view (Figure 8b). They were asked to see how the MechAnimation moved. Then, the students were asked to construct a blueprint on a paper pegboard (Figure 8c) that would show how the MechAnimation could have been built so that it would move similarly. In addition to assessing student propensity to design a useable blueprint, this item also diagnosed their propensity to show mechanistic reasoning through their design.

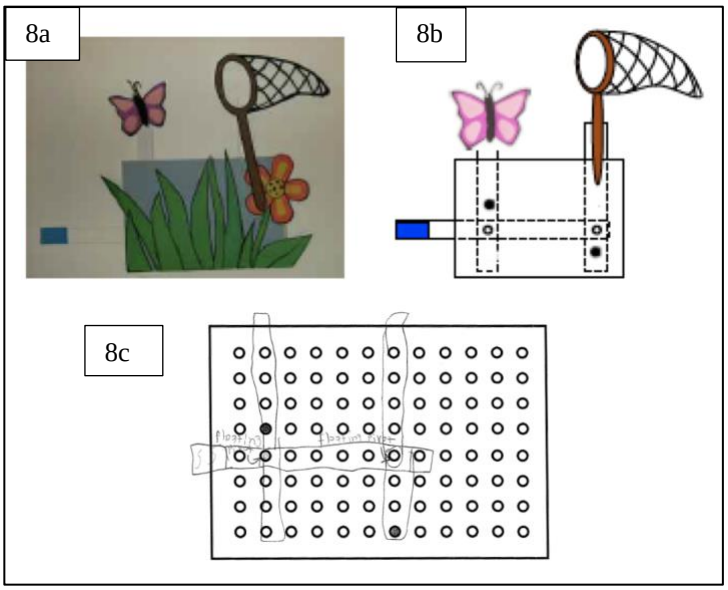


Figure 8. The blueprint item: Butterfly and Net MechAnimation (8a), model (8b), and student blueprint (8c).

Assessment of Mechanistic Reasoning Project (AMRP). The development and scoring of the AMRP relies upon IRT modeling. The calibration of this instrument is described in Weinberg (2018). Each assessment administration was completed during one day and lasted an average of 35.7 minutes (ranging from 23 minutes to 76 minutes). The assessment was presented to participants across one form. The AMRP items required respondents to draw predicted motion. There were 21 items in which *rotation* could be scored. In addition, there were eleven items in which *constraint via the fixed pivot* could be scored. The AMRP also assessed other mechanistic elements; however, the consideration of these elements is beyond the scope of this article.

Analysis

Coding categories for talk and gesture. This study tracks the development of student discourse (talk and gesture) (Gee, 2004) as indicators of mathematical and mechanistic learning (Sfard, 2007). Accordingly, student talk and gesture were coded during two days of instruction. Transcripts were made from video, indicating all talk and gesture during the first two class

sessions. The tree diagram (Figure 9) shows the hierarchical coding scheme developed for this study. At the top level (Figure 9, far left) are three categories: Direction, Reference to the Mathematics of Circles, and Reference to Embodiment.

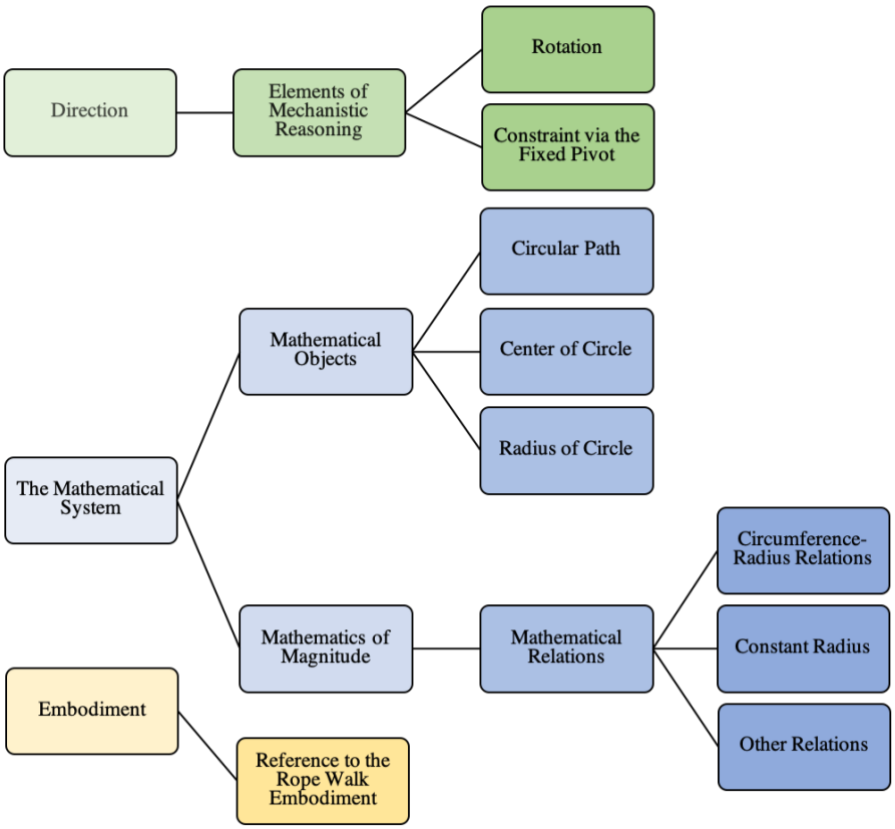


Figure 9. Coding categories used in analysis of talk and gesture during instruction.

Direction includes the Elements of Mechanistic Reasoning that can be deployed to characterize simple systems of levers (Bolger et al., 2012; Weinberg, 2017a; 2017b; 2019) (Table 3).

Table 3
Elements of Mechanistic Reasoning with descriptions and examples.

Mechanistic Element	Description	Example
Rotation	Attention to the rotary motion of the levers.	"The output goes around."
Constraint via the Fixed Pivot		

Attention to the causal relation between the pivot being fixed to the board and the resultant motion.	"Because the brad is stuck to the board, the lever is going to move that way."

The *Reference to Mathematics of Circles* and the *Mathematics of Magnitude* categories are presented below (Table 4)

Table 4
The Mathematics System codes

Categories	Example
Describing or Noticing Mathematical Objects	
Circular path	"this lever goes in a circle."
Center of circle	"this circle is centered around this point."
Radius of circle	"the rope and the link go from the center of the circle to the outside point."
Mathematics of Magnitude	
Circumference-radius	"the farther you go out on the rope, the bigger your path will be."
Constant radius	"as you move around on the rope, the Walker is always going to be the same distance from the Holder."
Other relations	"when you do the Short Hold, you go faster than when you do the Long Hold."

Finally, the *Reference to Embodiment* category indicates when a student spontaneously referenced the Rope Walk.

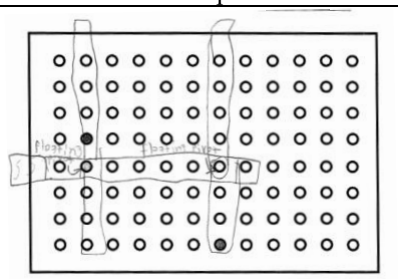
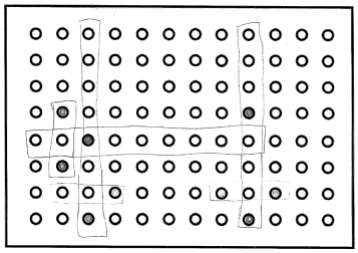
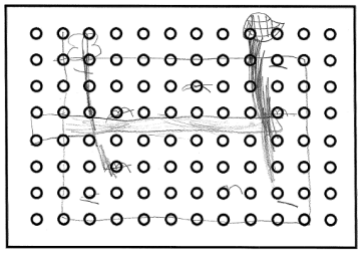
Instances were defined as students turns of talk during whole group discussion. There were 94 instances across the first two classroom sessions. The first author coded 100% of the corpus. The second author coded 10% of the total instances. Their agreement was 87%.

Radius-circumference item. On this item (Figure 8), students were asked the following two questions: "If I pushed up on the input, which color would move the most?" and "Why?" The colors that the students indicated are presented. In addition, student rationales were

classified into the following thematic categories: (a) distance from the fixed pivot, (b) closest to the end, and (3) other. The two authors coded this assessment. Their agreement was 100%.

Blueprint item. This item (Figure 9) was scored according to an analytic framework (Table 5). The framework did not have a prescribed solution strategy; any strategy that generated the requisite motion was accepted. In addition, the framework discriminated between students who were reasoning about machine components, structure, or mechanism. Two researchers coded the assessments. Their agreement was 93%.

Table 5
Scoring guide for blueprint item, including category, description, and example.

Category	Description	Example
Mechanism	Illustrates how a system of linkages and fixed and floating pivots are organized in ways that support their function. However, the direction of lever motion may be incorrect.	
Structure	Illustrates how parts of a system of linkages are organized in ways that are relevant to its function.	
Components • Appearance • Individual Parts	Focuses on visual aspects of the system irrelevant to function. Notes or describes specific parts of the system (e.g., links, brads, holders) but without indication of how they contribute to the system's structure.	
Missing/Unclear	Student was unable to complete the task or student's blueprint indicated that student did not understand the nature of the task.	N/A

AMRP. The pre- and post-test AMRP were scored according to the developed exemplar (Appendix A, Table A). Two researchers coded the assessments. Their agreement was 89%. Student pre- and post-assessments were compared using a sign test for significance. In addition, IRT analysis modeling was used to compare student post-assessments to a comparison group of 112 individuals.

Results

The Results section is structured as follows: (a) selected transcripts from the first two days of instruction are presented. These transcripts have been coded to demonstrate individual student use of the embodiment as a resource for reasoning about the mathematical system, (lever motion, as well as coordinating all three systems; (b) codes from the first two days of instruction are presented in the aggregate; (c) findings from the circumference-radius item and AMRP pre- and post-test assessment data are presented; finally, (d) an IRT analysis of post-tests data from the AMRP is presented. The differences in scores between the target class and a comparison group are described.

The Embodiment as a Resource for Reasoning about the Mathematical System

Mapping from the Rope Walk embodiment to the mathematical system. In order to support student mechanistic reasoning about lever motion, the researchers leveraged embodiment and mathematical description. Here, the Rope Walk embodiment and its mathematical description are described. As the task began, AJ took hold of one end of the rope (Holder) and Jackson (Walker) took hold of another part of the rope, several feet away. PW instructed the class to “try walking in a straight line” (indicating direction) while keeping the rope taut. Jackson immediately sensed difficulty, at first hesitating and then asking whether he should “turn.” As Jackson walked, he described feeling constrained and then having to turn. He noted “I keep

going in circles (*circular path*).” This noting of circular path indicated a mathematical description of the Rope Walk embodiment.

Embodiment re-described as circular path, circumference-radius relations. Next, AJ switched to the Short Hold, walking the path with a shorter rope length. He was directed to walk straight, but immediately had to turn. AJ initially observed that the path was again a circle and indicated that, “it’d be [the path would be] smaller [than that experienced when walking the Long Hold].” (*embodiment*, *circumference-radius relations)*

Next, Jackson took on the role of the Holder. He described the constraint he felt in this role: “I’m staying in one position and [AJ] keeps trying to walk forward. The rope only goes in circles (*embodiment*, *circular path*). [AJ makes] a circle shape and he just goes around and around and around (circular gesture) (*embodiment*, *circular path*).” AJ describes the embodiment mathematically and relates the speed of rotation between the long and Short Hold (Transcript 1).

Transcript 1

PW: Make [the rope] longer then; make it go all the way to the end. (AJ transitions from the Short Hold to the Long Hold). Then can you walk straight? Try walking straight.

AJ: I start to turn ... at first you walk straight, but then you start to turn. But if you hold on the red spot (Short Hold), you turn faster than if you turn on the blue spot (Long Hold) (*embodiment*, *other relations*).

This consideration of angular velocity (“you turn faster”) will be a powerful resource when further considering the motion of systems of levers, particularly when reasoning about leverage.

Whole group discussion of the Rope Walk: Mathematical description of the embodiment. After students engaged in the Rope Walk embodiment, ES and PW facilitated a

discussion highlighting elements of the embodiment and further mapping them to the mathematics of circles (Transcript 2). In doing so, Parker indicates the correspondence between the Rope Walk and the mathematical system by highlighting the analogue between the Walker and the circle.

Transcript 2

ES: What is happening?...[W]e tried to go straight…What happened?

Parker: The force of the thing [the rope]. It was pulling back so you felt like you had to turn. And you had to try to go straight, but it was really hard (*embodiment*).

ES: You had to go straight and there was a force?

Parker: …Yeah, on the rope. That made you turn kindof like [inaudible] you were going in a circle (*embodiment*, *circular path*). That made you go around in circles (*embodiment, circular path*) or something. You were supposed to go forward, but the force made you go around (circular gesture) (*embodiment, circular path)*.

Mapping to and from the mathematical system: Representing the Rope Walk. Students were asked to create representations of the Rope Walk embodiment immediately following the experience and the subsequent discussion. PW asked the students to draw two pictures to show their experiences with both the long and the short rope. This activity further supported the mapping from the embodiment to the mathematical system. In addition, the whole group discussion that took place after students created their representations supported the reverse mapping from mathematical system to embodiment.

Jackson mathematized the forces he experienced during the Rope Walk (Figure 10). His representation included a physical model of the Rope Walk, including the Holder, Walker, and rope. However, it additionally showed the physical forces acting on the Walker. He had

mathematized the embodiment as a quasi-force diagram, indicating where the Walker felt the force of the rope (arrow pointing from the Walker to the Holder), the force counteracting the rope (arrow from the Walker to the left), the force that resulted from the Walker's attempt to walk straight (the force from the Walker to the bottom of the page).

Figure 10. Jackson's representation of the Rope Walk includes forces, represented as arrows.

PW asked Jackson to explain the arrows in his representation. Jackson described the forces he experienced when he was the Holder and AJ was the Walker. "My friend AJ was pulling me while I was holding still and he went straight and started to turn (*embodiment*)." PW probed Jackson further for an explanation (Transcript 3). Jackson described this representation of the Rope Walk as if it were the actual embodied experience

Transcript 3

Jackson: They [the arrows] mean that he's turning (circular gesture) (*embodiment*, *circular path*); he's turning in a circle (*embodiment, circular path*). That means he's walking forward and then he turns (circular gesture) (*embodiment*, *circular path*).

Jackson mapped back and forth between the embodiment and mathematical system as he described how the arrows (mathematical notation) described the Walker's path (embodiment), which is circular (mathematical object). Owen's representation (Figure 11) showed his

mathematical description of the embodiment. It included neither the Holders nor the Walkers. Instead, the diagram showed two circles traced out for both the Long Hold and Short Hold. Owen indicated that the Short Hold was "eeser [easier]" than the Long Hold.

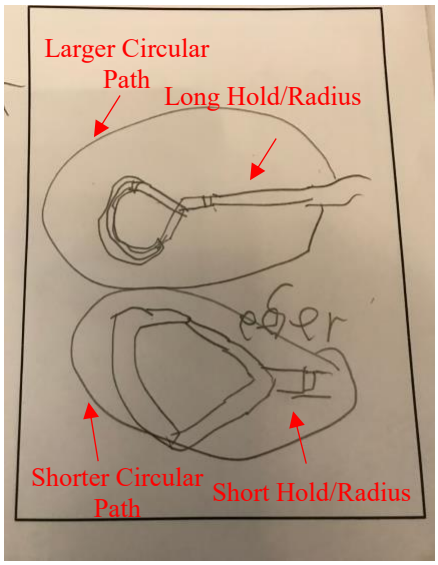


Figure 11. Owen's representation of the Rope Walk includes the circular path. Red text added by the author.

PW initiated a discussion of Owen's representation to support a bidirectional mapping between Owen's representation and the Rope Walk (Transcript 4).

Transcript 4

PW: Does anybody…want to tell me what they think Owen showed [through this representation]?...Jackson?

Jackson: It's just the rope (*embodiment*).

PW: …So…you say that he's showing just the rope.

Jackson: Because he wants to show how it [the rope or Rope Walk] works and stuff (*embodiment*).

PW: …Yeah. JT. So, … JT, what do you see here (indicates the representation of the larger circle)?

JT: I think he's showing the Long Hold (*embodiment*)

PW: …Anya?…What's this thing (indicates the Long Hold)?...

Anya: A circle (*circular path*).

Jackson and J.T. use Owen's representation of circular paths and radii (mathematical objects) to describe the corresponding embodiment. Then, Anya supports a mapping back from the embodiment to the mathematical system.

Supporting reasoning about geometric objects and relations. Next, students were presented with a representation from a the curriculum (Bolger, Kobiela, & Lehrer, 2013) (Figure 12). PW and ES introduced this inscription into instruction in order to discriminate between circles (generated by the Rope Walk) and the mathematical objects in Owen's representation, which appeared to be more elliptical. This representation (Figure 12) supported students to reason about the center of circle, circular path, radius of circle, circumference-radius relation, constant radius, and other relations.

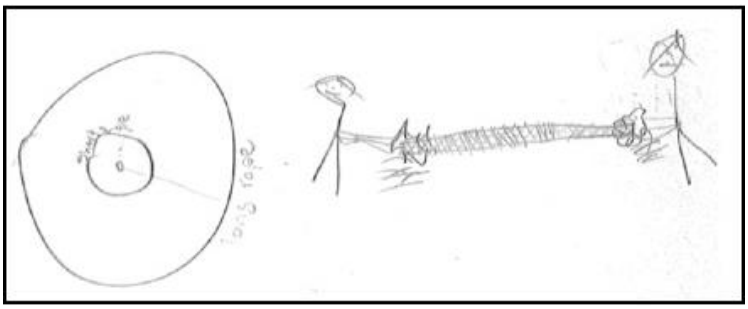


Figure 12. Representation of the Rope Walk with concentric circles clearly depicted.

Samantha's description of this representation highlighted the circular path, radius of circle, and circumference-radius relation (Transcript 5).

Transcript 5

Samantha: The small one [circle] came from the small one [rope/radius] and the big [circle] came from the big one [rope/radius] (*embodiment*, *circular path, radius of circle, circumference-radius relation*).

PW asked the students if both a circular and elliptical path could have been traversed by a Walker walking the Long Hold (Figure 13). This further supported the need for students to reason about objects and relations within the mathematical system.

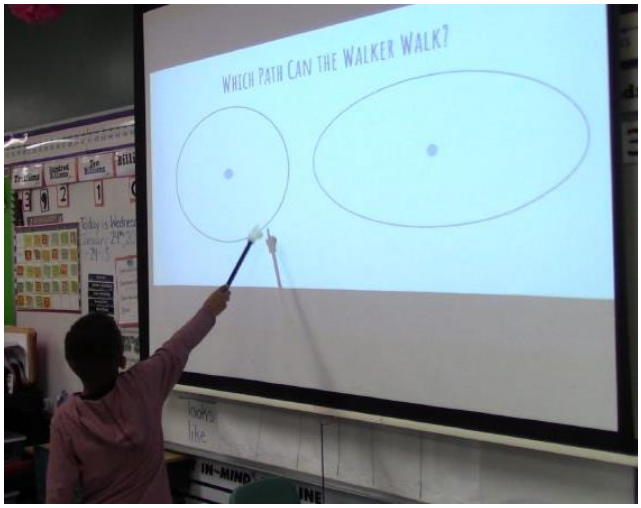


Figure 13. AJ indicates that the Walker could traverse a circular path, but not an elliptical path.

AJ began arguing that the Rope Walk could not generate an elliptical (non-circular) path. He noted: "Because it's (indicates major axis) (*radius of circle*) too long and the other (indicates minor axis) (*radius of circle*) one's too short…You see how that one (indicates circle's radius) (*radius of circle*) is shorter than that one (indicates ellipse's major axis) (*other relations*). This one (indicates ellipse), you can't walk (*embodiment*) the same because it's (indicates major and minor axes of ellipse) different lengths (*other relations*) (Figure 14a)…This one is the same (indicates the circle's diameter is constant) (Figure 14b) (*constant radius*)."

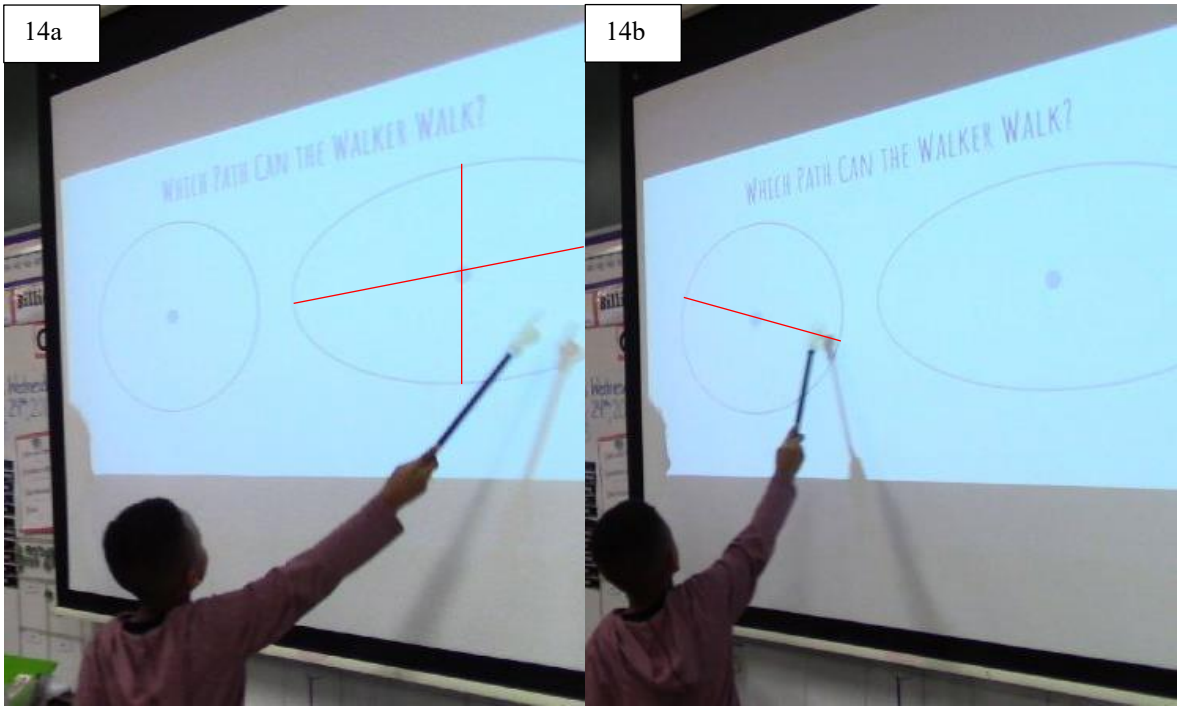


Figure 14. a. AJ shows that the elliptical axes are not congruent, b. while the circle's radii are.

The Embodiment as a Resource for Reasoning about Lever Motion

Constructing a correspondence between the Rope Walk and the system of levers. Next, students mapped the Rope Walk embodiment onto the pegboard system. Students worked in groups of three and were given a pegboard, a link, and a brad fastener and asked to attach the link to the pegboard any way they wanted. Each group placed the pivot at the end of the link, creating a mechanism resembling the Rope Walk (Figure 15).

Wilshawn describes the similarities between the physical system and the embodiment. "[The system of levers and the Rope Walk are] the same because [in both] the person tries to walk straight and then go[es] sideways…I think it is the same because of the forces moving in the same direction."

Figure 15. AJ, Jackson, and Parker construct a pegboard mechanism model of the Rope Walk.

Next, students were asked to place a "Person Pen" (dry erase markers were decorated as people to highlight the mapping from the Rope Walk to the system of levers) into a hole on the link that would create that largest possible path (Figure 16) (Transcript 6). In order to develop the correspondences between the embodiment and the system of levers, students explicitly constructed the following correspondences: rope/link, fixed pivot/Holder, point on link/Walker.

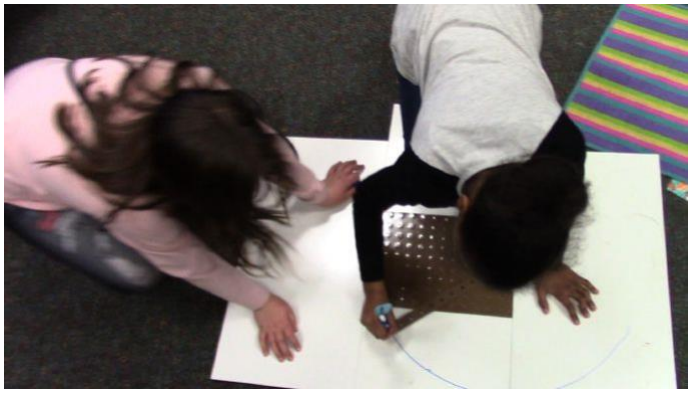


Figure 16. Samantha and Autumn trace the path of the lever to create the largest path.

Jackson created a mapping from the Rope Walk embodiment to the pegboard system.

Transcript 6

PW: Tell me, what part of what you're doing [on the pegboard] is like the rope?

Jackson: This (Jackson touches the link) [is like the rope] (*reference to the Rope Walk embodiment*).

Samantha also leveraged the Rope Walk embodiment to support her reasoning about the system of levers (Transcript 7). PW asked her what was preventing the lever from moving in a straight path.

Transcript 7

Samantha: The thing that's pulling it down [embodies the Holder with her right arm as the rope] (Figure 17a) (*reference to the Rope Walk embodiment*). When it [the fixed pivot] holds it down (indicates the fixed pivot) (Figure 17b) (*reference to the Rope Walk embodiment*, *constraint via the fixed pivot*), it [output] can't go any farther. It has to turn (*rotation*).

PW: Can you take that board and show us the thing that's preventing it from going straight? The thing that's like the Holder?

Samantha: Like if it's (indicates the fixed pivot with her right hand) holding it down, it [the output lever] tries to go straight, but it can't anymore, so it has to turn (*constraint via the fixed pivot*, *rotation*).

Figure 17. Samantha (a) embodied the fixedness of the Holder as the Walker curved around her and (b) demonstrated the similar motion made by the pegboard system.

Coordinating the Rope Walk, mathematical system, and the physical system. Students were supported to create mappings between the Rope Walk, the mathematical system, and the system of levers. For example, AJ describes a correspondence between the Walker's path, the circle, and rotation (Transcript 8). After stating that the Walker is like the Person Pen, PW asks him, "why?"

Transcript 8

AJ: (Indicating the Person Pen on the lever) Because the person who's going in circles (*circular path, rotation*), they're the one with the rope around them [Holder] (*reference to the Rope Walk embodiment*), where there is the yellow square [the yellow square indicated a hole where students were to place the fixed pivot] and they're holding onto the rope (*reference to the Rope Walk embodiment*), that makes them go in circles (*reference to the Rope Walk embodiment*, *constraint via the fixed pivot*, *rotation, circular path*)…That the person who's going around in circles is the person with the rope around them (*reference to the Rope Walk embodiment*, *circular path, rotation*).

Students were asked to describe how the mechanisms moved. Charissa constructed a correspondence between the Walker's path (embodied system), circle (mathematical system), and lever motion (system of levers) (Transcript 9).

Transcript 9

Charissa: [W]hen you move it like this (pushes the link), it moves in a circle again (*circular path, rotation*). Like it did in the rope…when the rope was there (gestures circular path) (*reference to the Rope Walk embodiment, circular path*, *rotation*).

Next, Jackson constructed a correspondence between the Holder, center of the circle, and fixed pivot (Transcript 10). In addition, his mechanistic explanation causally connected the mechanistic elements of rotation and constraint via the fixed pivot.

Transcript 10

Jackson: It kindof goes like… like a circle (rotates the link) (*circular path, rotation*)...In the center it goes just like a (indicates the fixed pivot) ... kindof, but somebody's not staying there [the fixed pivot is fixing the link even though there is no Holder in the lever system] (*reference to Rope Walk embodiment, center of circle*, *constraint via the fixed pivot*).

Coding Talk and Gesture: Learning as Discourse Change

During the first two days of instruction, the most popular code was *reference to the Rope Walk Embodiment* (n = 45; 48% of instances) (Table 6). This suggests that the Rope Walk was an important resource for student reasoning. *Circular path* was also popular (n = 31; 33%). Another mathematical object code that students used frequently were *radius of circle* (n = 10, 11%). The mechanistic element of *rotation* was cited frequently (n = 14, 15%); *constraint via the fixed pivot* (n = 7, 7%) was coded less frequently. The frequency of these codes across instruction indicates that student learning developed across the embodiment, mathematical system, and system of levers.

Table 6

Codes of talk and gesture for the embodiment, mathematical system, and pegboard system

Codes	Frequency (94 instances)	Percent
Rotation	14	15%
Constraint via the Fixed Pivot	7	7%
Circular Path	31	33%
Center of Circle	4	4%
Radius of Circle	10	11%
Circumference-radius Relations	4	4%
Constant Radius	1	1%
Other Relations	11	12%
Reference to the Rope Walk Embodiment	45	48%

Coordinating the Rope Walk with the Mathematical or Pegboard Systems

The Rope Walk embodiment was a powerful resource for reasoning about both the mathematical and physical systems. The majority of discourse (i.e., talk and gesture) that cited mathematical objects and/or mathematical relations also involved a reference to the Rope Walk embodiment (Table 7). For example, 100% (n = 4) of student references to the *center of the circle* involved a reference to the *Rope Walk embodiment*. In addition, 76% (n = 25) of references to a *circular path*, 80% (n = 8) of references to *radius of circle*, 100% (n = 4) of references to *circumference-radius relations*, 100% (n = 1) of references to *constant radius,* 73% (n = 8) of references to *other relations* were cited with *references to the Rope Walk embodiment*. This shows that student reasoning was supported by the coordination of the embodiment and the mathematical system.

In addition, the Rope Walk embodiment also supported students to reason mechanistically (Table 7). For example, 71% (n = 10) of references to *rotation* and 83% (n = 5) of references to *constraint via the fixed pivot* were cited with *references to the Rope Walk*

embodiment. This shows student reasoning was supported by the embodiment and the system of levers.

In addition, 100% (n = 7) of references to *constraint via the fixed pivot* were also made with references to *rotation.* This is important because the curriculum was developed to support student reasoning about these mechanistic elements

Table 7
Codes showing references to the mathematical system and the Rope Walk embodiment

Codes	*Mathematical System*	Center of Circle (4)	Circular Path (33)	Radius of Circle (10)	Circumference-Radius Relation (4)	Constant Radius (1)	Other Relations (11)
Rope Walk Embodiment							
Reference to the Embodiment		4 (100%)	25 (76%)	8 (80%)	4 (100%)	1 (100%)	8 (73%)
Pegboard System							
Rotation		4 (100%)	13 (39%)	4 (40%)	4 (100%)	1 (100%)	1 (9%)
Constraint via the Fixed Pivot		4 (100%)	5 (15%)	2 (20%)	2 (50%)	1 (100%)	0 (0%)

Coordinating the Rope Walk with the Mathematical and Pegboard Systems

Student discourse (Table 8) showed that students were reasoning about the three analogue systems (the Rope Walk embodiment, the geometry of circles, and the pegboard system) while working with the pegboard system. Of the four instances of *referencing the Rope Walk embodiment* and *center of circle*, 100% (n = 4) also cited *rotation* and *constraint via the fixed pivot.* Of the 25 references to *referencing the Rope Walk embodiment* and *circular path,* 40% (n = 10) also cited *rotation* and 20% (n = 5) also cited *constraint via the fixed pivot.* Of the 25 instances of *referencing the Rope Walk embodiment* and *circular path,* 40% (n = 10) also cited *rotation* and 20% (n = 5) also cited *constraint via the fixed pivot.* Of the eight instances of *referencing the Rope Walk embodiment* and *radius of circle,* 50% (n = 4) also cited *rotation* and 25% (n = 5) also cited *constraint via the fixed pivot.* Of the four instances of *referencing the Rope Walk embodiment* and *circumference-radius relation,* 100% (n = 4) also cited *rotation* and 50% (n = 2) also cited *constraint via the fixed pivot.*

Table 8
Codes showing the simultaneous student citations of references to the Rope Walk embodiment, the mathematical system, and the pegboard system.

Coded with Embodiment	*Mathematical System*	Center of Circle (4)	Circular Path (25)	Radius of Circle (8)	Circumference-Radius Relation (4)	Constant Radius (1)	Rotation (10)
Pegboard System							
Rotation		4 (100%)	10 (40%)	4 (50%)	4 (100%)	1 (100%)	
Constraint via the Fixed Pivot		4 (100%)	5 (20%)	2 (25%)	2 (50%)	1 (100%)	5 (50%)

Radius-circumference Relations

The Rope Walk embodiment was intended to support reasoning about the mathematics of circles (focusing on mathematical objects and the mathematics of magnitude). In order to characterize student reasoning about this mathematical system, the Radius-circumference item was used (Figure 7). For this task, the researcher presented the student with a pegboard mechanism, with three pieces of tape on the output lever (blue, green, and red), with each farther from the fixed pivot. The participants were asked the following questions, "If I pushed up on the input, which color would move the most?" "Why?" This was intended to determine if participants could see that the red, green, and blue tape would travel in a rotary (and circular path) and that the red tape's path would be the largest because it was the farthest from the fixed pivot.

Thirteen (87%; Table 9) students correctly indicated that the red tape would generate the longest path. This indicates a relationship between the embodied experience, its mathematical description, and the physical system. Of these thirteen students, three (23%) indicated that the distance between the red piece of tape and the fixed pivot (radius) was what would determine that longest path (circumference).

Table 9
Radius-circumference item.

Questions	Response		
If I pushed up on the input, which color would move the most?	Red 13 (87%)	Green 0 (0%)	Blue 2 (13%)
Why?	Distance from Fixed Pivot 3 (20%)	Closest to the End 10 (67%)	Other 2 (20%)

Engineering Practices

This after-school STEM program supported many practices that are central to the engineering design process. For example, constructing blueprints is a critical engineering practice. Accordingly, students created blueprints to motivate or explain MechAnimation design. Here, they were presented with an unfamiliar MechAnimation (Figure 8a) with all parts hidden. Students were asked to move the MechAnimation. Then, they were asked to construct a blueprint for a mechanism that would move in the same way as the MechAnimation (Figure 8c). Student blueprints indicated that three students (20%) were reasoning about mechanism, 10 (67%) students were reasoning about structure, and two (13%) students were reasoning about components (Table 10).

Table 10
Blueprint item

Category	Number of Participants
Mechanism	3 (20%)
Structure	10 (67%)
Components	2 (13%)
Unclear	0 (0%)

Assessing Mechanistic Reasoning

Student propensity to reason mechanistically was characterized according to the Assessment of Mechanistic Reasoning Project (AMRP). Twelve students indicated a higher AMRP score on their post-test than they had on their pre-test ($p < 0.05$, sign test). In addition, students in our after-school engineering program deployed mechanistic reasoning more adeptly and had higher person ability scores ($M = 0.09$ logits) than a comparison group who had not participated in the MechAnimations Curriculum ($M = -0.22$, $p < 0.05$). For example, the highest performing student, Ryan, had a person ability estimate of 0.59 logits.

DISCUSSION

A challenge facing K-12 STEM education is integration (National Research Council [NRC], 2007). One particular challenge with STEM integration has been meaningfully supporting concepts across domains (Honey, Pearson, & Schweingruber, 2014). The instruction in this study has supported concepts in mathematics (i.e., geometry of circles) and science (i.e., simple machines). In addition, this study has also supported disciplinary practices (National Research Council, & Mathematics Learning Study Committee, 2001) in science (e.g., mechanistic reasoning) (Bolger et al., 2012; National Research Council, 2011; Russ, Scherr, Hammer, & Mikeska, 2008; Weinberg, 2017a; 2017b; 2019) and engineering (e.g., the engineering design process, modeling, constructing blueprints) (NRC, 2009). The NRC described modeling in engineering as follows: "Engineers use models to help visualize potential solutions to design problems and/or as an interim step in the development of working prototypes." (p. 87) In the MechAnimations Curriculum, modeling was an integral part of the design process. Students modeled their MechAnimation designs with blueprints and pegboard mechanisms as well as developed blueprints for their designs. In addition, the MechAnimation Curriculum meaningfully integrated mathematics with science and engineering by mathematically describing the lever systems and supporting mathematical description through these modeling practices.

This study shows that embodiment serves as a resource to support STEM integration within mathematics, science, and engineering. The study followed one class within an after-school STEM program. During the first two days of instruction, students experienced a physical embodiment of a link and pivot system, mathematically re-described this model, and applied this model to a link and pegboard system. Student learning was assessed through the coding of student talk and gesture from transcripts taken from video of classroom sessions, the radius-

circumference item, the blueprint item, and the Assessment of Mechanistic Reasoning Project (AMRP).

Discourse Change: The Embodiment, Mathematical System, and System of Levers

Participant discourse (Gee, 2004; Sfard, 2007) shows that students are reasoning in sophisticated ways about the Rope Walk embodiment, the geometry of circles, and the pegboard system. The Rope Walk embodiment was an important and popular resource for students, referenced in 43% of instances (Table 6). In addition, student references to the embodiment were coordinated with references to the mathematical system. For instance, 76% (n = 25) of references to a *circular path* were cited with *references to the Rope Walk embodiment*. This was also seen in the transcripts: "The small one [circle] came from the small one [rope/radius] and the big [circle] came from the big one [rope/radius] (*embodiment*, *circular path, radius of circle, circumference-radius relation*)." (Samantha) In addition, students also used the Rope Walk to support reasoning about the system of levers and its mechanistic elements. For example, 71% (n = 10) of references to *rotation* and 83% (n = 5) of references to *constraint via the fixed pivot* were cited with *references to the Rope Walk embodiment* (Table 7). This is also seen in the transcripts: "This (Jackson touches the link) [is like the rope] (*reference to the Rope Walk embodiment*)." (Jackson) Finally, students coordinated the three systems (embodiment, mathematical system, and system of levers). For instance, of the 25 references to *referencing the Rope Walk embodiment* and *circular path,* 40% (n = 10) also cited *rotation* and 20% (n = 5) also cited *constraint via the fixed pivot.* This is noteworthy because the curriculum was designed to support student reasoning about rotation and fixed pivot constraint. This is shown in the transcripts: "(Indicating the Person Pen on the lever) Because the person who's going in circles

(*reference to the Rope Walk embodiment, circular path, rotation*), they're the one with the rope around them (*reference to the Rope Walk embodiment*)." (AJ)

Disciplinary Practices

AMRP. Discourse from the drawing of student predictions of machine motion were analyzed with the Assessment of Mechanistic Reasoning Project (AMRP) assessment (e.g., Appendix A). Twelve students indicated a higher AMRP score on their post-test than they had on their pre-test ($p < 0.05$, sign test). In addition, students in our after-school engineering program deployed mechanistic reasoning more adeptly and had higher person ability scores ($M = 0.09$ logits) than a comparison group (n = 112) that had not participated in the MechAnimations Curriculum ($M = -0.22$, $p < 0.05$). This comparison group was composed of a diverse population of elementary, middle, and high school students as well as college undergraduates and adults without college education.

Blueprint assessment. The design and use of blueprints are an important disciplinary practice within engineering (NGSS Lead States, 2013; NRC, 2009). In the post-test administration, 87% (n = 13) of students were able to construct blueprints that indicated an understanding of mechanism or structure.

STEM Education

This study presents instruction and curriculum that supports embodiment, mathematics, and mechanistic reasoning across disciplinary content and practices. Accordingly, STEM integration could and should more frequently be supported based on the capacity of disciplinary content and practices to be embodied, described mathematically, and applied within a scientific system (e.g., simple machines). This study described a theory of action-based STEM learning

across disciplines through the use of physical embodiment and intentional mathematical re-description within a mathematical and pegboard system.

STEM integration was supported within this study. This work was accomplished within one third-grade classroom during an after-school STEM program. Our future work attempts to support this (and other embodiment-based) instruction across multiple classrooms during science instruction. In order to accomplish this, K-12 teachers will experience professional learning. STEM integration of this type (involving embodiment and mathematical re-description) requires that teachers have sufficient knowledge of the target STEM domain, mathematics, and practices within these disciplines. In addition, the classroom teacher must be able to anticipate what mathematical analogues students will likely construct, understand the value in supporting particular analogues over others, and effectively facilitate learning through these analogues.

Supporting Marginalized Students

Three students in this study had Individualized Education Programs (IEPs). In addition, one student was reading at a Kindergarten level. The activities presented allowed these students to be successful. One student, Ryan, with a tested intelligence of 85 performed more successfully on the AMRP than any other student with a person ability score of 0.59 logits, which is higher than the mean of person ability scores for both the target classroom and the comparison group. A future article (Weinberg, in process) will focus on how this curriculum supports learning across STEM disciplines for all students. In addition, it addresses principles for effective STEM learning all students.

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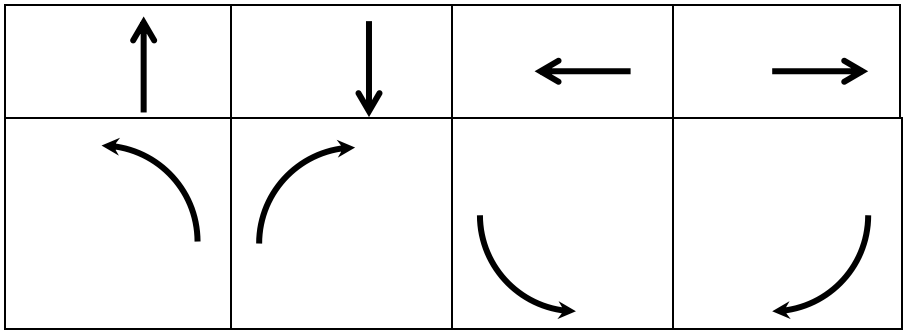
Appendix A

Item

Key: Fixed Pivot (attaches link(s) to base) ●
Floating Pivot (attaches link to link) ○
★

Draw an arrow, like one of these below, to show how each star would move if you pushed up on the black handle. (Draw an arrow starting at EACH star and show how they will move)



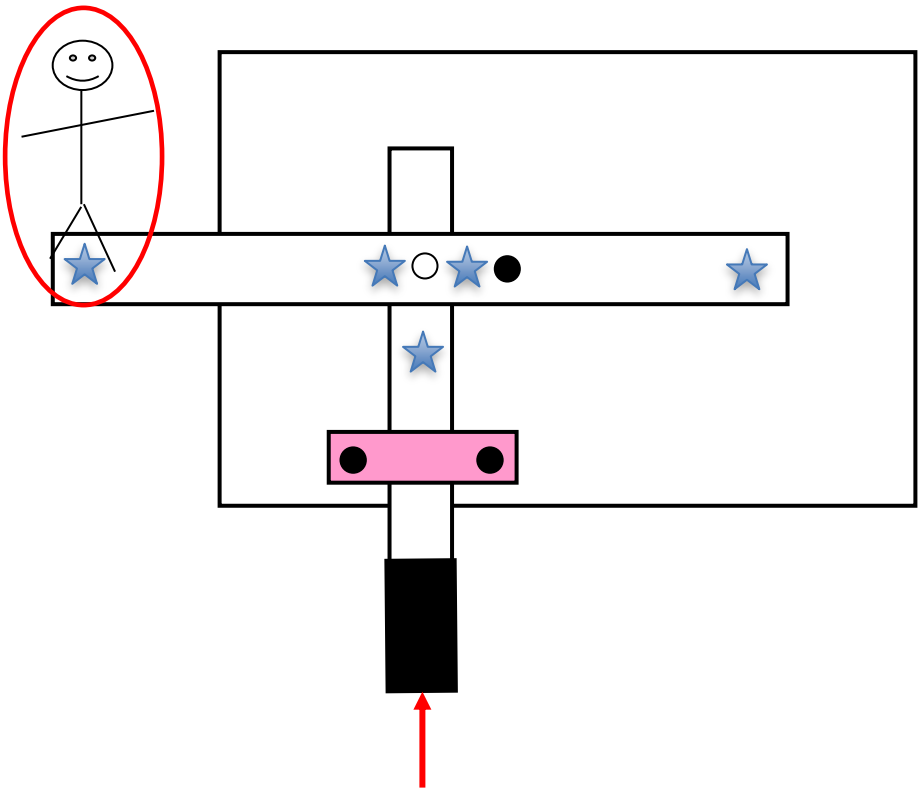
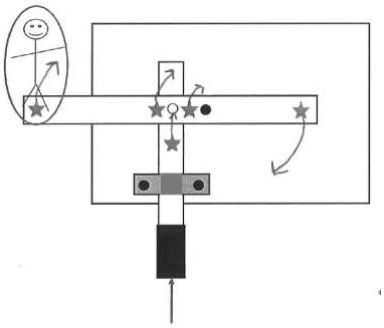
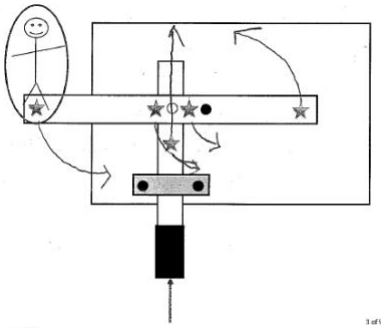
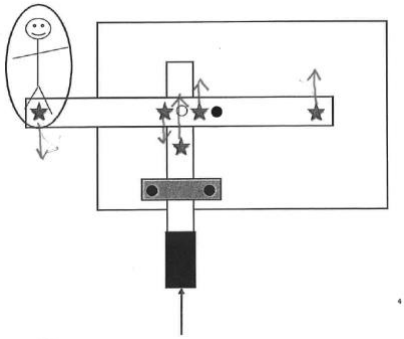
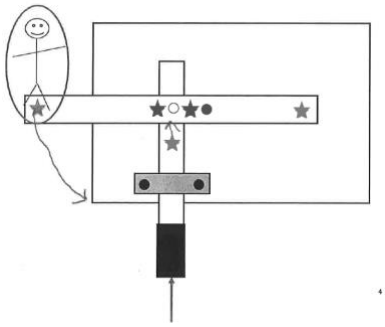
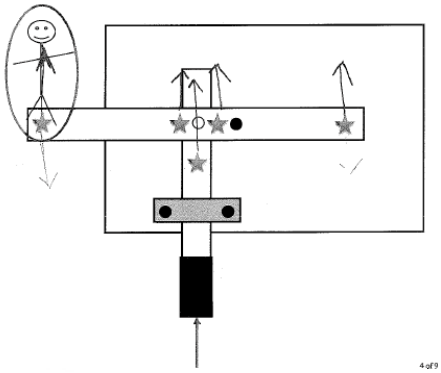
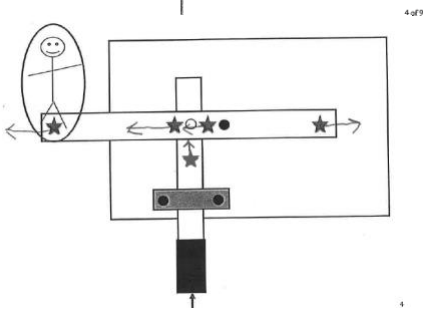


Figure A. Item Sequential Tracing A1 (STA1).

Table A.
Exemplar for Sequential Tracing A1 (STA1).

Level	Mechanistic Element	Mechanistic Element Descriptions	Mechanistic Element Example
5	Tracing	Participant diagnoses all mechanistic elements (without gaps) from input to output.	
4	Constraint via the Fixed Pivot	Participant draws the opposite motion of the two closest points on opposite sides of the fixed pivot.	
3	Lever Arms	Participant draws arrows with opposite directions from stars on opposite sides of a lever's arms.	
2	Rotation	Participant draws arced paths. However, the location of these paths must reasonably approximate fractions of circles either centered around the fixed or floating pivot. *Note: Although these paths are centered around the fixed pivot,*	

		this element of mechanistic reasoning does not make this distinction.	
1	Related Direction	Participant draws the coordinated input/output motion.	
0	Student Diagnoses No Mechanistic Elements	No mechanistic elements are shown.	
NL	No link	It is not clear if the participant understood the nature of the task.	"I don't know"
M		Missing Response	

Note. This item assesses students' ability to diagnose the mechanistic elements of *related direction*, *rotation*, *lever arms*, and *constraint via the fixed pivot* as well as *tracing*. No link (NL) indicates an item response that does not provide any evidence of mechanistic reasoning (i.e., diagnosis of no mechanistic elements). "Missing" indicates that the item was left completely blank. The "stars" have been placed on the levers to allow participants to indicate lever motion. A "little person" has been included on the output lever to make the system output salient.