AC 2008-533: A TEACHING TOOL FOR DESIGN AND ANALYSIS OF CAM AND FOLLOWER MECHANISMS

Mina Hoorfar, University of British Columbia Okanagan

Mina Hoorfar received her Ph.D. from the Department of Mechanical and Industrial Engineering at the University of Toronto in 2005. In the course of her graduate studies at the Laboratory for Applied Surface Thermodynamics, University of Toronto, Dr. Hoorfar worked in the area of surface and interfacial engineering. Her research mainly focused on the development of methodologies for accurate measurement of interfacial tensions, contact angles, and line tension. After completing her Ph.D. research, Dr. Hoorfar joined the Case Advance Power Institute at the Case Western Reserve University as an NSERC Postdoctoral Fellow. Her research involved in the enhancement of water management in the proton exchange membrane (PEM) fuel cells. Dr. Hoorfar is currently an Assistant Professor in the School of Engineering at the University of British Columbia Okanagan.

Homayoun Najjaran, University of British Columbia Okanagan

Homayoun Najjaran received his M.A.Sc. and Ph.D. degrees in Mechanical Engineering from the University of Tehran in 1996 and University of Toronto in 2002, respectively. He worked on different research projects in the area of mechatronics at the National Research Council (NRC) Canada from 2003 to 2006. Currently, he is an Assistant Professor in the School of Engineering at the University of British Columbia Okanagan. His area of research includes robotics, mechatronics and control systems.

William Cleghorn, University of Toronto

William Cleghorn graduated from the Department of Mechanical Engineering, University of Toronto. In 1980, he received his Ph.D. from the same department. He subsequently worked in industry. In 1986, he joined the Department of Mechanical Engineering, University of Toronto. Dr. Cleghorn has authored numerous publications related to vibrations. In 2001, he was appointed the Clarice Chalmers Chair of Engineering Design.

A Teaching Tool for Design and Analysis of Cam and Follower Mechanisms

Abstract

This paper presents a software tool for teaching the kinematics, design and analysis of cam and follower mechanisms used in the undergraduate mechanisms course that is generally included in the Mechanical Engineering curriculum. The software tool has mainly been developed to enhance student learning, but it can readily be used to design and analyze cam mechanisms for industrial applications. The software includes numerous combinations of follower types (e.g., knife-edged, flat face and roller) and follower motions e.g., constant velocity, constant acceleration and harmonic. In order for the students to gain insight into the subject, the software generates detailed information about the displacement, velocity and acceleration of the follower in tables and graphs more so than the other cam and follower design tools used in industrial applications. It also provides graphical representations and animation of the cam and follower mechanism. Specifically, the emphasis of this work has been on having interactive software that can enhance student learning by exposing them to theoretical and practical aspects of the design of cam and follower mechanisms. The software provides design tips in the form of warning and error messages whenever the users attempt to enter invalid values of input parameters, and suggests fault-recovery steps that help the users optimize their designs. The paper includes two simple design projects, generated by the software program to demonstrate important problems such as elevated pressure angle and undercutting that can occur in the design of cam mechanisms. The software tool was used in a lecture where positive feedback from students enrolled in the mechanisms course has encouraged the authors to continue the development of such educational software tools for other applications.

Introduction

Cam and follower mechanisms were traditionally designed using graphical layouts, intuition and experience. As a result, the measurement of the radius of curvature was often inadequate for modern machines[1] where accurate contact stresses between the cam and the follower were required. Clearly, the problem is more complicated for complex machines with several types of follower motion in which the radius of curvature over every particular segment of the cam profile must be considered separately. Besides the practical issues, a major drawback of the traditional methods was in regard to learning of the analysis and design of the cam and follower mechanisms, which was often overlooked in the mechanisms course of Mechanical Engineering curriculum.

With the availability of digital computers, however, the design of cam and follower is more straightforward provided that appropriate mathematical and numerical schemes are available to evaluate all of the design criteria[2,3]. In general, the design of a cam mechanism involves the determination of the cam profile such that a full 360° rotation of the cam results in a full cycle of the desired motion of the follower. A powerful and versatile example of such computer-aided cam design tool is the CamDesign program[4]. This program not only facilitates the analysis and design of cam and follower mechanisms but also provides extensive feedback to help the user learn the steps of the design.

The program carries out the design by creating a high-resolution displacement diagram and converting it to the shape of the cam profile. The inputs to the program are the geometrical parameters of the cam and the follower and the kinematics parameters of the cam and the follower. The former include the base circle diameter, type of the follower motion (i.e., translating or pivoting), shape of the follower (i.e., knife edge, flat face, roller and spherical), offset, and diameter of the roller. The latter include the cam speed, number of intervals, and follower motion properties in each interval. The motion properties depend on the follower motion type that is selected from a list of common motions including dwell, constant velocity, constant acceleration, simple harmonic, cycloidal and modified trapezoidal, and modified parabolic.

The output of the program includes the displacement, velocity, acceleration, and jerk of the follower as well as the cam profile. Another output of the program is the pressure angle, which is defined as the angle between the instantaneous direction of the follower motion and the normal to the pitch curve (i.e., the path of the follower trace point generated by rotating the follower about a stationary cam).

This paper focuses on a new feature of the CamDesign program that has been added to tackle the undercutting problem. The undercutting problem refers to the condition in which it is physically impossible for the follower to follow the desired cam path. Procedures for detection and prevention of undercutting are based on the radius of curvature of cam profiles and have been implemented into the CamDesign program to improve the program further by preventing undercutting conditions, automatically. In view of the pedagogical goals of the CamDesign program, this new feature emphasizes students' learning by detecting and describing the design problems and suggesting necessary modifications to resolve them. Examples of warning messages and corresponding solutions in regards to the intercutting problem have been demonstrated in the Design Projection section.

Undercutting

This condition is attributable to the radius of curvature of the cam, the follower acceleration and the cam size, which is directly related to the cam base circle radius[5]. A common way for avoiding undercutting and sharp corners is to select a very large cam base circle. However, this solution may not be practical because the size of a mechanism is typically a major design constraint. Thus, it is desirable to determine the minimum base circle radius for given follower motion without undercutting. In this work, several optimization techniques have been implemented into the CamDesign program to calculate the optimal radius of curvature. Subsequently, the undercutting condition for each design is evaluated, and if the result is positive, optional preventive measures are recommended to the user. For example, the program may recommend both an increase of the base circle radius and decrease of the roller diameter.

Fig. 1 shows a typical undercutting condition in which a part of the cam profile forms a fish tail. In practice, it is not possible to manufacture an undercut in a planar mechanism. Undercutting can be assessed based on the value of the radius of curvature, ρ, at each point of the cam.

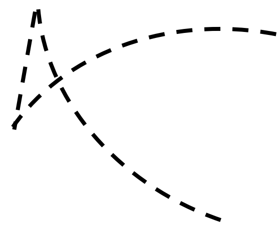


Fig. 1 – Undercutting (fish tail)

In essence, undercutting occurs when the radius of curvature switches sign from positive to negative. On the verge of undercutting, ρ becomes zero and hence the cam will have a sharp corner. This may usually result from attempting to achieve a large lift within a small rotation angle of a small cam. Although it is possible to avoid this problem by decreasing the desired lift or increasing the cam rotation angle, the solution may not be as straightforward when the mechanism motion is predetermined. Another solution is to use the same displacement characteristics but change the design parameters including the radius of the cam base circle and/or roller radius. In the following, appropriate equations are introduced for the calculation of ρ for each type of the follower. Then, the required design conditions are suggested to prevent undercutting.

Flat face follower – For the case of flat face followers, the radius of curvature for each value of cam rotation θ is calculated from the following equation[5,6]:

$$\rho = R_0 + s + \frac{a}{\omega^2} \tag{1}$$

where R_0 presents the radius of the cam base circle, ω is the cam angular velocity, and s and a present displacement and acceleration for a given type of the follower motion at each cam rotation, respectively. On the verge of undercutting, ρ will be zero for some value of θ. Thus, for given displacement and acceleration, R_0 must be chosen large enough to avoid this problem. Based on Eq. 1, it is required that

$$R_0 + s + \frac{a}{\omega^2} > 0 \tag{2}$$

Since R_0 and s are always positive, the critical situation occurs where a has its largest negative value. Denoting the minimum value of acceleration as $a_{\min}$, the following condition must be met to prevent undercutting

$$R_0 > -s - \frac{a_{\min}}{\omega^2} \tag{3}$$

In this equation, s corresponds to the cam angle θ where the acceleration is minimum, i.e. $a_{\min}$. The values of s and $a_{\min}$ are obtained using the CamDesign program. Based on the above equation, the program suggests a minimum value of R_0 in order to avoid undercutting.

Roller/spherical follower – For the case of roller (and spherical) followers, the radius of curvature for each value of cam rotation θ is computed based on the following equation[5,6,7]:

$$\rho = \frac{[(R_0 + r_f + s)^2 + \left(\frac{v}{\omega}\right)^2]^{3/2}}{(R_0 + r_f + s)^2 + 2\left(\frac{v}{\omega}\right)^2 - \frac{a}{\omega^2}(R_0 + r_f + s)} \quad (4)$$

where r_f presents the radius of the roller/spherical follower and v presents velocity for a given type of the follower motion at each cam rotation. Undercutting occurs if $|\rho| \le r_f$. The condition must be checked for both convex (where $\rho > 0$) and concave (where $\rho < 0$) portions of the cam. In the case of the roller followers, undercutting can be resolved in two ways: 1) by increasing R_0, or 2) by decreasing r_f. In the CamDesign program, optimum values are suggested for R_0 and r_f on the verge of undercutting, i.e. $|\rho| = r_f$. In this situation, Eq. 4 becomes

$$\begin{aligned} &[(R_0 + r_f + s)^2 + \left(\frac{v}{\omega}\right)^2]^{3/2} \\ &\pm r_f[(R_0 + r_f + s)^2 + 2\left(\frac{v}{\omega}\right)^2 \\ &-\frac{a}{\omega^2}(R_0 + r_f + s)] = 0 \end{aligned} \quad (5)$$

CamDesign program uses the Newton-Raphson optimization method to suggest the minimum value of R_0 and maximum value of r_f for preventing undercutting.

Knife-edge follower – The radius of curvature is given by Eq. (4) except that $r_f = 0$ for the case of knife-edge followers. In this case, fish tale undercutting does not occur, but an optimal design may require preventing high contact stresses associated with sharp corners. For this purpose, a minimum radius of curvature, $\rho_{\min}$, is defined. The design is carried out such that $|\rho| \le \rho_{\min}$. Again undercutting can be resolved by increasing R_0. The optimum value of R_0 can be obtained for the situation that $|\rho| = \rho_{\min}$. Thus, in this situation

$$\begin{aligned} &[(R_0 + r_f + s)^2 + \left(\frac{v}{\omega}\right)^2]^{3/2} \\ &\pm \rho_{\min}[(R_0 + r_f + s)^2 + 2\left(\frac{v}{\omega}\right)^2 \\ &-\frac{a}{\omega^2}(R_0 + r_f + s)] = 0 \end{aligned} \quad (6)$$

The CamDesign program uses the Newton-Raphson optimization method to suggest the minimum value of R_0 for preventing undercutting.

Design Projects

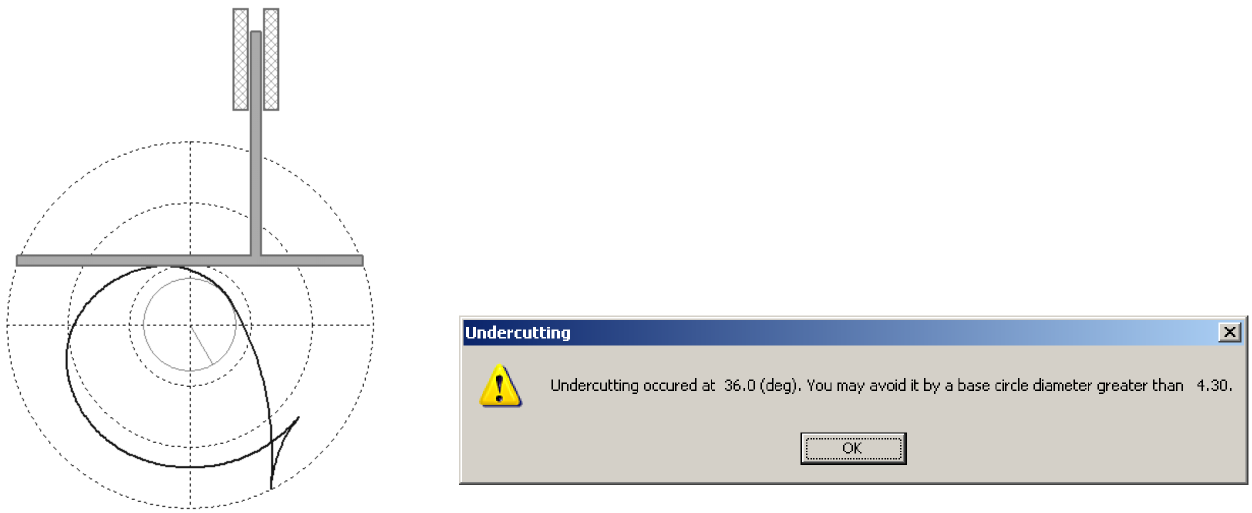
The effect of the CamDesign program on the enhancement of students' learning has been evaluated in the lectures focusing on the design of cam and follower systems in the undergraduate Mechanisms course. The CamDesign is currently used as a peripheral tool for the study of this topic that is covered in Chapter 7 of the textbook[9] of the course. The software is provided with the textbook free of charge. Prior to the development of the CamDesign, this topic was often neglected due to time consuming calculations involved in the analysis of cam and follower mechanisms.

The CamDesign has also been used by a group of the undergraduate students in a design project to facilitate the design of a cam and follower mechanism used in the Tracking System of Smart Antennas. Given the interactive features of the CamDesign program that provides error and warning messages and also suggests solutions for these problems, the software helps the students understand the concepts and the sources of the problems that occur in the design of cam and follower mechanisms. The two examples have been presented in the classroom to demonstrate the "undercutting problem," which is a complicated design problem that typically puzzles the students. Both examples begin with an undercut condition, which is detected automatically in the CamDesign program. Subsequently, appropriate changes are suggested by the program that are used to resolve the undercut design.

Example 1 - The first example involves a flat face translating follower. The design requirements in terms of geometrical and kinematic properties of the cam and follower mechanism are shown in Table 1. Fig. 2(a) shows the resulting cam profile, and Fig. 2(b) is the warning message generated by the program that reports the undercutting problem and provides a solution for resolving the undercut design.

Table 1 – Design properties of Example 1

Base circle diameter: 1.0 (u)* Follower: Translating and flat face Offset: 0.7 (u) Cam Speed: 1 (u/sec)			
Int.	Int. end	Motion type	Lift (u)
1	70°	Constant Acceleration	1
2	135°	Dwell	0
3	360°	Modified Trapezoidal	-1
*(u) is the unit length e.g., centimeter.			



(a) (b)

Fig. 2 – (a) Undercutting for flat face follower, (b) warning message and suggested solution

The main idea is to check the undercutting condition and prescribe appropriate changes to the properties of the cam so that they result in equivalent motions but prevent undercutting. In this example, the program detects that undercutting has actually occurred and the worst case is at the 36° rotation of the cam based on a search among the radii of curvature of the cam profile. Then, the program uses an optimization technique to determine the minimum base circle diameter that can remove undercutting. In this case, the minimum base circle diameter is 4.30 for the given follower motions. Fig. 3 shows that the cam base circle diameter is increased to resolve undercutting at the cost of a larger cam.

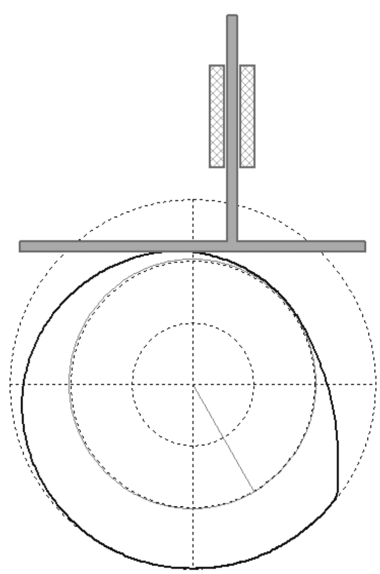
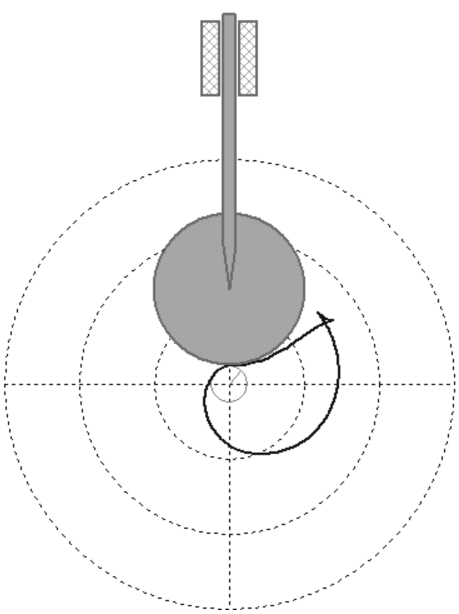


Fig. 3 – Larger base circle resolves undercutting

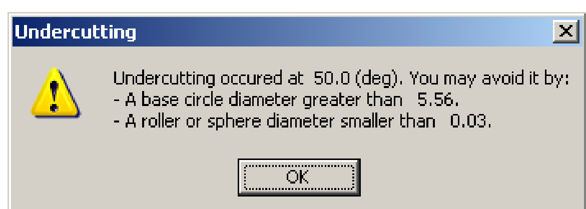
Example 2 – The second example involves a roller follower. In general, undercutting of roller followers are because of small base circle diameter or large roller diameter. Thus, undercutting can, by definition, be prevented in two ways. Now, the idea is to find a range of roller radii that prevents undercutting for the same base circle diameter. On the other hand, one can find a range of base circle radii that prevent undercutting while using the same roller follower. Table 2 summarizes the design geometrical and kinematic properties of this example. Fig. 4(a) shows the cam profile of Example 2, and Fig. 4(b) shows the dialog box that reports the undercutting problem and provides two solutions for resolving it.

Table 2 – Design properties of Example 2

Base circle diameter: 1.0 (u)* Follower: Translating and roller follower Offset: 0 Roller diameter: 4.0 (u) Cam Speed: 1 (u/sec)			
Int.	Int. end	Motion type	Lift (u)
1	50°	Dwell	0
2	100°	Simple Harmonic	2.5
3	360°	Modified Parabolic	-2.5
*(u) is the unit length e.g., centimeter.			



(a)



(b)

Fig. 4 - (a) Undercutting for roller follower, (b) warning message and suggested solution

In Fig. 5, the cam base circle diameter is increased to 5.60, and undercutting is removed completely. It is noted that based on the radius of curvature the cam will be at the verge of undercutting if the cam base circle diameter is less than 5.56, Figure 4(b).

Another way for preventing undercutting is to reduce the diameter of the roller follower. The objective is to find the maximum roller diameter for the original base circle diameter (i.e., 1.0). The program calculates the maximum allowable roller diameter as 0.03. Therefore, in this example the roller follower must be almost a knife-edge follower to prevent undercutting. Fig. 6 show the new design with a knife-edge follower where the follower maintains the design kinematic requirements.

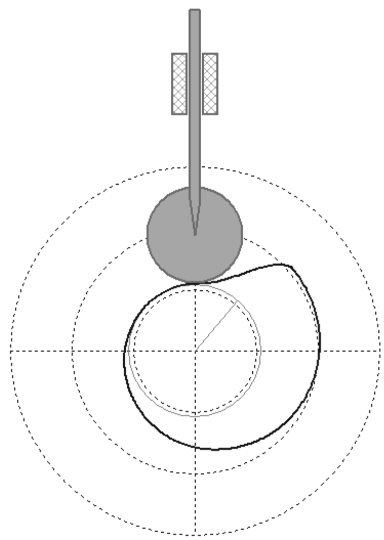


Fig. 5 - Undercutting for roller follower

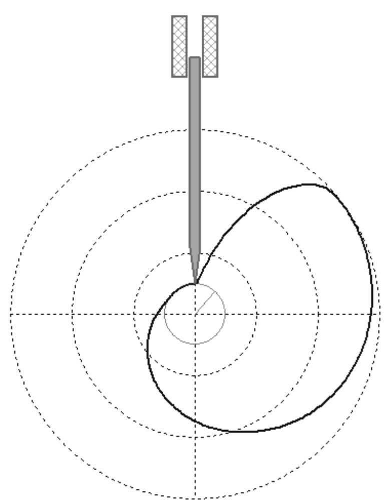


Fig. 6 - Undercutting for roller follower

Summary

This paper discusses the undercutting problem of disc cam mechanisms. However, general derivations of the radius of curvature for translating followers with offset are mathematically involved and textbooks on kinematics do typically mention them[7,9]. This study focuses on different types of translating followers including knife edge, flat face, roller, and spherical. Procedures for detection and prevention of undercutting are based on the radius of curvature of cam profiles that are determined using a software program called CamDesign, a versatile software tool that can be used for both teaching and industrial purposes. In this work, the CamDesign program has been used to study the mathematical and numerical methods used to prevent undercutting problem. The algorithms for determination of the radius of curvature and detection of undercutting condition in cam and follower mechanism were added to the software.

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