



A Practical Guide to Graphical Statics

Casey Allen Halbmaier, University of St. Thomas

Casey Halbmaier is a junior mechanical engineering student at the University of St. Thomas in St Paul MN.

Dr. Sarah Baxter, University of St. Thomas

Dr. Baxter is a Professor of Mechanical Engineering in the School of Engineering at the University of St. Thomas in St. Paul, MN. She received her PhD in Applied Mathematics from the University of Virginia School of Engineering and Applied Science.

Dalton Irving Humphrey

Dalton Irving Humphrey is a Junior Mechanical Engineering major at the University of St. Thomas in St. Paul, MN.

Dr. Bethany Fralick, University of South Carolina - Aiken

Dr. Fralick is an Assistant Professor of Engineering in the Department of Mathematical Sciences at the University of South Carolina Aiken in Aiken, SC. She received her Ph.D. in Mechanical Engineering from the University of South Carolina College of Engineering and Computing.

A Practical Guide to Graphical Statics

In previous work, the inclusion of Graphical Statics modules in undergraduate Statics class was proposed as a novel method to reinforce fundamental concepts and re-engage students via active learning. Here, Graphical Statics refers to the use of geometric constructions to visualize the solution of two-dimensional Statics problems; fundamental concepts include force and funicular polygons. The hypothesis posed was that Graphical Statics components could strengthen visualization skills and would encourage drawing as a conceptual aid.¹ The response to the talk given on this work was very positive; however, while admitting the potential for these outcomes, the audience was more interested in specific problems that could be directly inserted into the class as active learning exercises. In a companion paper,² this discussion is continued and Graphical Statics is examined, in more detail, within the framework of its role as an element of active learning and its relevance in the curriculum.

In this poster presentation, the focus is on the practical aspects of using Graphical Statics techniques. The poster will identify fundamental problems that can be solved using Graphical Statics and illustrate the visual components of the technique. A short textbook of problems has been constructed and will be offered at the poster session and via email from scbaxter@stthomas.edu. Each problem includes both the analytic and graphical solution.

In this accompanying paper, rather than emphasizing the curricular aspects of Graphical Statics, the authors identify the visual components and concepts that have emerged from the users, faculty and students, points of view.

Fundamental concepts and application

There are two major analysis tools used in Graphical Statics. The first tool is the force polygon. Force polygons can be found in most Statics textbooks in the early sections that discuss how to add vectors; the most familiar application is the force triangle resulting from 3 forces. The second tool is the funicular, or equilibrium, polygon. Force polygons deal with the balance of forces and take into consideration their magnitudes and direction. Funicular polygons keep track of the spatial position of the forces, focusing more on moment equilibrium. The underlying analysis can be related to the forces applied to a hanging cable. An excellent, recent, textbook reference for applications of Graphical Statics is Allen et al.³ While mostly aimed at Civil Engineers and Architects, the drawings, explanations and comparisons to analytic techniques are excellent. However, while the analysis is appropriate for undergraduates, many of the applications are focused on higher-level structural design. There are two, much older, references online that offer a more focused undergraduate presentation, the 1906 "Text Book on Graphical Statics"⁴, a thesis for the "Degree of Civil Engineer" (M.S.) from the University of Illinois, by Charles Wesley Malcom and the "Graphical Analysis, A textbook on Graphic Statics"⁵, by William S. Wolfe, 1921. Both books look more at forces and equilibrium than design, making them more appropriate for topics in Statics for undergraduate mechanical and civil engineering students. Additionally, a detailed explanation and illustrations of force and funicular polygons can be found in^{1,2}.

The non-student authors of this paper have included Graphical Statics in their classrooms primarily as a follow-up or extension activity. As an example, students are initially taught to analytically decompose concurrent forces into component form, assemble and solve two equilibrium equations. In a separate presentation, they are then provided incomplete information or information on more than 2 forces. If there are more than 2 unknowns, or if the resulting equilibrium equations are nonlinear, then analytic methods are cumbersome, ineffective, or result in non-unique solutions. However, graphical methods, where students draw the force vectors to scale and orientation, graphically add their sum, (tail-to-head), can often provide the solution(s) and verify equilibrium. The additional care that must be taken in drawing and measuring highlights the differences between a measured solution (experiment) and an analytic solution (mathematics). There is a potential for the drawing experience to reinforce subsequent analytic work.

Visual components and concepts

While preparing the short textbook, the faculty and student authors, who were already experienced in solving two-dimensional static equilibrium problems, began to identify some less obvious benefits to learning the techniques of Graphical Statics and drawing the solution to equilibrium problems. These included a wider range of concepts that are reinforced, ‘ah-hah’ moments resulting from completed problems, and some general observations.

Concepts that are reinforced

1. A vector with known magnitude, but unknown direction, can be visualized by the points on a circle with radius equal to the magnitude.
2. Vectors with known direction, but unknown magnitude, lie on a line of action; lines of action can be extended in both directions, infinitely.
3. Coordinate axes could be put anywhere as long as all the vectors are drawn with the same reference.
4. Scaled drawings can be scaled in between the dimensions, i.e. length to length, 1 in = 1 ft; or scaled between quantities with different dimensions, 1 cm = 100 N.
5. The question can be asked in both directions; what would make this system in equilibrium? Is this system in equilibrium?
6. Solutions to problems where the angles are unknown, rather than the magnitudes, can be easily solved graphically (these problems result in a non-linear system of equations).
7. With Graphical Statics it is easy to see why reaction forces are needed.
8. Force polygons have to do with force equilibrium – but that is not the whole story.
9. If the sum of the forces is equal to zero, then either the system is in equilibrium or there is a resultant couple.
10. Funicular polygons have to do with spatial positions of the forces and, therefore, moment equilibrium.

‘Ah-hah’ moments

1. While all of the participants clearly understand that in equilibrium the sum of the forces is zero, drawing the solution to the sum of three or more forces, and seeing the vector-loop close, was a profound visual of equilibrium.

2. Similarly, a funicular polygon, when used to calculate reaction forces; clearly shows how moving the forces, or changing their magnitudes, changes the reactions.
3. The Graphical solutions obtained were not exact, in comparison with the math.
4. The math makes more sense when you can draw it.
5. The larger the drawing the less error in your measured solution.
6. There can be a visual interpretation of different kinds of loadings; you can see patterns in the pathways of the forces.

Observations

1. The solution instructions in the older textbooks were sometimes easier to follow than the recent work.
2. Sketching emphasized the power of ‘back of the envelope’ answers, encouraged estimates, e.g., minimum or maximum resultant, and validated good guesses.
3. Students were more willing to draw and re-draw to improve, or check, their answer; they remain less willing to repeat calculator calculations.
4. Students drew better free-body diagrams (or were more willing to draw them) after using the graphical techniques on simple problems.
5. Scaled drawings helped students better understand and see proportional/relative magnitudes of forces.
6. The ideas of direction and magnitude became physically real concepts when the problem was drawn to scale.

Conclusions

In developing the set of problems to include in the short textbook, the student co-authors explored early examples of Statics textbooks^{4,5} that emphasized graphical analysis. Not only was the style of exposition distinctly different from today’s textbooks, but these texts effectively blended analytical and graphical techniques for solving engineering problems. Realizing that, in both these cases, calculators were not available to students, professors or working engineers, served to emphasize the usefulness and power of the graphical techniques. These graphical approaches, while grounded in mathematics, had a great deal of embedded visual and physical intuition.

References

- [1] Baxter, S. C., & Johnson, A., & Fralick, B. S. (2015, June), “Revisiting Graphical Statics” Paper presented at 2015 ASEE Annual Conference and Exposition, Seattle, Washington. 10.18260/p.24681.
- [2] Baxter, S. C., & Fralick, B. S. (2016, June), “Graphical Statics Redux” Paper accepted for presentation at 2016 ASEE Annual Conference and Exposition, New Orleans, Louisiana.
- [3] Allen, E., Zalewski, W., Foxe, D.M, Anderson, J., Hriczo, K. Ramage, M.H., Ochsendorf, J.A., Block, P., and Iano, J. “Form and Forces: Designing Efficient, Expressive Structures” John Wiley & Sons, Inc., Hoboken, NJ 2010.
- [4] Malcolm, C.W. “Text Book on Graphical Statics”, a thesis for the “Degree of Civil Engineer” (M.S.) from the University of Illinois, 1906.
- [5] Wolfe, William S. “Graphical Analysis, A textbook on Graphic Statics”, McGraw-Hill Book Company, NY, 1921.