

A simple, concrete, and effective teaching method suitable for online courses

Jan Cannizzo, Stevens Institute of Technology

Jan Cannizzo, PhD is a Teaching Assistant Professor in the Department of Mathematical Sciences at Stevens Institute of Technology. His research interests include group theory and dynamical systems, as well as mathematics education.

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Abstract: We report on the adaptation of a flipped-classroom teaching method due to David Pengelley. Our main contribution is the development of a teaching strategy for managing class time in synchronous online courses, which have surged in number in the wake of the coronavirus pandemic. Among our findings are that Pengelley's method is particularly simple to implement in online courses and is a significant improvement over online lectures.

Introduction

In the wake of university lockdowns put into effect in March 2020 due to the coronavirus pandemic, thousands of instructors in the United States and throughout the world have been forced to teach their courses online. On the one hand, this has posed myriad challenges for instructors, many of whom have no prior experience with online teaching. On the other hand, we believe it is a unique opportunity to transition from a traditional lecture-based teaching method to evidence-based practices that promote *active learning* which, according to a large and growing body of research, leads to deeper learning than listening to a lecture. See, for instance, [1], [2], and [3].

Following an underwhelming online teaching experience in the spring of 2020, we looked for ways to improve our upcoming summer and fall courses (which were also to be taught online) by incorporating more active learning into class periods. At some point, we discovered a paper of Pengelley [4] in which he describes a particularly simple *flipped classroom* teaching method that he developed for in-person mathematics courses over many years. Broadly speaking, a flipped classroom has students make first contact with new material outside of class while devoting the large majority of class time to active learning activities (cf. the definition given by Talbert [5]). Inspired by the teaching method described in [4], which we shall henceforth call *Pengelley's method* and which Pengelley refers to as his *ABC method*, we thought about whether it might be suitable for online courses.

The purpose of this paper is to report on our experience in adapting Pengelley's method to two synchronous online mathematics courses. One of the great advantages of the method, as we see it, is that it is particularly easy to implement. Indeed, we were able to adapt the method to a course that we had previously taught using a lecture-based approach with only a few days to prepare. As such, we believe that the method has potential for widespread adoption, especially among instructors looking to improve the quality of their online courses.

Our main contribution is the development of a teaching strategy for managing class time in synchronous online courses in accordance with Pengelley's method. Instead of having students discuss and refine their pre-class work in small groups, which can be difficult to implement effectively in an online classroom, we devised a method for discussing pre-class work as a class, with plenty of active involvement among students and whereby students were the drivers of the discussion.

Our findings are that Pengelley's method can be simple to implement in synchronous online courses and that it is a significant improvement over online lectures. We also give a general description of Pengelley's method that explains how it may be applied to many courses (not just

mathematics courses), discuss observations from our online classrooms, and discuss the issue of academic integrity, which is a concern for many instructors.

A general description of Pengelley's method

Although Pengelley developed his teaching method for mathematics courses, we believe it is suitable for courses on a variety of subjects, notably in STEM. In particular, we believe Pengelley's method is suitable for courses that meet the following conditions:

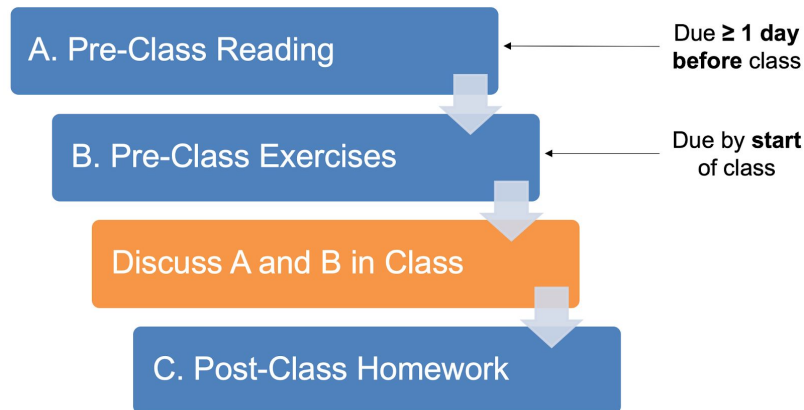
1. Most topics in the course are associated with reading material, e.g. a (section of a) textbook, a scholarly paper, an essay, or high-quality lecture notes.
2. Students are given regular homework assignments that consist of exercises spanning a range of difficulty levels.
3. The course meets synchronously (i.e. during a scheduled block of time) on a regular basis.

What follows is a general description of Pengelley's method that assumes only that a course meets Conditions 1-3 above. All of the underlying ideas are taken from [4].

Pengelley's method requires students to engage in reading, writing, and problem-solving on a topic before that topic is discussed in class. In class, lecturing is eliminated. Instead, students and instructors discuss and refine pre-class work, with the goal of achieving a thorough understanding of this material by the time class ends. Students should then be prepared to complete higher-level work, which they are required to do via a post-class homework assignment. To be more precise, the method entails the following:

1. *Before* class, students complete *two* assignments related to that day's topic:
 - a. A *reading assignment* due at least one day before the start of class
 - b. A *set of exercises* due by the start of class
2. *During* class, students discuss and refine these pre-class assignments.
3. *After* class, students complete a third, more advanced assignment related to the day's topic.

This structure is summarized in the following diagram:



All together, this work should account for the large majority of a student's grade (according to Pengelley, at least 60% of the final grade [4]). In addition, there may or may not be quizzes or exams. One goal behind such a grade distribution is to make students take seriously the day-to-day work of reading, writing, practicing, discussing, and problem-solving.

Creating assignments

To create a pre-class reading assignment, the instructor directs students to reading material associated with the topic(s) to be addressed in class. In addition, they require students to answer, in writing, a handful of reading questions that ask them to think more deeply about what they have read. Students must submit their answers to the reading questions at least one day before the start of class. The instructor may also wish to have students write down questions that they have about the material, and to indicate how much time they spent on the assignment. In our experience, reading assignments take approximately 15 minutes to create.

To create a set of pre-class exercises, the instructor should think of the homework exercises that they would usually assign to their students but select those of easy to medium difficulty. This is not to say, of course, that they shouldn't be interesting or conceptual. The number of exercises should be enough to occupy students for the bulk of one class period, since most of class time will be devoted to discussing and refining students' work on the pre-class exercises. Students' solutions to the pre-class exercises should be due by the start of class.

To create a post-class homework assignment, the instructor should again think of the exercises that they would usually assign to their students but select a small number (2-3 may be sufficient) of more challenging, higher-level exercises. These should be due well after, e.g. a day or two after, class.

Grading assignments

Pre-class work should be graded for completion and seriousness of effort only. A simple +/✓/– grading scale (where the +, ✓, and – are assigned appropriate numerical values in the gradebook) is sufficient for this purpose. Assigning a grade to a reading assignment thus takes only a few seconds. As we will explain, however, instructors need to carefully read (a good portion of) students’ reading assignments in order to prepare for class. In the course of doing this, instructors may wish to provide students with brief feedback on their work.

Grading pre-class exercises, which can wait until after class, is again very straightforward: Students’ work should be graded for completion and seriousness of effort. A +/✓/– grading scale should again be sufficient for this purpose. Since the pre-class exercises are thoroughly discussed in class, there is no need to provide feedback when grading. It thus takes seconds to grade each assignment.

Post-class homework, which is intended to reflect the final level of achievement of each student, should be graded carefully for completion, correctness, and clarity of reasoning. Providing detailed feedback is highly recommended.

Conducting class

To prepare for class, the instructor does not prepare a lecture. Instead, they read students’ responses to the pre-class reading assignment, which contain detailed information about students’ understanding of the material. They make a note of common misconceptions, questions, and particularly insightful, interesting, or creative answers. These notes will serve as the basis of a brief discussion to be held with students at the beginning of class. Since each pre-class reading assignment is due at least one day before the start of class, there is ample time to review students’ responses. For large classes, reading every student’s work is not necessary. Reading a random sample of, say, 20-30 submissions is sufficient.

The instructor begins class by leading a brief discussion based directly on students’ responses to the pre-class reading. Pengelley’s method as described in [4] dictates that, following this discussion, students be placed into small groups to discuss and refine the answers to the pre-class exercises. The instructor, in turn, should move about the classroom, interacting with different students by asking questions, providing feedback, having students present some of their work on the board, pausing to discuss a question with the entire class, etc.

In an online classroom, which is not considered in [4], it is not obvious whether class may be conducted as just described. The challenges we encountered in thinking about how to adapt

Pengelley's method to our online courses and the system we developed to address those challenges are described in the next section.

Adapting Pengelley's method to an online course

We adapted Pengelley's method to two online mathematics courses: MA 124, a fast-paced course on multivariable calculus taught in the summer of 2020, and MA 120, a half-semester course on precalculus and introductory calculus taught in the fall of 2020. MA 124 had an enrollment of approximately 20 students, and MA 120 had an enrollment of approximately 100 students who were split across three lecture sections.

In adapting Pengelley's method to our synchronous online courses, the structure of pre-class and post-class assignments was modified slightly. In MA 124, which met three times per week for two-hour periods, students were given a pre-class reading assignment and pre-class exercises for every class period. With the exception of the first week of the course and the week of the midterm exam, students were given two post-class homework assignments per week. We judged three post-class homework assignments per week—one for every class period—to be too much work to ask of students.

In MA 120, which met three times per week for 50-minute periods on Monday, Wednesday, and Friday, students were given a pre-class reading assignment and pre-class exercises for their Wednesday and Friday class periods only. Monday class periods followed a more traditional lecture format that incorporated a brief active learning activity or two—usually clicker-style questions administered as an online poll and followed by discussion. Students were given one post-class homework assignment per week. Here too we judged additional assignments (in this case, more than five assignments per week) to result in too much work for students, as well as too much grading for the instructor and teaching assistant.

Our primary challenge in adapting Pengelley's method to our online courses was determining how class time would be structured. We initially envisioned splitting students into online breakout rooms so that they could discuss their pre-class assignments in a small group, but we judged this to be problematic for several reasons:

1. Unlike in a physical classroom, in an online breakout room students may not have the means to perform calculations or draw pictures on a common space that is visible to everyone in their group. We could not rely, for instance, on students having a tablet and stylus that they could share with their peers, and we felt that a virtual whiteboard that students had to operate with a touchpad or mouse was inadequate for writing detailed calculations or drawing complex images. Asking students to write on a piece of paper and

then hold their work in front of their webcams likewise felt inadequate.

2. Unlike a group of students in a physical classroom, students in an online breakout room are cut off from the rest of the class. Depending on its makeup, a group of students in an online breakout room might feel lost, or deprived of broader discussions. With the instructor only able to spend a few minutes at a time interacting with a given group—and remaining invisible and inaudible at other times—it may be tempting for students to lapse into tangential, unproductive discussions, or to engage in no discussion at all unless the instructor is present to elicit responses from them.
3. Depending on the stability of their internet connections and their physical environments, some students may be unable to speak to their peers via a microphone or turn on their webcams, thus significantly limiting discussion within a group.

Whether the above concerns are truly warranted remains an open question to us; in any case, they felt serious enough that we decided to abandon the idea of splitting students into online breakout rooms. We instead developed an approach that entailed the following:

1. The instructor, who is equipped with a tablet and stylus, shares a virtual whiteboard with the class.
2. At the beginning of class, the instructor leads a brief discussion tied directly to students' answers to the pre-class reading questions. Some reading questions, together with some student responses (which are kept anonymous) are put on the virtual whiteboard and discussed. These questions and responses are written on the virtual whiteboard before the start of class so that discussion may begin immediately. The instructor invites feedback from the rest of the class, either via the chat or by allowing students to unmute themselves and speak.
3. Following this opening discussion, the instructor puts one of the pre-class exercises on the virtual whiteboard (to save time, pre-class exercises are also written on the virtual whiteboard before the start of class). The instructor then uses a *random name selector* to call on one or more students and invites them to unmute themselves and explain their solution(s) to the exercise. In case a student is unwilling or unable to unmute themselves, they are encouraged to provide feedback via the chat and another student is called on in their place.
4. As a student explains their solution, the instructor writes down the student's work on the virtual whiteboard for everyone to see, pausing from time to time to confirm with the student that their work has been accurately represented. Once the solution has been

written down, the other student who was called on (if there is another student) is asked to comment on the first student's solution. If they obtained a different solution, they are invited to share it and it is likewise written down by the instructor.

5. While one student dictates and explains their solution, the rest of the class is free to provide feedback via the chat. The instructor may engage in further discussion with one of the students who dictated a solution by asking them to explain aspects of their work. The instructor then invites other students in the class to unmute themselves or to respond in the chat. The instructor may ask the class questions such as "Is this solution correct?", "Has the solution been fully explained?", "Did anyone come up with a different solution?", or "Why is this step of the argument correct?" The instructor may also launch an impromptu poll in order to obtain a response from everyone in the class, especially on a question about which there seems to be significant confusion.
6. Once the solution to an exercise has been fully discussed and explained to the instructor's and the students' satisfaction, the instructor moves on to another exercise, uses a random name selector to call one or more other students, and the process repeats until class time is over.

Further remarks

The random name selector used to call students is a simple tool that is freely available online; see [6]. It has the benefit of eliminating bias on the part of the instructor when calling students, thereby ensuring that every student in the class has an equal chance of being called. In particular, it ensures that class discussions will not be dominated by the input of those students who are naturally eager to speak.

When interacting with students during class, we believed it was important to create a friendly and forgiving environment in which students felt comfortable expressing themselves and making mistakes. The instructor told students as much, emphasizing that making mistakes is a natural part of learning. If a student who was called on was shy or said they weren't sure how to begin a solution, the instructor encouraged them to contribute something even if it wasn't a complete answer, e.g. by asking a simple question or by asking the student to explain why they got stuck. The instructor made sure to thank each student who was called on after they shared their work, regardless of whether their work was incomplete or contained errors.

We also believed that it was important for students to be the drivers of the problem-solving process. The instructor therefore tried to act like a stenographer while students were explaining their solutions, writing down exactly what they said. An effort was made to resist the temptation

to immediately correct mistakes, instead giving students and the rest of the class a chance to find and correct mistakes for themselves.

Findings and discussion

Broadly speaking, our adaptation and implementation of Pengelley's method to our online courses went very smoothly. Engagement during class time increased dramatically compared with our spring online courses, and class time became more enjoyable for the instructor. The use of a random name selector seemed to keep students on their toes and, given the sizes of our class sections, ensured that 20-25% of students actively participated in discussions and problem-solving during each class. The true total was higher, as many other students participated via the chat and still others voluntarily unmuted themselves to ask and answer questions.

We felt that student preparation in advance of class was essential. Because of it, all students had a stake in the exercises discussed during class. Students who had successfully completed pre-class exercises on their own were often eager to assist their peers who had been called on but were unsure of how to complete an exercise. On the other hand, students who struggled with the pre-class exercises had more reason to pay attention and ask questions, and they were usually eager to understand the solutions. Students in general seemed to us to be more outspoken, and more inclined to ask deeper questions. Sometimes, they volunteered alternative solutions to those presented by their peers.

Given the short period of time available to us to implement Pengelley's method, we did not design a rigorous study to measure the effects of the teaching method on student learning outcomes or their general performance. Moreover, our significant changes to how students were assessed and graded made comparing their scores to the scores of students who took MA 124 or MA 120 in previous years difficult. Our general impression, however, is that students performed no worse and, especially in MA 120, quite possibly better than in previous instances of the course.

Finally, we observe that the time it took us to implement Pengelley's method, even in a course of about 100 students, was not unreasonable. In fact it was comparable to the amount of time that we had spent teaching these courses in previous years. This makes sense, since the time needed to prepare a lecture is roughly equal to the time needed to read and take notes on student responses before class, and since creating pre-class and post-class assignments essentially amounts to repackaging the homework assignments that we would otherwise be creating for students.

Academic integrity

With post-class homework assignments accounting for the largest proportion of students' grades, we were concerned about academic integrity violations, given that students might copy answers from one another or solicit solutions online. In an effort to ward off cheating in MA 120, students were told on the first day of class to abide by the following rule when completing their homework, a close variant of which is advocated by Pengelley [4]:

When writing down your solutions to homework assignments, you must do so on your own. You may never copy solutions from another source, and your work should not look like anyone else's.

This rule was also emphasized in the course syllabus, and students were told that the instructor and teaching assistant would be actively looking for violations when grading.

Among the four components (pre-class reading assignments, pre-class exercises, homework, and exams) that determined students' grades in MA 120, we found upon the conclusion of the course that students' average scores were lowest on post-class homework assignments. As expected, students' scores on pre-class work were high, as this work is graded for completion and effort; the average exam score was approximately 3% higher than the average homework score. We view this as an encouraging sign, as it suggests that widespread cheating on homework assignments was not an issue, or at least not very effective.

We hypothesize that the difficulty level of homework problems, the fact that homework problems were created from scratch by the instructor, and the fact that students were required to explain their reasoning in their own words on homework assignments acted as further barriers to cheating.

Further considerations

A teaching method is not a formula to be applied indiscriminately. When teaching a course, an instructor must be sensitive to the particular circumstances of their students, classroom, schedule, and more, and they must make adjustments to accommodate them. We do not pretend that the teaching method described here can be applied to any online course without modifications.

The frequency with which a course meets may be an important factor. If a course meets only once or twice per week, then it is natural to assign pre-class reading, pre-class exercises, and post-class homework for every class period. If a course meets three or more times per week, then limiting pre-class and post-class assignments is likely necessary.

When teaching an online course using Pengelley's method, there are doubtless other options for managing class time than the system we have described here. Online breakout rooms, for instance, may indeed be an effective means for in-class discussions, especially in courses where students need not make many calculations when completing exercises. We imagine an instructor splitting students into online breakout rooms to discuss a question for several minutes, then closing the breakout rooms and calling on a representative from a group to share their group's work with the class.

For some courses, the substantial preparatory work that students do before class and their high level of engagement in the classroom may allow recitations to be eliminated or reduced. This may simplify the course and allow the instructor to focus their teaching assistants' efforts on other tasks, like assisting during class time (especially in larger classes), doing more grading, or helping with the creation of assignments.

Conclusion

The coronavirus pandemic has created many problems for higher education, but as the saying goes, a crisis should not go to waste. Given that instructors are being forced to make radical changes to how they teach, we feel it is important to address the question *Why not make changes that will significantly improve student learning?* We hope that this paper provides a useful starting point for implementing such changes, especially for instructors who are teaching online courses.

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