# A Structured Problem Solving Model for Developing High-Level Skills

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### Abstract

In professional practice, engineers commonly solve problems that are highly complex and openended. Since good engineering requires high-level thinking, classroom activities should lead students to develop and improve appropriate skills. To foster this aim, we have adapted the <u>Professional Decision Making (PDM)</u> process for application in engineering science courses. The PDM process uses seven basic elements: affirmation, define the situation, state the goal, generate ideas, prepare a plan, take action, and review. Within each element, thinking skills are described using a small number of heuristics. The amount of detail is purposefully limited so that the complete model may be implemented in an engineering science course.

To assess the PDM process, we examined three types of data acquired during a recent implementation in a fluid mechanics course. These data support the hypothesis that the PDM model as described herein (a) promotes effective problem solving, (b) appeals to users, (c) builds skills for professional practice, and (d) promotes communication.

### Introduction

Landis (1995) presents a working definition of problem solving: "... the ability to identify and define a problem, develop and evaluate alternative solutions, and effect one or more designs to solve the problem." While development of effective problem-solving skills is a primary goal of engineering education, reaching this goal is very challenging. Most engineering science classes require substantial coverage of content, leaving little time for teaching problem solving skills. Moreover, problem solving involves high-level skills, and most students cannot learn these skills in a short time. Another issue is that many students have learned "dead-end" problem solving techniques. That is, they use techniques that are effective for textbook problems, but ineffective for practical problems. Examples of dead-end skills include (a) using example problems as templates, (b) plugging numbers into formulas with little thought of the concepts, and (c) working backwards from known answers.

To address the aforementioned issues, we have developed a model of the problem solving process. This model is designed for application in engineering science courses. The goals are:

- Promote effective problem solving (fast, applicable to many types of problems, etc.)
- Appeal to users (appeal to a diverse range of students)
- Build skills for professional practice (use general principles, develop skills for open-ended problems)
- Promote communication (develop professional documentation skills, communicate fundamental steps of problem solving, foster communication in a team environment)

## Literature review

Woods (1987) notes that during a four-year degree program, students observe professors work 1000 or more example problems, and the students themselves solve more than 3000 problems. However, the students "show negligible improvement in problem solving skills … what they did acquire was a **set of memorized procedures** for about 3,000 problem situations that they could, with varying degrees of success, recall." Based on findings such as these, Woods has spent many years developing a problem-solving strategy. In his most recent paper, Woods (2000) reviews basic problem-solving strategies and presents the MPS (McMaster Problem Solving) program. For problem solving, five sets of knowledge/skills are identified: (1) subject knowledge, (2) tacit knowledge (3) links and clues to subject knowledge, (4) pattern recognition skills, and (5) generic problem-solving skills. Regarding skills for problem solving, Woods (2000) presents an extended list of metacognitive, cognitive and attitudinal skills. Woods stresses the importance of learning problem solving as a nonlinear process.

Polya (1973) presents a well-known model for problem solving in the context of mathematics education. He uses the concept of heuristic. Modern heuristic is an attempt to illuminate the process of solving problems by focusing on the useful mental operations. In simple terms, a heuristic is a description of the steps that would be used by an expert to accomplish a specific problem-solving operation. Implicit in the heuristic are the higher-order thinking skills used by the expert. Polya presents a one-page chart labeled "How to solve it" that summarizes his model.

Based on extensive classroom experience, a group led by Dr. Charles E. Wales concluded that thinking skills can and should be taught. The Wales group recognized that experts organize higher-level thinking operations into a process. The Wales group labeled this as the Professional Decision Making (PDM) process—see the series of books and articles (Wales et al., 1972a, 1972b, 1979, 1986, 1987, 1990). The PDM process structures problem solving using a 5-element process. To use this process requires many thinking skills; details are presented in the aforementioned references.

Dr. Wales taught the PDM process and thinking skills in a freshman engineering class at West Virginia University. To assess outcomes, he compared student data for ten years: five years prior to teaching thinking skills and five years after. The data showed that when thinking skills were taught, the number of students who ultimately graduated increased by 32%. Also, the average GPA at graduation was up by 25% (Wales, 1979).

The model proposed here was developed following the PDM process of Wales et al. We also use concepts of Polya (global steps with specific heuristics; summary on a single page). Details of the thinking skills and heuristics were adapted from Wales et al., Woods et al., as well as other authors. The unique aspect of our work is presenting a model that is specifically designed for teaching thinking skills in the context of an engineering science course.

## **Basic PDM process**

The basic PDM process, summarized in Table 1, is comprised of seven elements. The term *element* identifies a fundamental of good problem solving. The elements in Wales et al.'s model

extend from "define the situation" to "take action" (elements 2 to 6 in Table 1). We have enriched and expanded Wales et al.'s model by adding an affirmation and a review element.

The affirmation is a process for recognizing and confronting the affective components of problem solving. Especially in professional practice, problems are difficult and problem solvers need to manage ambiguity, as well as emotions such as fear and anxiety. In professional practice, problems are complex and ill-structured. It is the engineer's job to perform the investigative work necessary to understand the situation and then to establish an appropriate goal. These processes are in elements 2 and 3.

To reach a specified goal, engineers usually develop alternatives, analyze these alternatives against criteria and then select a best alternative (element 4). An engineer will typically make a detailed plan for implementing a selected method (element 5). Action and review are the last elements in the model.

## TABLE I Basic elements of the PDM process

- **1. Affirmation.** Make statement(s) that promote effective psychological management.
- **2. Define the situation.** Ask questions and gather appropriate information with an intent of clarifying, interpreting, and understanding the situation.
- **3. State the goal**. Determine the appropriate or best goal or combination of goals. The goal should be concrete. That is, the goal should be presented with enough specificity so different people would agree when the goal is reached.
- **4. Generate ideas**. Generate many possible ways to reach the goal. Analyze these ideas, and then select the best idea or combination of ideas.
- 5. Prepare a plan. Carefully plan the steps needed to make the best idea a reality.
- 6. Take action. Implement the plan.
- **7. Review and Reflect.** Check the solution to assess quality. Analyze the problem solving approach in order to identify what worked and what did not work. Seek ways to refine or improve one's problem solving approach. Clarify what was learned during the complete experience.

## PDM process adapted for an engineering science course

To use the PDM process in a typical engineering science classroom, we have added specific heuristics that are useful for textbook problems. These heuristics are listed in the left column of Table II. The right column lists practices to be avoided. Regarding terms, we use *concrete* to mean existing in reality or in real experience. That is, perceptible by the senses. We use *visualize* to mean the act of imaging and picturing something (an object or concept) as if this object or concept existed in real life. We use *analyst* or *problem solver* to identify the person engaged in problem solving. We use *experiential knowledge* to identify factual information about typical values of engineering parameters. Examples include (a) a natural convection heat transfer coefficient in air is about 6 W/m<sup>2</sup>•K, and (b) a small airplane often flies at a speed of 40 to 100 m/s. Experiential knowledge is sometimes described using "rules-of -thumb."

Some examples of an affirmation are (a) I think I can solve this, (b) I will work systematically and trust my process to guide me to a solution, (c) breaking a problem down into easy-to-do steps makes the problem much easier, and (d) I can do this!

When constructing meaning (heuristic 2a), the analyst is extracting and interpreting information and then reformulating it into her own words. She is changing the abstract problem statement to a meaningful and concrete description. Imagine that the analyst and the author of the text are holding a conversation with the aim of a shared understanding of the nature of the problem. When visualizing (heuristic 2b), the analyst formulates a concrete image and compares this image with experiential knowledge. To apply the "one year rule," the analyst documents her work well enough to allow her to retrieve this work in one year and recall the situation without any reference to the original problem source. In its entirety, define the situation (element 2) emphasizes that it is the job of the analyst to clarify, interpret and document the situation.

When stating the goals (element 3), the analyst interprets and documents the problem goal(s). In this process she will make explicit use of units (heuristic 3b). Throughout the PDM process (elements 2, 3,6,7) we emphasize using units and unceasingly thinking about the meaning of these units. For example, we teach students to inquire about units (e.g. what does a watt really mean?) and then to formulate concrete answers (e.g. a watt is rate; a 60 W light bulb emits 60 joules of thermal energy per second). In its entirety, state the goal (element 3) emphasizes the importance of formulating a specific, clearly understood goal

Generating ideas (element 4) is based on the concept that each engineering science course has a set of basic principles. By using these principles, an analyst can solve nearly all the problems in a textbook. Thus, the problem solver begins by brainstorming in order to create a list of the basic concepts that might be useful for the given problem. Next, he analyzes and selects these concepts most likely to be useful.

In addition to selecting basic principles (heuristic 4a), the problem solver visualizes the underlying concepts (heuristic 4b). For example, she might visualize forces using a free-body diagram. Or, she might visualize energy flows and work terms associated with the first law of thermodynamics. Unfortunately, only a few visual tools exist in the textbooks. So we create visual tools and teach our students this process. The importance of visual (nonverbal) thinking to engineering is well documented by Ferguson (1997). Often an analyst will need ideas from previous courses or a sketch to clarify geometry (heuristic 4c). After generating ideas, the analyst should have a good idea about how to solve the problem, and she is ready to begin planning.

One means to develop a plan is to use the GENI heuristic developed by Wales and Stager (1990). GENI is a mnemonic device that stands for Goal, Equation, Need and Information. The GENI heuristic is a systematic approach in which the analyst uses equations to reason out a solution. An example of the GENI approach will be presented in the next section of this paper. Sometime, the GENI method is not the most appropriate tool, and the analyst might formulate a plan as a numbered set of steps (heuristic 5b). The plan element emphasizes the importance of planning to the engineering process.

Action (element 6) typically involves calculations. We teach students to carry and cancel units. To review (element 7), one can follow the heuristic CARL: <u>Check</u>, <u>Analyze</u>, <u>Refine</u>, and <u>Learn</u>. Checking the answer may involve looking in the back of the book. Other skills include a simple estimate, or use of experiential knowledge. In the analyze step, the problem solver identifies what worked (strengths) and what did not work (areas for improvement). In the refine step, the problem solver seeks to improve his or her general approach to problem solving. The learn step of CARL involves active inquiry about what one learned with an intent of extending experiential knowledge and reinforcing key concepts of subject knowledge.

Good practice	Avoid
1. Affirmation	
2. Define the situation	
(a) Extract information and construct meaning	Trying to solve the problem
(b) Visualize real situation and sketch	Recopying problem statement
(c) One year rule	Rote transfer of information
3. State the goal(s)	
(a) List symbol	Too abstract
(b) List units	Too lengthy
(c) Define with words	Trying to solve problem
4. Generate ideas	
(a) Brainstorm and select: basic principles	Trying to solve the problem
(b) Visualize baisc principles	Too detailed
(c) Other ideas?	Sloppy layout
5. Prepare a plan	
(a) Plan using GENI heuristic GENI table	Too detailed
or	
(b) Plan using a list.	
6. Take action	
(a) Perform calculations	
(b) Carry and cancel units	
7. Review	
(a) Check (magnitude?, estimate?, assumptions?, professional issues?)	Answer looks reasonable
(b) <u>A</u> nalyze (what worked?, what did not work?)	
(c) <u>R</u> efine. (how can I improve my problem solving?)	
(d) Learn (concepts? methods?)	

## Table II PDM model (textbook problems)

## An Example Problem

Fig. 1 shows a problem from a fluid mechanics text, and Fig. 2 shows PDM documentation. PDM documentation is based on the principle that the analyst is responsible for communicating and documenting their work. Thus, the analyst must include technical details; for example in Fig. 2, the specification of the control volume, identification of parameters and identification of sections.

**5.52** Water is forced out of this nozzle by a piston moving at a speed of 5 m/s. Determine the force F required to move the piston and the speed of efflux of water from the nozzle. Neglect friction on the piston and assume irrotational flow. The exit pressure is atmospheric; D = 6 cm and d = 2 cm.



Figure 1 Problem 5.52; from Crowe et al. (2001)

In defining the situation, the analyst performed substantial interpretation and reformulation. "Efflux" was interpreted to mean outflow and "assume irrotational flow" was interpreted to mean that the Bernoulli equation applies to this nozzle. During visualization, the analyst (a) pictured the actual size of the nozzle (about 2.5 in. narrowing to about 1 in.; about the size of a tube of silicone caulk), (b) envisioned the speed of the piston (about 10 mph, like a fast run) (c) realized that the piston will likely hit the end wall in a short time period, and (d) realized that the flow is unsteady. Since unsteady flow invalidates the common form of the Bernoulli equation, the analyst formally assumed steady flow and documented this assumption.

When generating ideas, the analyst identified general concepts and listed the three ideas most likely to be useful. Notice the use of visualization.

In planning, the analyst formulated a solution path using the GENI heuristic. The only logical place to begin seeking a solution is with the goal. Since force (*F*) is a goal, he started by identifying that the equilibrium equation involves force. This equation was written as line 1 of the GENI table. Next, he analyzed the equilibrium equation to identify knowns (information) and unknowns (needs). Since area ( $A_1$ ) is known, it is written in the information column. Since pressure ( $p_1$ ) is unknown, it is written in the needs column. Next, the analyst selected pressure  $p_1$  as the new goal, and he identified that the Bernoulli equation involves  $p_1$ . Thus, this equation was written in line 2 of the GENI table. Analysis of the Bernoulli equation revealed that in order to find  $p_1$ , a value for  $V_2$  is needed. Next, the analyst identified that  $V_2$  is a parameter in conservation of mass. When the analyst examined the third line of the GENI table, he noted there are no longer any unknowns. Thus, the number of equations equals the number of unknowns and the problem is closed.

The solution was reviewed using the CARL heuristic. In statement 1, the analyst noted the values of pressure and force. In statement 2 (both analyze and refine), she noted that the GENI method worked well and thus should be done more often. Also, she noted that her problem-solving approach can be improved by double checking the given information. In line 4 (learn), she reinforced a primary concept—the Bernoulli equation usually works well when fluid accelerates through a contraction.



Figure 2 PDM documentation of the problem given in Fig. 1

#### Assessment

Assessment was based on triangulation of three data types: user survey, exam performance, and user written response. The context was an engineering fluid mechanics course taught during the fall semester of 2000. This course was taught at the junior level to a mix of mechanical, civil, electrical and agricultural engineering majors. Of the thirty students who started the course, twenty-eight finished.

<u>User survey</u>. An in-class assessment (26 respondents) was performed on the last day of the course, with results shown in Table III. The evaluation form contained fourteen statements that were evaluated on a scale from 4 (strongly agree) to 0 (strongly disagree). The assessments were anonymous, and the professor emphasized that honest feedback was valued. Table IIIa lists the fourteen statements and the average student response. For example, in response to the statement "I will use the PDM process in the future," thirteen students indicated that they strongly agreed (scale = 4), ten students indicated that they agreed (scale = 3), and three students indicated that they strongly as the last column of Table IIIa.

*Exam performance*. To assess performance on an exam, the final exam contained a problem that was difficult (about 10% of students in a traditional engineering science course could have solved it). Problem difficulty was based on several factors: (a) the technical content had not

Category (highlighted) and Statement	Avg. Response <sup>1</sup>
Promotes effective problem solving	
Using the PDM process improves my problem solving	3.5
The PDM process can be used on many different types of problems	3.4
Using the PDM process improves test performance	2.8
Using the PDM process allows me to solve problems faster	2.7
Appeals to users	
I will use the PDM process in the future	3.4
I would recommend the PDM process to other students	3.3
I would like other professors to use the PDM process	3.0
The PDM process fits my style	2.7
Builds skills for professional practice	
Using general concepts to solve problems is important	3.8
The techniques in the PDM process are useful for realistic engineering problems	3.5
The PDM process works on open-ended problems	3.2
Promotes communication	
A solution that is documented using the PDM process is easy to understand	3.5
The PDM process illuminates and communicates the fundamental steps of problem	
solving	3.3
The PDM process works well for group problem solving	3.0

## Table III Data from classroom assessment<sup>1</sup>

<sup>1</sup>Scale: 4 = strongly agree, 3 = agree, 2 = neutral, 1 = disagree, 0 = strongly disagree

been recently covered, (b) the problem was unlike others that students had solved, and (c) the solution path featured coupled, nonlinear equations that led to an iterative (implicit) solution. On the test, students were asked to complete the PDM process through the plan stage. Fig. 5 shows a solution typical of the upper 1/3 of the class. Table VI presents an analysis of the types of errors made. What is notable in Table VI is that all but two students had a good grasp of the concepts. The student work in Fig. 5 is also notable—prior to implementing the PDM process, most student work on exams was very disorganized and difficult to follow.

<u>Written assessment</u>. We analyzed three different collections of student writing: answers to openended questions on an in-class survey, essays from student homework portfolios and a report from a design project. For example, on the report for the design project one team stated

"These basic principles allowed us to define the situation and state the goal of the design project. Once we had done this, we started working on the project by taking small pieces at a time, so that we did not get ahead of ourselves and miss something important along the way. We began this by brainstorming to come up with a number of possible designs that could work for the project, mathematical ways to estimate the information that was not known, methods to construct the parachute, and ways we could test our design. All of these ideas were used for our generating ideas step of the PDM."

Examples such as this suggest that if students are taught the PDM process in the context of textbook problems, they develop skills that can be used on open-ended problems.

Other student writings illuminated a variety of interesting points. Nearly all students initially disliked the PDM process. However, at the end of the course students identified many positive attributes, e.g. organization, GENI heuristic, simplification of complex problems, use of general concepts, documentation skills, increased confidence and retention of knowledge. Many students stated that the PDM process was cumbersome on simple problems and a number of students stated that they should be allowed more flexibility to adapt the PDM process to fit their style. The most common areas in which students have problems are the thinking skills associated with "define the situation," "generate ideas" and "review." Regarding an affirmation, approximately ¼ of the students commented favorably on this element and ¼ commented unfavorably.

Number of	I ype of error
Students	
14	no errors
3	correct solution; failure to realize that number of equations equals number of unknowns
5	presumed tank was emptying (not filling)
	presumed tank was emptying (not filling); located section 1 at inlet to pump, not at free
4	surface of river
1	poor documentation; unclear what the student was thinking
1	overall incorrect approach

TADLE IV Analysis of errors on final exam problem shown in Fig. 3
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#### Problem statement

A pump is used to fill a tank 5 m in diameter from a river as shown. The water surface in the river is 2 m below the bottom of the tank. The pipe diameter is 5 cm, and the head loss in the pipe is given by  $h_L = 10V^2/2g$ where V is the mean velocity in the pipe. The flow in the pipe is turbulent, so  $\alpha = 1$ . The head provided by the pump varies with discharge through the pump as  $h_p = 20 - 4 \times 10^4 Q^2$  where the discharge is given in cubic meters per second (m<sup>3</sup>/s) and  $h_p$  is in meters.



Find the discharge when the depth in the tank is 3 m.

Student solution AFFIRM: I can do this - Iwill rely on my general principles D.S  $h_{L} = \frac{10V^{2}}{2g}$   $h_{p} = 20 - (4 \times 10^{4})Q^{2}$ water flows [s] - discharge into tank when tank depth is 3m GOAL Therpy Lg  $z_{1}^{\prime} + a_{2}^{\prime} \frac{v_{1}}{z_{2}} + h_{p} = \frac{p_{1}}{2} + \frac{z_{2}}{z_{2}} + \frac{g_{2}^{\prime} v_{2}^{\prime}}{z_{2}} + h_{\tau}^{\prime} + h_{L}$  $= \sum h \rho = Z_2 + h L$   $20 - (4 \times 10^4) Q^2 = Z_2 + \frac{10 V^2}{2 Q^2}$   $(2) Q = VA = > V = Q/A \quad Volumetric Flow Rele$ PLAN <u>E</u> Q=VA  $\frac{L}{A = \frac{11}{10005}} = 0.0019 \text{ m}^{2}}{4}$   $g = 9.81 \text{ m/s}^{2}$ G Q  $h_{L} = \frac{10V^{2}}{2g^{2}}$   $h_{p} = Z_{z} + h_{L}$   $h_{p} = 20 - (4 \times 10^{4})Q^{2}$ Z2 = 5m Q => 4 unknowns, 4 equations - can be solved

#### Figure 3 Example of student work on the final exam

## Conclusions

We have presented an adaptation of the PDM model for application to teaching problem solving in the context of an engineering science course. The model is comprised of seven elements. For each element, we use specific heuristics to communicate appropriate skills. The details of the model are purposefully limited, thereby allowing the model to be introduced and implemented in an engineering science course.

Assessment data from a recent implementation supports the hypothesis that the PDM model as described herein (a) is effective for problem solving, (b) is appealing to users, (c) builds skills and knowledge for professional practice, and (d) fosters communication.

Wales et al. (1986) state that "*Professional decision-making reveals the height, depth and breadth of the human potential.*" We agree. Furthermore, we have found that teaching the PDM process elevates teaching to a new level, and provides a way to dramatically improve education outcomes.

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