A Unique Liquid-Vapor Thermodynamic Property Measurement Apparatus For A Hands-On Undergraduate Laboratory Experience

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Abstract

This paper describes a fluid property experiment used in an introductory level *Thermodynamics* course. The *Thermodynamics* course is geared to introducing students to fundamental principles and their applications, including fluid property relationships. This paper presents a unique experimental apparatus, designed and built at Oakland University, to introduce students to indirect measurement techniques for measuring properties that cannot be directly measured; specifically, properties such as saturated vapor density and heat of vaporization as a function of saturation temperature. Typical results of students' experiences will be presented.

I. Introduction

A primary philosophy of the undergraduate fluid and thermal science curriculum at Oakland University is a seamless integration of laboratory experiences in every undergraduate course. This includes the introductory thermodynamics course where the laboratory is carefully integrated into the lecture material as a supplemental learning experience. Property relationships are very important in any thermodynamic analysis, requiring the use of functional relationships, property tables and graphs that relate properties that are not directly measurable, such as specific internal energy, entropy, enthalpy and heat of vaporization, to directly measurable properties such as specific volume, temperature and pressure. It is desirable to provide students with insight as to how these property relationships might be developed for a liquid-vapor refrigerant mixture.

In this light, an experimental apparatus was designed and developed to directly measure pressure, and indirectly measure saturated vapor density and heat of vaporization as a function of temperature. Two different refrigerants are used, refrigerant-12 and refrigerant-134a. The mixture temperature is controlled by a simple flow-through water jacket, providing a means for changing refrigerant state-points by changing the water jacket temperature. A transparent viewing section allows the liquid and the liquid-vapor interface of the refrigerant to be observed during the experiment. Boiling can be observed when the water temperature is increased, and film-condensation observed when the temperature is decreased. The only direct measurements made are the refrigerant temperature, pressure and liquid level.

Using these three direct measurements, saturated vapor density and heat of vaporization can be indirectly measured using the conservation of mass principle and Clapeyron's model. The measurements are in good agreement with commercially available data.

II. Background of a Typical Thermodynamics Class

The typical *Thermodynamics* class is made up primarily of sophomore-level undergraduate engineering students in the mechanical, electrical, computer and systems engineering disciplines. The course is a four-credit class, and involves both a lecture and a laboratory component. The lectures introduce fundamental principles, such as the conservation of mass and energy, and the second law of thermodynamics. In addition, constitutive relationships, such as the ideal gas law, fluid property relationships and differential equations of state, are introduced, along with the methodology for problem solving. The laboratory component is then geared toward hands-on experiences that give students experiential knowledge to reinforce lecture disseminated theoretical principles.

As a four-credit course, the class meets twice a week for approximately an hour and a half. The lectures consist of a variety of techniques to promote interest, motivation and retention of material. For example, the lecture is broken up to include regular breakout sessions involving active learning techniques, student-centered learning and collaborative learning. Homework is assigned regularly to develop and hone skills.

The laboratory component is designed to nurture visualization, understanding and stimulate creativity. Rather than rely purely on theoretical simulation, the laboratory provides a hands-on experience. Some students without such prior experiences appear to be initially intimidated by having to connect-up different flowmeters, pressure regulators and flow control valves, along with pressure gages and thermocouples. At the end of the course, however, these same students are typically the ones most enthusiastic about what they have learned.

III. Overview of Course Goals and Objectives

Catalogue Description of Course:

> *"This course introduces the fundamental concepts and analytical techniques of classical thermodynamics. This includes various forms of energy, its conversion from one form to another, and the effects of both energy conversions and energy transfers on various system and material properties. Macroscopic properties and thermodynamic property relationships are studied, along with the fundamental laws of thermodynamics. About half of the course is directed at applications of the basic concepts to engineering systems and processes, which are governed by thermodynamic principles."*

The singular goal of the Thermodynamics course is to expose students to, and challenge them to think about, the fundamental principles, their implications and their applications to a variety of energy related systems and processes. The lectures, including student-centered and active learning techniques, promote knowledge, comprehension and application. Regular homework, frequent small quizzes and six or seven hands-on laboratory experiences further promote the learning process.

IV. Experimental Apparatus

The specific experimental apparatus in focus in this paper was designed and developed for the purpose of indirectly measuring saturated fluid properties. Refrigerant-12 and R-134a were chosen as the working fluids because of their common usage, and favorable properties in being able to control vaporization or condensation utilizing a water jacket with temperature ranges that would allow for the use of a mixture of hot and cold tap water. The apparatus was designed and developed by a senior-level student doing a senior engineering design project – a current requirement for all senior-level students.

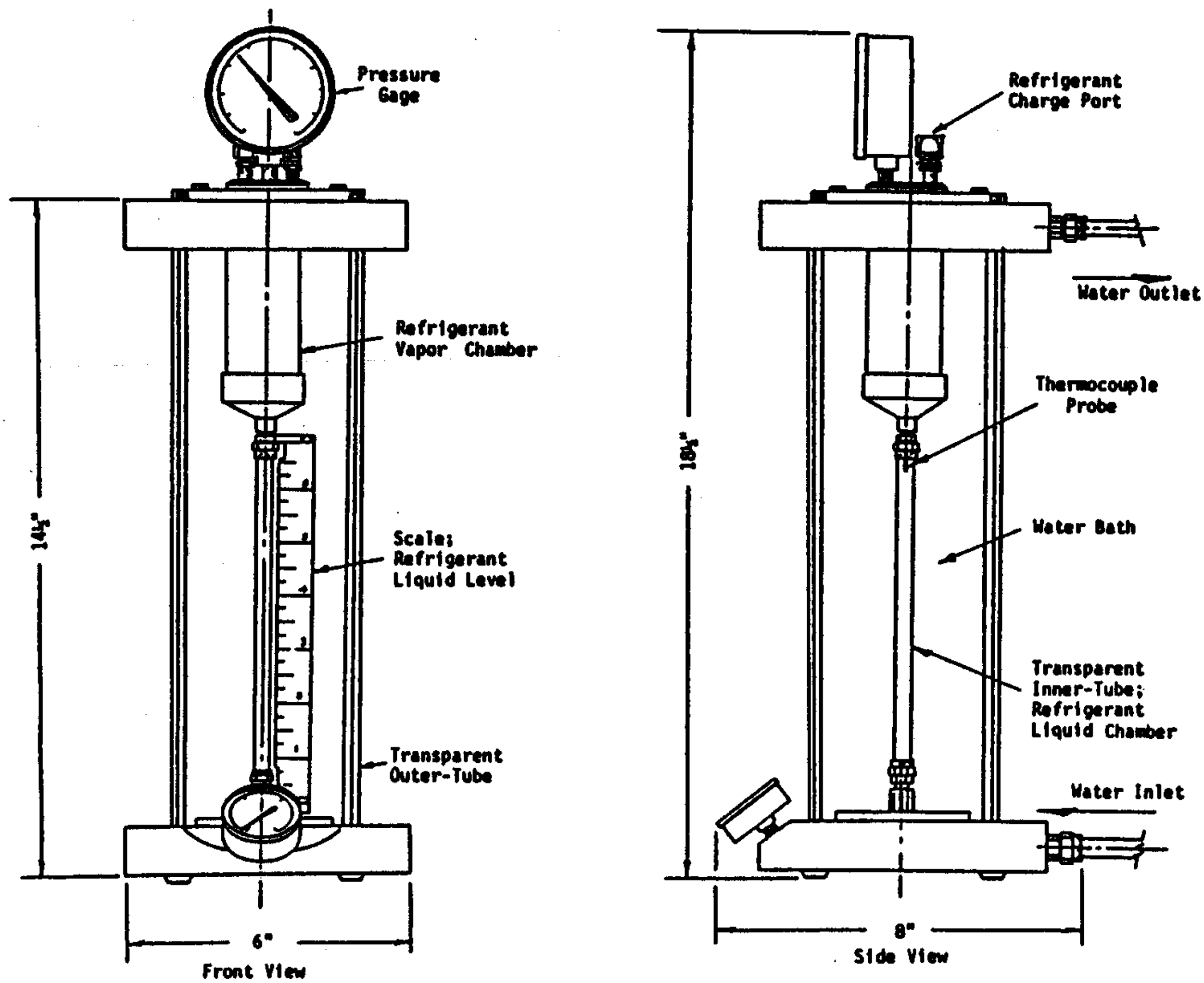

Figure 1: Schematic of Experimental Apparatus.

Figure 1 depicts a schematic of the experimental apparatus. The refrigerant is contained within a sealed stainless-steel vapor chamber. A semi-transparent nylon tube is attached to the bottom of the vapor chamber, where the liquid will naturally collect. A Bourdon-tube pressure gage is attached at the top of the vapor chamber to measure saturation pressure. A small resistive heating element is attached to the backside of the pressure gage to prevent condensation from occurring within the Bourdon-tube itself. A stainless-steel scale mounted on the outside of the nylon tube allows for the measurement of the refrigerant liquid level. Also, a small thermocouple is mounted within the nylon tube to measure the saturation temperature.

The entire vapor-chamber/nylon-tube system is mounted inside a transparent polycarbonate outer-tube within which water is circulated. Adjusting the water temperature, and waiting for steady-state to be reached alters the state of the refrigerant mixture. Steady-state is reached when the pressure, temperature and liquid level are no longer changing with time. A small pressure gage is mounted near the base of the system to prevent over-pressurizing the water jacket.

Figure 2: Picture of Experimental Apparatus

V. Experimental Specifications

Figure 3 represents a typical laboratory assignment and experiment specification sheet describing the purpose of the laboratory experiment, and the direct and indirect measurements, experimental techniques and empirical correlations involved. Students work in teams of three to four students each. They perform the experiments during a two-hour block of time when the apparatus was reserved for their team. The required technical report is the combined effort of the team.

Winter 2001	**Thermodynamics** **ME 241** **Laboratory Assignment #4**	**G.L. Wedekind** **C.J.Kobus**

Measurement of Thermodynamic Property Relationships; Refrigerant-12

The experimental apparatus shown in the accompanying sketch was designed for the purpose of measuring the relationships between various thermodynamic properties of a liquid/vapor mixture of Refrigerant- 12; properties such as temperature, T, pressure, p, vapor density, ρ_g, and heat of vaporization, h_{fg}.

Experimental Specifications

1. **Direct Measurements**
   Experimentally measure and tabulate the liquid/vapor mixture pressure, p, psig, and liquid level, L, in, as a function of temperature, T, °F, for the full range of temperatures available (50-130"F). Obtain at least 8 to 10 data points (more if you have time), starting with the lowest temperature. Be sure steady-state conditions exist and that the pressure gage heater is plugged in and on to prevent condensation inside the pressure gage. Also, be sure to read the barometric pressure so you can determine absolute pressure;
   $p_{abs} = p_{gage} + p_{bar}$.

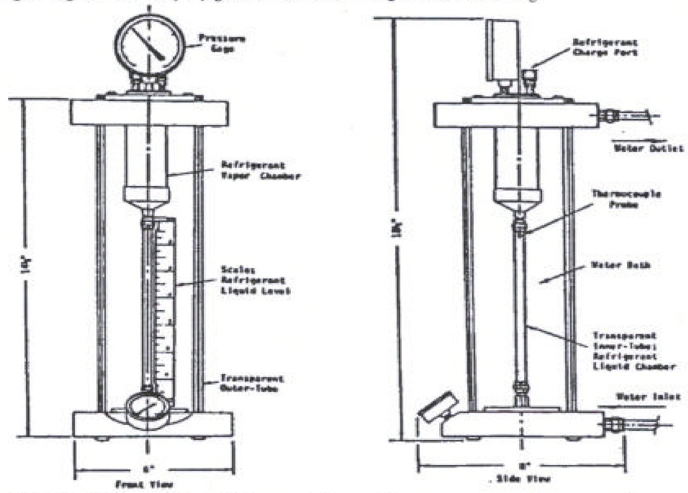

Fig.1: Schematic of Thermodynamic measurement apparatus; liquid-vapor mixture

2. **Pressure-Temperature Relationship**
   Tabulate and graph (as small open circles) the measured absolute pressure, p, psia, as a function of the measured temperature, T, "F. Also, include absolute temperature, T, °R, in the above table. From a best-fit function, find the coefficients, α and β, of the following empirical relationship which represents the "best fit" to the experimental data:

$$p(T) = \alpha e^{\beta/T} \text{ ; } p = \text{psia, } T = {}^{o}R \text{ (absolute temperature, } {}^{o}R = {}^{o}F + 459.59)$$

   Superimpose the above empirical correlation (as a solid curve) on the above graph. Also, superimpose commercially tabulated property data as small solid circles (see Table B-5, Reynolds and Perkins).

3. **Density-Temperature Measurement**
   Saturated vapor density, ρ_g, can be indirectly measured in terms of the liquid level, L, using the following simplified theoretical model.

$$\rho_g = \rho_{g,i} + \rho_{f,i}\left(\frac{\pi d^2}{4\forall_g}\right)(L_i - L)$$

where: ρ = liquid density, d = inside diameter of liquid tube (0.284 in)

V_g = total volume of vapor other than that in liquid tube (7.81 in^3)

i = subscript referring to initial value of physical quantity at the lowest temperature, T_i.

Note: $\rho_{g,i}$ and ρ_g are found from commercially tabulated property data at the initial lowest temperature, T_i.

Note: The system sketched out in Fig. 1 is designed such that the vapor volume is of sufficient extent such that it essentially remains constant, even when the liquid level changes (it is a *negligible* change in vapor volume).

From the conservation of mass principle, obtain the above model, making appropriate simplifications.

4. Density-Temperature Relationship:

Tabulate and graph (as small open circles) the measured vapor density, ρ_g, lb_m/ft^3, as a function of temperature, T, °F. From a best-fit function, find the coefficients, a and b, of the following empirical relationship which represents the "best fit" to the experimental data:

$$\rho_g(T) = ae^{b/T}\ ;\ \rho_g = lb_m/ft^3,\ T = {}^{o}R$$

Superimpose the above empirical correlation (as a solid line) on the above graph. Also, superimpose commercially tabulated property data (as small solid circles).

5. Heat of Vaporization Measurement

Heat of vaporization, h_{fg}, can be indirectly measured in terms of measurable properties, T, p, ρ_g and ρ_f using Clapeyron's model:

$$h_{fg} = T\left\{(\frac{1}{\rho_g}) - (\frac{1}{\rho_f})\right\}(\frac{dp}{dT})$$

where, since it is relatively constant, $\rho_f = \rho_{f,i}$ = saturated liquid density at initial temperature, T_i.

Using the empirical correlations developed for $\rho_g(T)$ and p(T), show that the heat of vaporization, h_{fg}, can be predicted as a function of temperature by the following empirical relationship:

$$h_{fg} = -\alpha\beta\frac{1}{T}\left\{\frac{1}{a}e^{-bT} - (\frac{1}{\rho_{f,i}})\right\}e^{-\beta/T}$$

6. Heat of Vaporization-Temperature Relationship:

Tabulate and graph (as a solid line) the empirically determined heat of vaporization, h_{fg}, Btu/lb_m, as a function of temperature, T, °F (be careful of units). Superimpose commercially tabulated property data (as small solid circles). Discuss.

Figure 3. Assignment Specification Sheet.

VI. Results of Experimentation

This section describes the results from actual student laboratory reports, and will be outlined in the same order as given in the assignment specification sheet (Figure 3). Specification 1: Direct Measurements. The first specification involves a series of tests, adjusting the temperature of the water flowing through the transparent water jacket, which in turn controls the saturation temperature of the liquid-vapor mixture within the refrigerant chamber (Figure 1).

Measurements are taken when steady-state is reached. The only direct measurements taken are barometric pressure, mixture gage pressure, mixture temperature, and the refrigerant liquid level in the nylon tube.

Specification 2: Pressure-Temperature Relationship. The objective of this specification is an empirical correlation for the pressure-temperature relationship of the form:

$$p(T) = \alpha e^{\beta / T} \tag{1}$$

where the pressure, p, is in units of psia (or Pa), and temperature, T, is in units of Rankine (or Kelvin). Since the temperature is in the denominator of the argument of the exponent, the students must reason that the plot should be of the pressure versus the inverse of the absolute temperature.

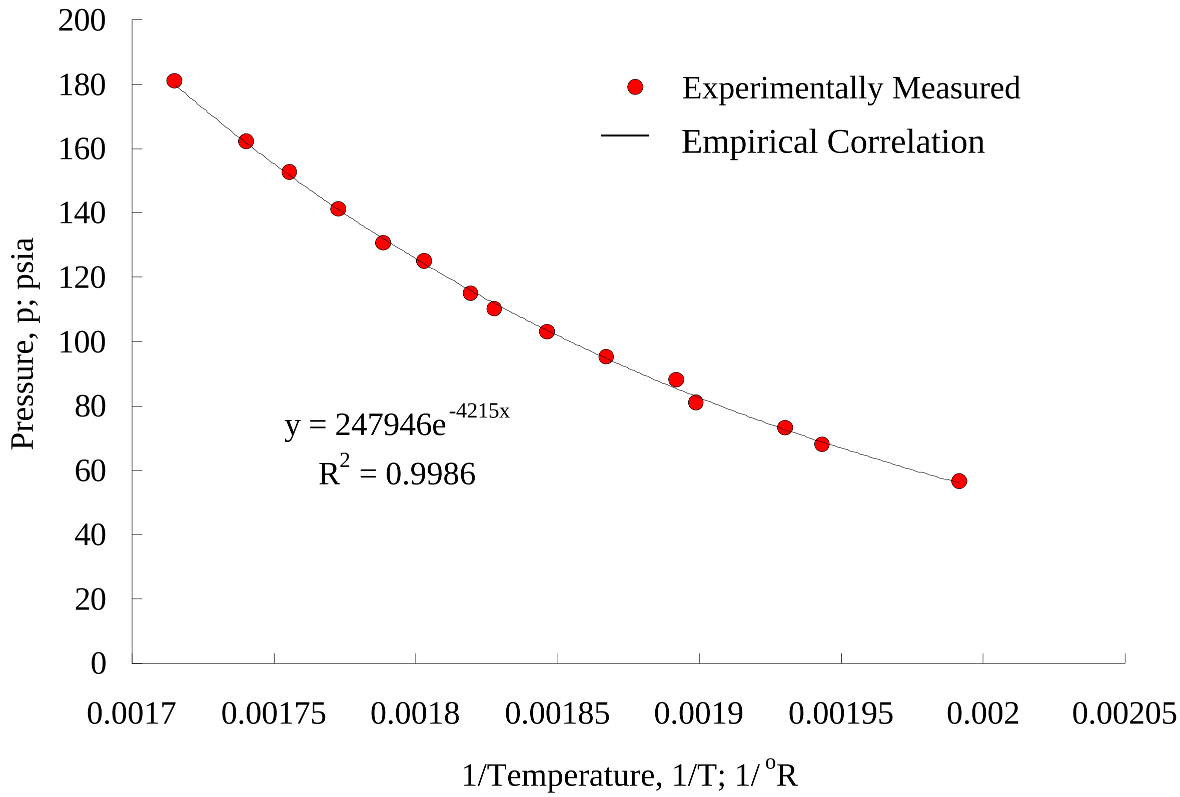

Figure 4. Empirical correlation for the pressure-temperature relationship of refrigerant-12

Referring to Figure 4, a correlation is obtained from a best-fit curve of the experimental data plotted in the correct form. Based on the form of the correlation in Eq. (1), α = 247946 psia, and β = -4215 ^{o}R. It should be noted that at this stage of the laboratory assignment, students often forget to designate units on α and β. Not having the appropriate units, or in fact any units, on these parameters will be detrimental later in this laboratory assignment. However, the experience forces students to appreciate the value of being careful with units.

Once the empirical correlation is obtained, a graph can then be made of the pressure versus temperature, ^{o}F. In addition to plotting the experimental data and the empirical correlation, commercially available data are superimposed for comparison purposes. This data is tabulated in most Thermodynamics texts. The results are plotted in Figure 5.

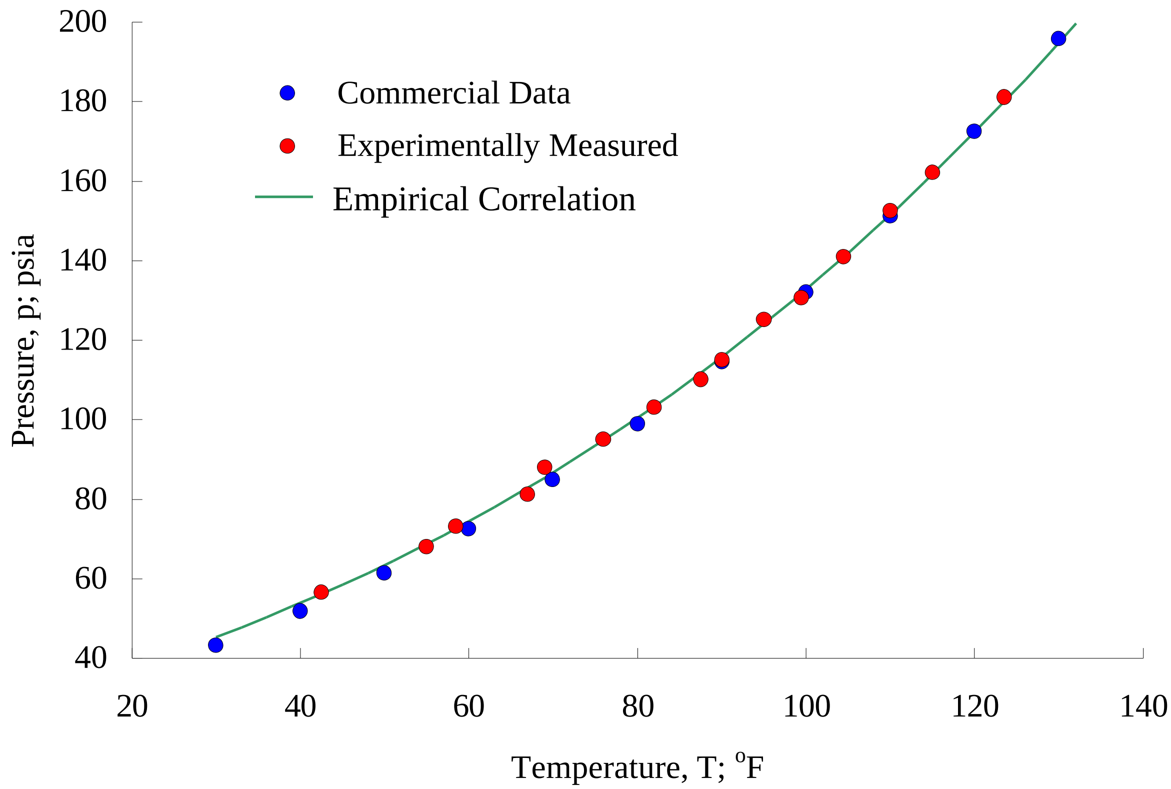

Figure 5. Pressure-temperature relationship of refrigerant-12; comparison with commercially-available data.

Referring to Figure 5, excellent agreement is seen to exist between the experimental data, the empirical correlation that was based on the experimental data, and the commercially available data.

Specification 3: Density-Temperature Measurement. In specification 3, students are asked to derive a model for indirectly measuring the vapor density in terms of directly measured quantities. The apparatus was designed so that changes in the liquid level will have a negligible effect on the total vapor volume, which includes the vapor volume in the nylon tube as well as that in the much larger vapor chamber. Therefore, the vapor volume is relatively constant and given to the students. The student teams then derive the model (the final model is given, but the student teams must perform the analysis to obtain it) sing the conservation of mass principle and given geometry. The resulting relationship can be expressed as:

$$\rho_g = \rho_{g,i} + \rho_{f,i}\left(\frac{\pi d^2}{4V_g}\right)(L_i - L) \qquad (2)$$

where ρ_f is the liquid density, d the inside diameter of nylon tube (0.284 in), V_g the total volume of vapor other than that in liquid tube (7.81 in^3), and the subscript i refers to the initial value of the physical quantity at the lowest temperature, T_i. The parameters $\rho_{g,i}$ and ρ_f are found from commercially tabulated property data at the initial lowest temperature, T_i.

Specification 4: Density-Temperature Relationship. Based on the model in Specification 3 above, Eq. (2), the vapor density can be indirectly measured and plotted so that an empirical correlation can be obtained in the following form:

$$\rho_g(T) = ae^{b/T} \tag{3}$$

where the density, ρ, is in units of lb_m/ft^3 (or kg/m^3), and temperature, T, is in units of Rankine (or Kelvin). Similar to Specification 2 above, since the temperature is in the denominator of the argument of the exponent, the students must reason that the plot should be of the pressure versus the inverse of the absolute temperature.

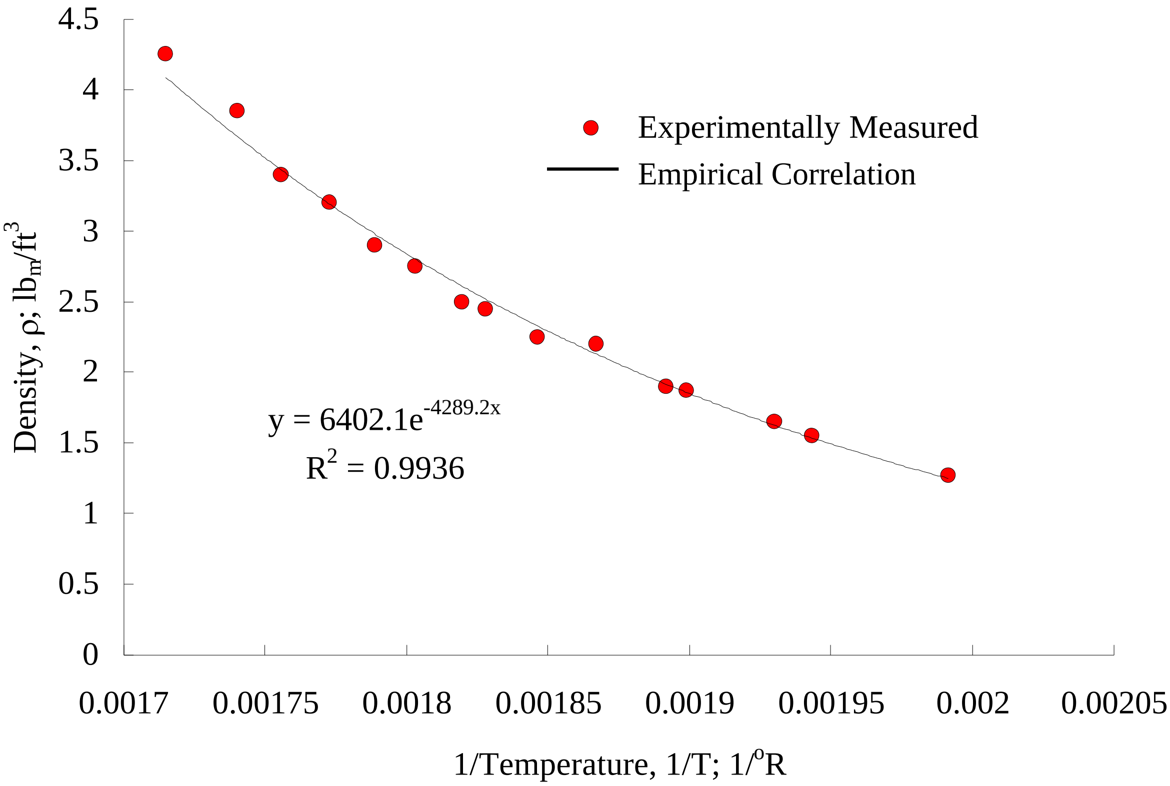

Figure 6. Empirical correlation for the pressure-temperature relationship of refrigerant-12

Referring to Figure 6, a correlation is obtained from a best-fit curve of the experimental data plotted in the correct form. Based on the form of the correlation in Eq. (3), a = 6402.1 lb_m/ft^3, and b = -4289.2 ^oR. Once the empirical correlation is obtained, a graph can be then made of the density versus temperature, ^oF. In addition to plotting the experimental data and the obtained empirical correlation, commercially available vapor data can be superimposed. The results are plotted in Figure 7.

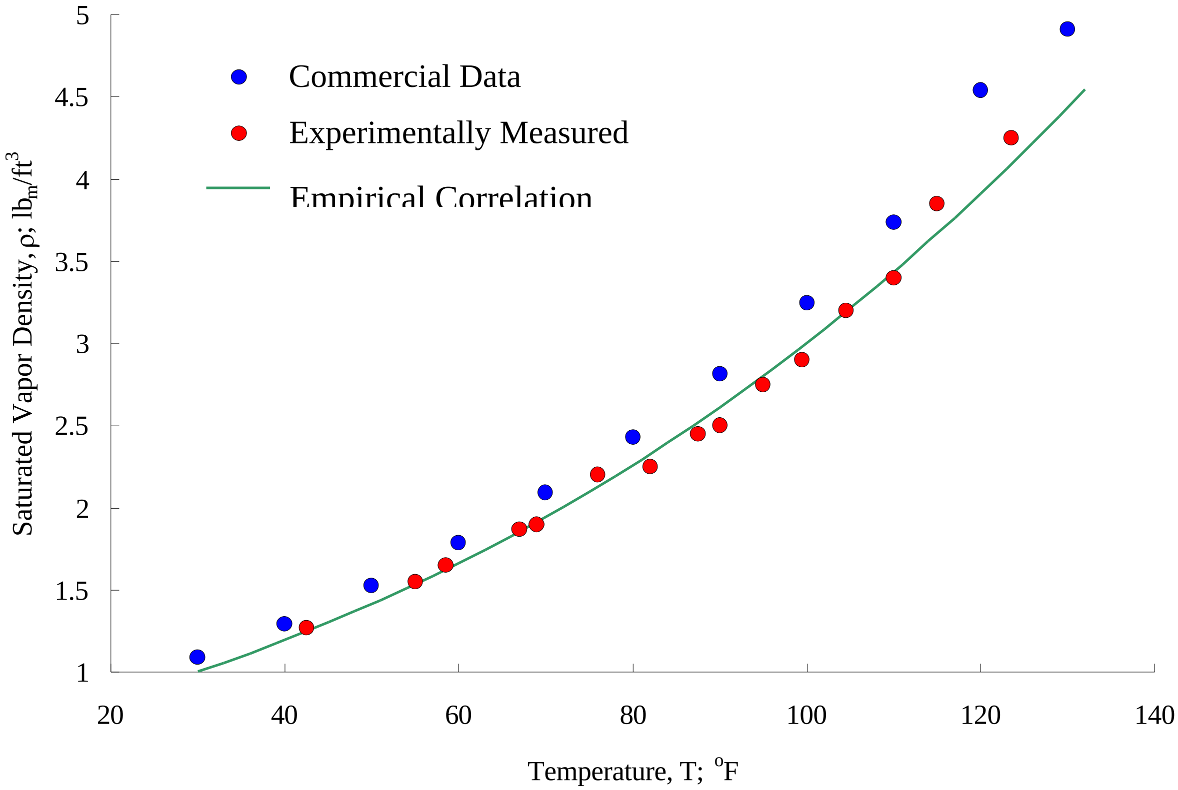

Figure 7. Density-temperature relationship of refrigerant-12; comparison with commercially available data.

Referring to Figure 7, good agreement is seen to exist between the experimental data, the empirical correlation that was based on the experimental data, and the commercially available data. The vapor density predicted by the empirical correlation is slightly lower than what is tabulated commercially, and is due to simplifications in the model to indirectly measure the vapor density, such as the vapor volume and the liquid density each remaining constant.

Specification 5: Heat of Vaporization Measurement. Heat of vaporization, h_{fg}, can be indirectly measured in terms of measurable properties, T, p, ρ_g and ρ_f using Clapeyron's model:

$$h_{fg} = T\left\{\left(\frac{1}{\rho_g}\right) - \left(\frac{1}{\rho_f}\right)\right\}\left(\frac{dp}{dT}\right) \tag{4}$$

where, since it is relatively constant over the temperature range involved, $\rho_f \cong \rho_{f,i} \cong$ saturated liquid density at the initial temperature, T_i. Utilizing the empirical correlations developed for $\rho_g(T)$ and p(T), Eqs. (1) and (3) above, and Clapeyron's model, Eq. (4), the heat of vaporization, h_{fg}, can be predicted as a function of temperature by the following empirical relationship:

$$h_{fg} = -\alpha\beta \frac{1}{T} \left\{ \frac{1}{a} e^{-bT} - \left(\frac{1}{\rho_{f,i}}\right) \right\} e^{-\beta/T} \tag{5}$$

Specification 6: Heat of Vaporization-Temperature Relationship. Utilizing the above model, the heat of vaporization can be plotted as an empirical model and compared to commercially available data.

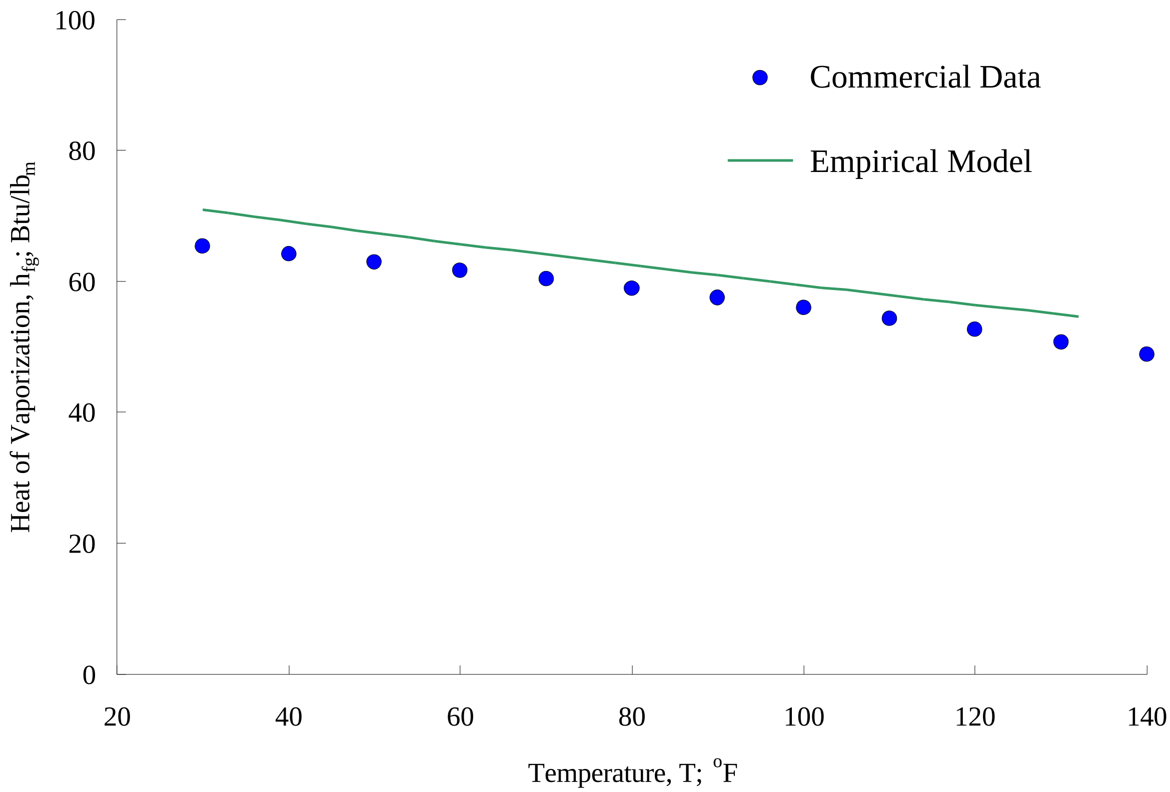

Figure 8. Heat of vaporization-temperature relationship of refrigerant-12; comparison with commercially available data.

Referring to Figure 8, it is seen that the empirical model shows good agreement in comparison with commercially tabulated data. The small (less than 10 percent) error is most likely due to simplifications in the empirical model for the vapor density. In any case, the empirical model is close enough to the commercial data such that experimental measurement uncertainty can cover the spread.

What is impressive here, and students recognize this, is that the heat of vaporization (energy content) was indirectly measured without any direct energy measurement. The only direct measurements taken were pressure, temperature and liquid height. Not only was the heat of vaporization measured, but it was measured quite accurately, especially when consideration is given to the complexity of measuring the heat of vaporization in multiple steps as outlines in this paper, and the relative simplicity of the experimental procedure.

VII. Student Feedback and Comments

Over the many years that this laboratory project has been assigned, the student response has been positive. Although the concept of indirectly measuring fluid properties that cannot be measured directly is mostly new to students, they become aware of the experimental precision needed to accurately measure these properties. In addition, they now have some idea of how fluid property tables and diagrams are developed.

VIII. Acknowledgements

The authors would like to acknowledge Gregory Collier, who was a major contributor to the initial design and development of the first prototype of this experimental apparatus as a senior engineering design project. Gratitude is expressed to Forrest Wright, a Project Engineer at Oakland, his technical insights and machining skills contributed greatly to the aesthetic quality as well as functionality of the current version of the apparatus. Finally, acknowledgement is also given to the many students who have participated in this particular laboratory over the years, especially those that love to ask questions and continually challenge us.

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