# A Population Dynamics Model for Gender Diversification in Orthopaedic Surgery: A Case Study with Relevance to Engineering 

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# A SIMPLE MATHEMATICAL MODEL FOR GENDERDIVERSIFICATION IN ORTHOPAEDIC SURGERY: A CASE STUDY WITH RELEVANCE TO ENGINEERING 

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## Introduction

Despite widespread effort over the past decade on the part of K12 and university educators, the private sector, and state and federal agencies, women are still underrepresented at the undergraduate level in some STEM disciplines, such as physics, mathematics, and mechanical and electrical engineering, while simultaneously equally or even overrepresented in fields such as biosciences, environmental science, and biomedical engineering [1]. This unequal distribution of female talent persists, despite increased awareness and achievement in STEM amongst high school age women [2]. In many respects, this situation is similar to the medical profession, where women are entering and completing medical school at equal rates to their male colleagues, but they are concentrated in specific specialties, such as pediatrics and family medicine, while sparse in others [3-7]. Orthopaedic Surgery is one of the least gender diverse medical specialties, with $4 \%$ women in practice and $14 \%$ in residency [5], which is a 5 -year apprenticeship following medical school. The affected population is relatively small, with approximately 28,000 practicing orthopaedic surgeons and 670 new surgeons licensed annually [6].

Since 2009, our group has conducted targeted outreach and mentoring efforts amongst the high school, college, and medical school population to improve gender diversity in the orthopaedic surgeon population. The program directly reaches upwards of 1,700 young women annually at program locations across the country [8], with approximately 1,300 high school program participants and 400 medial students. Many more students are impacted through informal programming and in-school curriculum sponsored by our organization. Similar to gender diversification of STEM majors in college, there is a long lead-time between intervention at the K12 level and the desired impact on the field in terms of recruitment of female talent into orthopaedic surgery. Specifically, if the intervention is targeted at high school juniors or seniors, there will typically be a 9 to 12 year lag time before these students matriculate to residency, with 1-2 years to complete high school, 4-5 years to complete college, and 4-5 years for medical school.

There are numerous, well-documented "leaks" in this pipeline towards orthopaedic residency, the first of which occurs at the transition from high school to college with only $15 \%$ of the college bound female population choosing STEM majors [13]. From there, only 3\% of female STEM majors in college matriculate to medical school [1,7], and $0.9 \%$ of female medical students apply to orthopaedic residency [9,10]. Studies have shown that key time points for
exposure to maintain the orthopaedics trajectory are junior and senior year of high school [8] and during the first and second years of medical school $[9,10]$.

To track the progress of our organization towards our longitudinal goal of $30 \%$ females in the orthopaedic residency population - generally considered to be the target for self-sustaining minority populations [11] - we have developed a simple mathematical model that can be used to predict the timing and magnitude of the effect of our intervention on diversity in the field. Furthermore, we can use this model to understand and modify the various elements of our outreach programs, including the volume and age group composition of program cohorts, to maximize student "yield" at the terminal point in the pipeline. Taking a broader perspective, we assert that this type of mathematical model may be applicable to diversification efforts for individual colleges' engineering programs, which frequently offer a similar portfolio of outreach initiatives.

## Methods

We created a simple mathematical framework, inspired by commonplace models in population dynamics [12], to model gender diversity in the orthopaedic residency pipeline. A discrete, annual time-step approach was used to predict the annual percentage of females in the incoming residency class $\left(P_{f}^{i}\right)$. This outcome is a function of: (1) the recruitment and retention rates of our high school outreach program (The Perry Outreach Program, or POP); (2) the recruitment rates of our medical school program (The Medical School Outreach Program, or MSOP); and (3) the rate of females pursuing orthopaedic residency unaffected by our outreach programs, which can be assumed to be a constant $14 \%$ based on historical data [5]. The discrete, annual time-step approach allowed us to account for two important effects: (1) the time delay between the program intervention and matriculation into residency for different cohorts, e.g., 1-2 years for medical students (MSOP) and 9-10 years for high school students (POP); and (2) the exponential growth of both the high school and the medical school programs (Figure 1).


Figure 1: Growth in enrollment in our outreach programs. Perry Outreach Program (POP) is the high school program; and Medical School Outreach Program (MSOP) is for medical students. All participants are female.

The mathematical model is defined in Equations 1-4 below, and model variables and parameters are defined in Tables 1 and 2. Values for model parameters were determined from published program evaluation results $[5,6,8]$. Enrollment numbers in our outreach programs were based on historical data for 2009-2015 (Figure 1, [8]); and 2015 enrollment numbers were assumed for all future years, which is a conservative assumption.

$$
\begin{array}{ll}
\text { Eq. } 1 & P_{f}^{i}=\frac{N_{f}^{i}}{N_{T}} \\
\text { Eq. } 2 & N_{f}^{i}=\alpha_{0} N_{T}+V_{M S O P}^{i}+V_{P O P}^{i} \\
\text { Eq. } 3 & V_{M S O P}^{i}=\alpha_{M S O P} \frac{1}{2}\left(N_{M S O P}^{i-2}+N_{M S O P}^{i-1}\right)\left(1-\beta_{M S O P}\right) \\
\text { Eq. } 4 & V_{P O P}^{i}=\alpha_{P O P} \beta_{P O P} \gamma_{P O P} \frac{1}{2}\left(N_{P O P}^{i-10}+N_{P O P}^{i-9}\right)
\end{array}
$$

Table 1: Variables calculated by mathematical model. MSOP is medical school outreach program; POP is high school. Refer to Equations 1-4 above.

| $P_{f}^{i}$ | Percentage of females in incoming <br> orthopaedic residency class, $\mathrm{i}^{\text {th }}$ year | $N_{M S O P}^{i}$ | Number of alumnae from MSOP <br> program for year (i) |
| :---: | :--- | :---: | :--- |
| $N_{f}^{i}$ | Number of females in incoming <br> orthopaedic residency class, $\mathrm{i}^{\text {th }}$ year | $N_{P O P}^{i}$ | Number of alumnae from POP <br> program for year (i) |
| $V_{M S O P}^{i}$ | Number of alumnae from MSOP <br> program entering orthopaedic | i | Calendar year, e.g., 2014 |
| $V_{P O P}^{i}$ | residency, $\mathrm{i}^{\text {th }}$ year <br> number of alumnae from POP <br> program entering orthopaedic <br> residency, $\mathrm{i}^{\text {th }}$ year |  |  |

Table 2: Model parameters with specified values. Refer to Equations 1-4 above. . MSOP is medical school outreach program; POP is high school. Parameter values for "Baseline Value" are based on published data and program evaluation data $[5,6,8]$. "Worst-Case Value" assumes worst-case values for select parameters for recruitment and retention in orthopaedic residency pipeline.

| Symbol | Description | Baseline <br> Value | Worst-Case <br> Value | Reference |
| :---: | :--- | :--- | :--- | :--- |
| $N_{T}$ | Total number of incoming orthopaedic <br> residents | 670 | 670 | $[6]$ |
| $\alpha_{0}$ | Percentage females in orthopaedic <br> residency, unaffected by our outreach <br> interventions | $14 \%$ | $14 \%$ | $[5]$ |
| $\alpha_{M S O P}$ | Percentage MSOP alumnae who are <br> "Very Interested" in pursuing orthopaedic <br> surgery after attending MSOP program | $30 \%$ | $20 \%$ | $[8]$ |
| $\beta_{M S O P}$ | Percentage MSOP alumnae who are <br> "Very Interested" in pursuing orthopaedic <br> surgery a priori the program | $57 \%$ | $67 \%$ | $[8]$ |
| $\alpha_{P O P}$ | Percentage POP alumnae who matriculate <br> to 4-year college and major in STEM | $93 \%$ | $93 \%$ | $[8]$ |
| $\beta_{P O P}$ | Percentage POP alumnae who intend to <br> attend medical school | $56 \%$ | $56 \%$ | $[8]$ |
| $\gamma_{P O P}$ | Percentage POP alumnae who are "Very <br> Interested" in pursuing careers in <br> orthopaedic surgery | $23 \%$ | $13 \%$ | $[8]$ |

We used our mathematical model (see Equations 1-4) to conduct two unique simulations addressing critical issues for our programming efforts. First, we modeled the long-term effect of our programming efforts on the percentage of females in the incoming residency class assuming that we maintain our current level of programming indefinitely. We then adjusted select parameters in the model to reflect "worst-case" assumptions for recruiting and retention in orthopaedic residency (see Table 2). This analysis is particularly important because our current program evaluation data - on which the model parameters are based - reflect intermediate recruitment and retention outcomes, e.g., intention to pursue orthopaedic surgery for high school participants rather than actual matriculation rates into residency. Our worst-case assumptions were as follows: (1) $10 \%$ decrease in percentage of POP alumnae who are "very interested" in pursuing orthopaedic surgery $\left(\gamma_{P O P}\right)$; (2) $10 \%$ decrease in percentage of MSOP alumnae who are "very interested" in pursuing orthopaedic surgery after attending the program ( $\alpha_{M S O P}$ ); and (3) $10 \%$ increase in MSOP alumnae who are "very interested" in pursuing orthopaedic surgery a priori the program $\left(\beta_{M S O P}\right)$. The justification for these assumptions being "worst-case" is that the current conversion rate of POP alumnae into orthopaedic surgery ( $\gamma_{P O P}$ ) may be an over-
estimation, as it is based on intermediate (short-term) data because long-term data is not yet available due to the time delay in this cohort matching into orthopaedic residency. For the medical school program, the worst case assumptions would be that a priori interest in orthopaedics ( $\beta_{M S O P}$ ) will increase, most likely as a result of our high school program and that there may also be a decrease in the overall yield of the MSOP program ( $\alpha_{M S O P}$ ) as our intervention reaches full scale.

Our second simulation with the mathematical model investigated the effect of program duration, that is, how much longer that our organization continues to conduct outreach programs at our current scale ( 1300 high school students, 400 medical school). We are a small non-profit organization that relies on corporate and individual charitable giving for our operational budget as well as countless volunteer hours to conduct programming. While we are fiscally healthy and have a sustainable infrastructure, our organization was purposefully designed to address a specific issue, namely, underrepresentation of women in orthopaedics, and there has always been the long-term intention to "put ourselves out of business" by reaching a self-sustaining target percentage of $30 \%$ for women in orthopaedics [11]. We simulated the long-term effect of our programming efforts should we immediately cease programming efforts ("+0 years," ending in 2015), continue with programming for another 5 yrs ("+ 5 yrs," until 2020), and continue with programming for another 10 yrs ("+ 10 yrs," until 2025). Again, enrollment was assumed to be equal to 2015 numbers for future years.

All simulations were conducted in commercial software (MS Excel 2010, with preliminary results validated in Matlab R2016a). The primary outcome measure was percentage female in the incoming orthopaedic residency class by residency year $\left(P_{f}^{i}\right)$. Data were analyzed via graphical representation (\% female by year) and descriptive statistics.

## Results

The results of our mathematical model (Figure 2) suggest that our outreach programming efforts will increase the rate of women in orthopaedic residency to the critical threshold of $30 \%$ [11] within the next 6 years (by 2022) and eventually reach a plateau of $45 \%$ female within 10 years (by 2025). If we assume worst-case conditions - that is, we underestimate the rates of matriculation from our program into orthopaedics - we will achieve $30 \%$ female within 10 years, and this will be the approximate steady state value (see Figure 2).


Figure 2: Results of our mathematical model showing historical ( $<2015$ ) and projected (2016+) female enrollment in the orthopaedic residency class. "Baseline" represents model with parameter values reflecting our current program evaluation results. "Worst-Case" reflects worstcase assumptions for parameter values in terms of recruitment and retention in the orthopaedics pipeline.

Considering the duration of our programming efforts (Figure 3), if we were to cease all programming immediately (2015), our past programming efforts would yield a peak diversity of $27 \%$ female in 10 years (2025) before declining back to the $14 \%$ baseline within 12 years (2027). Similarly, if we were to continue programming only for 5 more years (until 2020), we would expect an identical peak of $27 \%$ female within 10 years. This peak would be sustained for 5 years (until 2030) before declining back to baseline. With 10 more years of programming, we will achieve the critical $30 \%$ threshold [11] within 10 years (2025); it will be sustained for approximately 10 years; and there will be a more gradual decay to baseline within 22 years (by 2037).


Figure 3: Results of simulation using our mathematical model that investigates the effect of differing durations of our outreach program intervention. " +0 yrs" assumes that programmatic activity ceases in 2015; " +5 yrs" assumes it continues until 2020 and stops; and " +10 yrs" assumes it continues until 2025. In all cases, program enrollment numbers for 2015 were continued into future years.

## Discussion

The results of our mathematical model suggest that our outreach efforts will have a substantial and sustainable impact on gender diversity in orthopaedics within the next 6 years. By 2022, we expect that we will achieve $30 \%$ female in the residency population, an accepted critical threshold for maintaining minority populations within professions [11]. Even with worstcase assumptions for our recruitment and retention results, we would nearly achieve this critical threshold by 2025. Again, planning for worst-case conditions, if we were to cease all of our programming efforts after 5-10 years, we would still achieve at or near 30\% female for a period of time (5-10 years) before the effects of our intervention wear off. This may be enough time for the culture of the field to shift enough, i.e., orthopaedics seen as more "female friendly" by medical students, to have a permanent effect on gender diversity without continued intervention.

The effectiveness of our programming efforts suggested by this mathematical model can be attributed to the scale of our programming efforts, rather than particularly high rates of recruitment into the pipeline. Presently, we reach 1,700 students annually, the majority of which $(76 \%)$ are high school students. Approximately $93 \%$ of these students major in STEM at a 4 -year university, with $56 \%$ intending to go to medical school and $23 \%$ being "very interested" in pursuing orthopaedics [8]. This gives us a compound recruitment rate of 7\% for our high school program. This means that the success of our program is build less on a high recruitment rate and more on a large number of program participants relative to the desired yield of the program. To provide a sense of scale, the increase from $14 \%$ to $30 \%$ female with an average residency class of 670 represents a gain of 107 individuals, which is definitely achievable given that we run a nationwide outreach program that reaches nearly four times this population size annually. The scale of our program also explains why there is mitigated sensitivity to program effectiveness,
specifically, that a $10 \%$ decrease in our high school program "yield" still results only a 3 year delay in achieving nearly $30 \%$ female in the residency population.

Our results from this case study of the orthopaedic surgery profession are particularly relevant to diversification efforts for individual colleges' engineering programs. Cohort sizes for most incoming engineering classes are similar to orthopaedic residency (on the order of $10^{\wedge} 3$ ), and multiple outreach efforts may exist at any given university with the mission to affect gender or racial diversification. We present our simple mathematical model inspired by commonplace models in population dynamics [12], as a straightforward method for gauging the potential impact of such outreach efforts on student body composition. Outcome measures, such as time to achieve a particular gender and racial composition for an engineering class, may be obtained by specifying the recruitment rate for each outreach program, with an appropriate time delays for the developmental age of each outreach population. We advocate that engineering outreach efforts at the college level can be "re-engineered" with the guidance of this simple model to reach desired diversity targets

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