



A proposal for using problem posing to connect learning of basic theory with engineering design

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Abstract

There is a need for educational methods that enable transfer of academic content to engineering practice. Such methods appear frequently in freshman and senior design courses (cornerstone and capstone courses), but not so often in basic theory courses, such as calculus, probability, and statistics. This paper presents a proposal for methods to achieve this connection, and the author's experience with using the proposed methods in applied probability courses. The essence of the proposal is that learners should be doing engineering while learning basic theory.

Introduction

The heart of engineering practice is design. Thus (ideally), engineering design should be omnipresent in the engineering curriculum, including basic theory courses. This paper discusses strategies for incorporating a design presence in applied probability courses. The content of these courses is basic probability, reliability models, Markov chains, the Poisson process, and queuing models. In a previous paper¹, it was proposed that engineering design activity be incorporated through

1. homework and test problems that emphasize parametric analysis, variations of standard models, and comparison of alternative systems;
2. loosely stated open-ended problems intended to allow creative response, with the hope that students will model the physics of the problem situation, identify economic and ethical constraints, and find ways to base decisions on quantitative analysis. These types of problems have helped students understand the nature of engineering problems and decisions. However, by themselves, they do not show students the process of transferring basic theory to engineering practice and incorporating it into the “making-of-meaning” required for addressing engineering design problems^{2,3}.

In this paper, it is proposed to augment these problems by asking the students (and instructor) to engage in problem posing and problem structuring. The goal is to suggest methods in concordance with established research about how learning occurs.

It is probably a mistake to lock into a highly structured educational model. Instead, the general principles listed after this paragraph are chosen as a guide. They are taken from Ambrose *et al*⁴. The author has taken the liberty of re-expressing the principles in terms of a joint venture between students and educators. This expresses a personal principle: the learning community includes both educators and students. Students, especially at college-age, must be co-constructors and co-maintainers of the learning environment. And, most content domains offer inexhaustible opportunities for new and deepening mastery for the instructor. Educators may be content-masters, but they are always learning more about transmission and communication.

1. Our prior knowledge can help or hinder learning.
2. How we organize knowledge influences how we learn and apply what we know.
3. Our motivation determines, directs, and sustains what we do to learn.

4. To develop mastery, we must acquire component skills, practice integrating them, and know when to apply what we have learned.
5. Goal-directed practice coupled with targeted feedback enhances the quality of our learning.
6. Our current levels of development interact with the social, emotional, and intellectual climate of the course to impact learning.
7. To become self-directed learners, we must learn to monitor and adjust our approaches to learning.

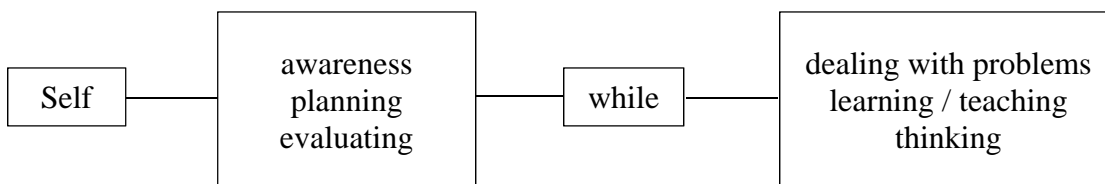
There are two educational processes that research has revealed to be essential to enabling transfer of thinking skills from one domain to another, such as from engineering classrooms to engineering workplaces. These two processes enable principles 2, 4, and 7, as discussed by Ambrose *et al.*⁴.

The processes are:

1. development of richly connected conceptual schemata,
2. development of metacognitive skills.

We must implement these principles and processes for particular content domains and professional activities. In this paper, the content domain is applied probability and the professional activity is engineering design. Learners need to develop conceptual schemes for applied probability itself, but also for the nature of problems. They should understand structuring problems and posing problems. They should be informed that there is a spectrum of problems, ranging from well-structured problems with definite answers and clear boundaries, such as are found in traditional textbooks (and nowhere else), and open-ended, ill-structured problems, such as are found in the engineering workplace. The essential and unique point is that learners must pose, clarify, and define problems, not simply solve them.

And, at the same time, learners should practice metacognitive skills such as reflecting on how they are building these schemes. Metacognitive activities are manifold and not easy to classify. However there is widespread agreement that consciously developing metacognitive habits significantly increases the ability to transfer academic learning to the workplace⁵⁻⁹ (The references given are only a small representative sample.). This diagram summarizes metacognition:



The rest of this paper focuses on problem posing and problem structuring, with little explicit mention of metacognition. However, thinking about the essential qualities of problems is reflective in nature, and hence metacognitive. Here is an example:

This week, I am choosing not to detail a specific problem, but rather discuss an overall problem I'm having with a certain area.

When we, as a class, are assigned certain problems, I am usually confident that I can solve them. But a few times this semester, some problems have caused concern on my part. The design problems and even certain regular assignments are those that I am eluding [alluding?]

to. I am not sure that my answers were incorrect, but I know I didn't learn anything by attempting to solve it. From talking with other students and from listening to your explanations, I learned that the answers were not really answers at all. For instance, with the design problem due on wed, the only answer I could come up with was a pair of inequalities. This gave me a range of answers with 2 and sometimes 3 variables at stake. It may be that I just don't know how to solve these problems, but I do know that from talking with other students, I am not alone with my feelings. I just am not learning anything from a problem that seems unanswerable. I know that not every problem has a distinct answer, but the few problems I am talking about seem to have infinite answers and don't seem to relate in the real world in any way.

This example displays a student's mild trauma when encountering ill-structured problems. In addition to helping the student, it shows the instructor that students may need fostering when presented with these types of problems.

Instructors Posing Problems

Most textbook problems posit a sharply defined initial state of knowledge from which the solver must move to a definite terminal state of knowledge. For engineering and mathematics textbooks, the most common task for the solver is to produce a number, a formula, or a set of formulas. For these tasks, there is only one correct answer. Such problems are very important in mastering a discipline, just as playing scales is important to mastering a musical instrument. But, also playing simple tunes will enrich the musical experience. In the same way, as stated in the introduction, instructors can enhance basic theory courses by constructing problems that are micro versions of engineering design problems. Here is an example:

Currently, a system is composed of two independent units connected in series with probabilities p_1 and p_2 of working. The cost of system failure is $\$C_f$. The following options are available:

- (1) A component identical to component 1 may be added in parallel with component 1 at cost C_1 .
- (2) A component identical to component 2 may be added in parallel with component 2 at cost C_2 .
- (3) Both a component identical to component 1 may be added in parallel with component 1, and also a component identical to component 2 may be added in parallel with component 2. The cost of adding these two components is $\$C_B$.
- (4) The system may be left as it is.

1. Choose numbers for p_1 , p_2 , $\$C_1$, $\$C_2$, $\$C_B$, and $\$C_f$. Then answer the following questions, stating any assumptions you feel forced to make:
 - Which of options 1 and 2 adds more reliability to the system?
 - Which of options 1 and 2 is preferred?
 - Which of options 1 through 4 is preferred?
 - If improvements must be made over time, which improvement should be made first?

2. Answer the following questions:

- Under what conditions does option 1 add more reliability to the system than option 2?
- Under what conditions is option 1 preferred to option 2?
- Under what conditions, if any, is each of options 1 through 4 preferred?
- If improvements must be made over time, give rules for deciding which improvement to make first.

Such problems are rare, and few of them engage learners in comparing alternatives in terms of performance measures. Even fewer introduce constraints such as economic factors, safety, reliability, aesthetics, ethics, and social impact. This is very different from discussing abstract theory and then presenting engineering examples.

Inspiration for such problems can come from considering issues involved in designing, maintaining, and improving human-technology systems. These include health care delivery, public health policies, sickness prevention, health education, energy, city management, environmental stewardship, quality control, inventory management, supply chains, workplace design, factory design, service delivery systems, and emergency room care. Also, since engineering design touches almost every aspect of daily life, ideas can be found in current news articles and in repositories such as The Probability Web (www.prob.berkeley.edu).

Discussing the Nature of Problems

As the first step in going beyond problems like those described in the previous section, the nature of problems is discussed. The learners that they will find at least four things in common textbook problems: **settings**, **scaffolding**, **challenges**, and **targets**. This language for problem anatomy is not universal and is possibly incomplete, but it provides a simple and useful way to talk about problems in the classroom.

A **setting** is the housing of the problem, the story in which the problem lives. It usually contains constant numbers, well-defined variables, and concepts that have recently been presented. Stories enhance learning by creating links between the learner's state of being and the concepts to be learned. Even a superficial link to daily life and/or engineering practice will deepen the learning. For most of us, it is difficult to connect to completely abstract problems without help, because we cannot see a purpose in finding the solution.

It is hard to judge the effectiveness of a setting. I have been surprised at positive reception of settings about break dancing, the intellectual adventures of octopi, the trials of university life, and the behavior of dysfunctional professors and students. Other useful types of settings are small-scale but life-like civil and mechanical engineering problems (as in Ang and Tang¹⁰) and mathematical models of scientific data (as in Olkin, Gleser, and Dynkin¹¹).

Scaffolding is material that the problem writer has included to help the solver. It takes several forms. It may be a suggested sequence of steps that will lead to the solution, or a ladder of component tasks that are connected and help the solver step through the production of requested

results. It may be chattiness, comfort-speech, weak academic humor, or attempts to make connections to a young person's sense of humor, curiosity, wonder, honesty, dignity, social life, future career, and/or passion for reform. It may call attention to the key concepts and tools needed to solve the problem. It may be embedded in the statement of the required tasks.

A **challenge** is the statement of what the solver is required to produce. It may be formulated with or without scaffolding.

A **target** is what the problem poser hopes will be accomplished when a learner engages with the problem. This could be the exercise or development of any intellectual skill, from fundamental abilities such as remembering concepts and building connections between them, to more complex activities such as deepening of intuition, creativity, and critical thinking. Here is an example of a problem dissected into these components.

A setting for an imaginary production process

A factory has a production area devoted to making wax blocks. Sometimes, when a wax block is unwrapped and put to use, it turns out to cause problems. Sometimes a wax block melts too fast. Sometimes a wax block crumbles away into powder. Wax blocks without either one of these two problems are Usable.

This production area has two shifts. Data has been kept on the performance of the shifts.

Shift number One produces 30% of all the wax blocks made.

Shift number Two produces the rest of the wax blocks.

The probability that a wax block will crumble away AND that it was made by shift number Two is known to be 0.01.

The probability that a wax block will be Usable AND that it was made by shift number One is known to be 0.27.

The probability that a wax block will crumble away is 0.02 (regardless of which shift made it).

The probability that a wax block will melt too fast is 0.03 (regardless of which shift made it).

A challenge that could be used with this setting

- a. Fill in the missing probabilities in the following table:

| | | |
|------|--|--|
| 0.01 | | |
| | | |

- b. Calculate the probability that a wax block will either crumble away OR melt too fast when it is put to use.
- c. You have been told that a particular wax block was not Usable, but not the reason that it was unusable. Calculate the (conditional) probability that the wax block melted too fast (given that it was unusable).
- d. Calculate the following three probabilities: the probability that a wax block will crumble away given that it was made by shift one, the probability that a wax block

will melt too fast given that it was made by shift one, the probability that a wax block will be Usable given that it was made by shift one.

- e. Suppose a wax block has just crumbled away. Given this information, calculate the probability that the block was made by shift two.
- f. Name at least one additional probability that you can calculate using the box in a.

Scaffolding

In the statements of the setting and the challenge, the capitalized OR's and AND's are intended to remind the solver that events are being considered and reinforce the connection between abstract concepts (unions and intersections) and informal probability language. The solvers are reminded more than once that conditional probabilities are requested. All of task a is scaffolding. It is a “warm-up” exercise that gives a step-up to the rest of the problem. More scaffolding could have been provided by labeling the rows and columns of the box. To make the challenge more challenging, task a could have been omitted. Successfully completing the second task produces information that can be used in completing the rest of the problem. The third task and fourth tasks require recognition that a probability conditioned on information is required and provide indirect and direct laddering for the fifth task. These tasks are easy if the box is used.

Targets

The goal is for the solver to learn to manipulate core probability concepts and see that they can be connected to industrial processes. There is also a slight hint about how probabilities might be assessed in practice.

The problem focuses on sample spaces, composite events, joint events, partitions, conditional probability, Bayes' theorem, and a tool for organizing probability information. The first and second tasks require recognition that the sample space can be partitioned in two ways and that four of the box's sub compartments satisfy the definition of the event whose probability is requested. This is meant to reinforce the concept of a union of events. The third task and fourth tasks require recognition that a probability conditioned on information is required.

The fifth task requires exercising Bayes' theorem directly or using the box as a tool to aid the calculation.

A working design engineer will never encounter a problem where the setting, scaffolding, challenges, and targets are clearly evident. For real problems, structure must be created. This involves clarifying more than the type of information typically given in traditional textbook problems. The current conditions and knowledge, the new knowledge needed, the resources needed, the solution method, and the nature of the terminal state must all be defined and clarified. The process may not be orderly all the way through a project, but when the time allotted is almost over, a successful project must produce something that can be clearly displayed. The nature of the end-product is not known when the problem-solving activity starts. There may be many acceptable solutions and most of them will not be applications of a previously encountered formula.

The problem posing and structuring strategies described in the next section are intended to give learners experience that, with proper reflection, can be transferred to engineering practice. As a byproduct, the problem structuring activities will encourage students to participate in design and construction of the learning environment. The essential point is that students should be encouraged to pose, clarify, and define problems, not simply solve them, and that this activity should be coupled with metacognition.

Learners Posing Problems

The inspiration for problem posing is a delightful book by Stephen Brown and Marion Walter called *The Art of Problem Posing*¹². They recommend producing a new problem by addressing an already formulated problem, interacting with it, and then making changes or additions. This opens up new avenues of thought that can lead to opportunities for exploration, deeper insight, better understanding, and simple joy.

Learners and instructor can take this general idea in any direction they wish. However, some recommendations should be given. Here are some recommended suggestions:

1. Make a slight structural change (an **ε-change**) to the story in the setting. For example, suppose the story is about a simple magazine delivery system with magazines delivered from a central supplier to independent sellers at isolated points in a large city. The story can be changed so that the sellers can communicate with each other and possibly share resources. This change will trigger adjustments to the setting, scaffolding, and challenge.
2. Make a substitution in the setting (a **σ-change**). This may (or may not) force other parts of the problem to change in interesting ways. For example:
 - a. Take any number and change it to a parameter or variable.
 - b. Take a parameter or variable and change it to a number.
 - c. Take a number and change it to a different number.
 - d. Remove the challenge from one problem and replace it with the challenge from another problem.
 - e. Substitute a frivolous context with a serious one.
 - f. Substitute a serious context with a frivolous one.
 - g. Substitute a system of one type with a system of another type --- either simpler or more complex.
 - h. Substitute an entity in the problem with a different entity. For example, a member of the class of 2012 suggested that a problem about automobiles (admittedly dull) be changed into a problem about sea gulls. This led other members of the class to incorporate ecological issues into reformulations of the problem.
3. Add engineering design concerns to the problem (**(ed)-changes**). Important themes in engineering design are policy formulation, policy evaluation, design changes, parametric analysis, definitions of boundaries, and incremental improvement.
4. Add missing elements of human-technology systems to the problem (**(ht)-changes**). For example, in a magazine delivery system, add the concerns and needs of magazine buyers, if they are not already present.
5. Imitate the setting in another context (an **i-change**). For example, take a story about monitoring a nuclear power plant and write a parallel story about monitoring an oil delivery system.

For example, the completed worksheets following this paragraph could be used to guide the first steps in such an activity. In the worksheets, the setting for the problem about wax blocks is broken into two parts, which are addressed separately. The first part creates an imaginary production process in which many different things could happen. The second part, targeted toward a particular content area, details possible aspects of the process. The settings are reproduced in the first columns, with possible responses and interactions in the second columns.

| <p style="text-align: center;">A general setting for an imaginary production process</p> | <p style="text-align: center;">Comments/Analysis</p> |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>A factory has a production area devoted to making wax blocks. Sometimes, when a wax block is unwrapped and put to use, it turns out to cause problems. Sometimes a wax block melts too fast. Sometimes a wax block crumbles away into powder. Wax blocks without either one of these two problems are Usable.</p> | <ol style="list-style-type: none"> 1. Is it a good story? Is the imaginary process realistic? <ul style="list-style-type: none"> • Is there such a thing as a wax block? • If so, is wax block production rare or commonplace? • Are wax blocks defective in the ways described? • Are there other problems with wax blocks? 2. Can the boundaries of the story be extended? <ul style="list-style-type: none"> • Are wax blocks produced in varieties, such as different colors? • What is the purpose of the wax blocks? • What raw materials are required? • What lies outside the boundaries of the production area? • What lies outside the boundaries of the factory? • What engineering design concerns can be imagined? 3. What are the attributes of the setting? What if they were contradicted or changed? <ul style="list-style-type: none"> • The story is about a substance (wax). • What if the substance were chocolate instead of wax? • <i>The Materials Science of Chocolate</i> Peter Fryer and Kerstin Pinschower, <i>MRS Bulletin</i>, December, 2000, pages 25-29. • <i>The Science of Chocolate</i>, S. Beckett, Royal Society of Chemistry, 2000. 4. For what content areas can the story be used, and how? <ul style="list-style-type: none"> • sequential events? • counting distributions? • lifetime distributions? • system dynamics? |

| A subsetting | Comments/Analysis |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>This production area has two shifts. Data has been kept on the performance of the shifts.</p> <p>Shift number One produces 30\% of all the wax blocks made.</p> <p>Shift number Two produces the rest of the wax blocks.</p> <p>The probability that a wax block will crumble away AND that it was made by shift number Two is known to be 0.01.</p> <p>The probability that a wax block will be Usable AND that it was made by shift number One is known to be 0.27.</p> <p>The probability that a wax block will crumble away is 0.02 (regardless of which shift made it).</p> <p>The probability that a wax block will melt too fast is 0.03 (regardless of which shift made it).</p> | <ol style="list-style-type: none"> 1. What could be changed? <ul style="list-style-type: none"> The number of shifts. The number of days. The probabilities. The classification of defects. The complete observability of the defects when a block is unwrapped. 2. What could be added? <ul style="list-style-type: none"> Parametric analysis. Probabilities for more than one wax block. Costs. Performance criteria. Random variables for counts. |

After completing the worksheets, the students should be asked to use the wax block problem as a starting point for creating new problems. Additional helps could include specific suggestions for posing new problems, collections of settings without scaffolding and challenges, and commentary on well-posed problems accompanied by commentary on their purposes, ways to personalize them, and their connections to applications. Learners would then have raw material for creating targets, scaffolding, and challenges.

But above all, with or without suggestions and examples, learners should be encouraged to assume independence, not listen and read passively, and remember that they are not necessarily required to solve their constructed problems. Ideally, the entire class or a small group should discuss them and after they are refined, produce a short booklet or blog that explains how they would be helpful. After learners have practiced all the above, they should be challenged with ill-structured problems and experience the way a working team of problem solvers might respond to them (not necessarily completely solve them).

Summary

This paper is based on the idea that educational material should be designed to increase the probability that engineering students will be able to transfer basic theory to engineering design and problem solving. Engaging learners in problem posing and explicit discussion of the nature of problem solving processes is proposed as a way to achieve this.

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