2006-1596: AN EXPERIMENT BASED STRUCTURAL DYNAMICS COURSE FOR ENGINEERING TECHNOLOGY STUDENTS

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Abstract

This paper describes a Structural Dynamics course for engineering technology students with emphasis on the understanding of the theoretical concepts by using lab experiments. The experiments involve a minimum amount of equipment.

The experiments are of increasing levels of theoretical complexity. The first two experiments involve single degree of freedom systems. The first one is a mass-spring system with translational motion, and the second is a rotational beam with an added mass and a spring attached to develop an understanding of the rotational inertia and resulting natural frequency.

The third experiment consists of a simply supported aluminum beam which is used to compare the theoretical and experimental values of the first three natural frequencies via a frequency-domain data analysis. The fourth and fifth experiments use a simply supported reinforced concrete beam and a simply supported prestressed concrete beam made by the students in the Reinforced Concrete and Prestressed Concrete courses. The first natural frequencies of the original beams were obtained experimentally and compared with the theoretical values showing good agreement. Also, the natural frequency shows a reduction, after testing of the beams up to the ultimate (cracked) condition. These experiments show that the first natural frequency changes with the integrity of the beams.

The sixth experiment consisted of finding the first two natural frequencies of a steel frame model. The properties of the frame were obtained using the geometry and results of static tests. The frame was then dynamically excited with an electromagnetic device actuated by an electronic signal generator. The theoretical values of natural frequencies were obtained using structural analysis software and they showed good agreement with the experimental results.

Engineering Technology students increase their understanding of how dynamic systems perform while also learning laboratory techniques for dynamic response testing. Verifying laboratory obtained natural frequency against theoretically computed values provides students with robust knowledge of dynamic system behavior.

Introduction

Structural Dynamics is an important subject for engineering technology students, because the principles are essential for understanding different conditions such as vibration control, earthquake and wind impact design. Structural Dynamics involves difficult theoretical and practical concepts for a typical under-graduate engineering technology student. However, if the course is taught using computer programs and experimental tests with data acquisition systems, the learning curve of the main concepts of structural dynamics may be improved. This approach is used in the Department of Engineering Technology, University of Houston – Downtown.

The first part of the course is a brief introduction to the theory of Structural Dynamics using the Finite Elements approach in order to obtain the matrices of mass, damping, stiffness, and forces. The mathematics of the dynamic equations is explained but emphasis is placed on finding the computer solution.

The computer program is explained to ensure that the students understand the input process and the results, particularly the computation of the dynamic parameters of the structure.

In order to reinforce the theory, a series of lab experiments are designed to determine the natural frequencies of different structures. The experimental structures are modeled using the computer program and the predicted responses are compared to actual experimentally measured responses.

Methodology

The theory of Stiffness, Mass, Damping and Forces matrices used in the dynamic equation is discussed following a textbook of Structural Dynamics [1]. The stiffness matrix components of a beam are obtained using a spreadsheet following a numerical integration method. This approach permits the review of fundamental concepts of structural analysis, such as the relationship of slope, deflection and acting force. The matrices are formed according to the degree of freedom (DOF) assumed for the structure being analyzed. The commercial computer program used handles 6 DOF per node, but the student can define fewer DOF to the desired level of simplification.

In order to understand and verify acquired knowledge, a set of experiments are designed for this course. The experiments are arranged in an increasing level of complexity, and basically consist of determining the natural frequencies of the structure. The following experiments were completed during one semester:

Experiment 1: Translational Single Degree of Freedom (SDOF)

The students construct a translational SDOF system using a tripod, a spring and a mass. A chart will be generated which illustrates the force applied to the mass versus its deflection. The stiffness of this system becomes the spring constant, k, which is defined as the slope of the force vs. deflection curve.

The system vibrates after the application of a small deflection and release. The frequency of the system is obtained visually by direct counting, which is independent of the initial deflection. The following equation is can be used for theoretical verification:

$$f = 1/2\pi\sqrt{K/M} \qquad (1)$$

where:

f = Natural frequency of the system, Hertz.

K = Stiffness Constant of the Spring, Newton / meter

M = Mass, kilograms

The error of this experiment is close to 5%. Figure 1 shows the setup for this experiment.
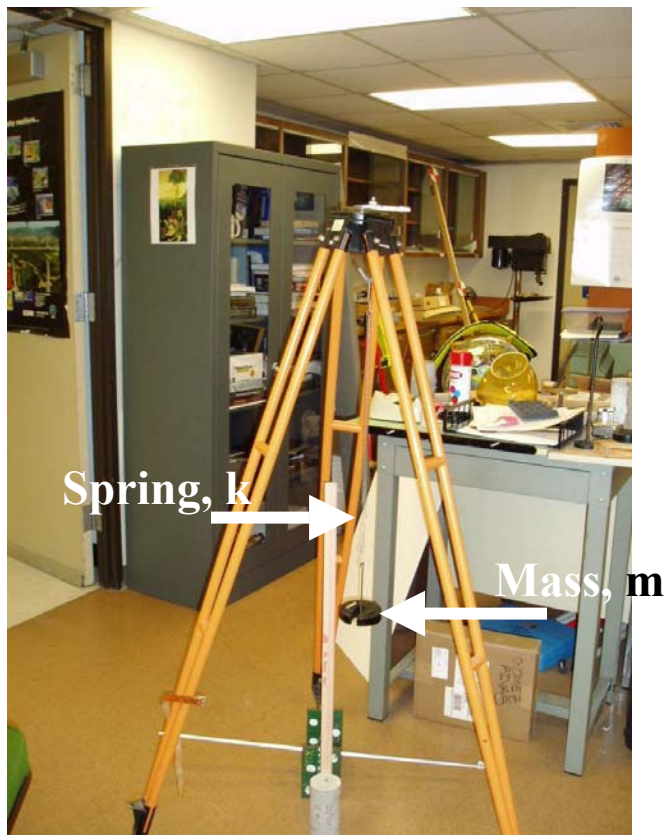

Figure 1 Setup for the Translational Single Degree of Freedom System

Experiment 2: Rotational Single Degree of Freedom System

The students constructed a rotational SDOF using a tripod, a mass, and an eccentric spring with respect to the center of gravity of the mass. The spring is the same as used in Experiment 1.

The dynamic equations are obtained from the dynamic equilibrium of the system shown in Figures 2 and 3:

$$I_o \ddot{\alpha} + m(R\ddot{\alpha})R = -k.d^2\alpha$$

or:

$$(I_o + mR^2) \ddot{\alpha} + k.d^2 \alpha = 0$$

Where:

$\ddot{\alpha}$ = Rotational acceleration of the system.

α = Rotational angle with respect to the axis of rotation.

k = Stiffness constant of a spring located at 'd' from the axis of rotation.

I_o = Rotational Inertia of the piece of W8x13 = mass * r_x^2

      mass = weight / g, where g is the gravity acceleration.

      r_x = gyration center respect to strong axis of W8x13 (from AISC manual)

m = additional mass located at a distance 'R' from the axis of rotation.

Defining:

M = $(I_o + mR^2)$ = Equivalent mass of the system

K = $k.d^2$ = Equivalent stiffness of the system

Then:

$$M \ddot{\alpha} + Kd^2\alpha = 0 \qquad (1)$$

And, finally, replacing M in equation (1) the natural frequency may be obtained:

$$f = 1/2\pi\sqrt{kd^2/(I_0 + mR^2)}$$
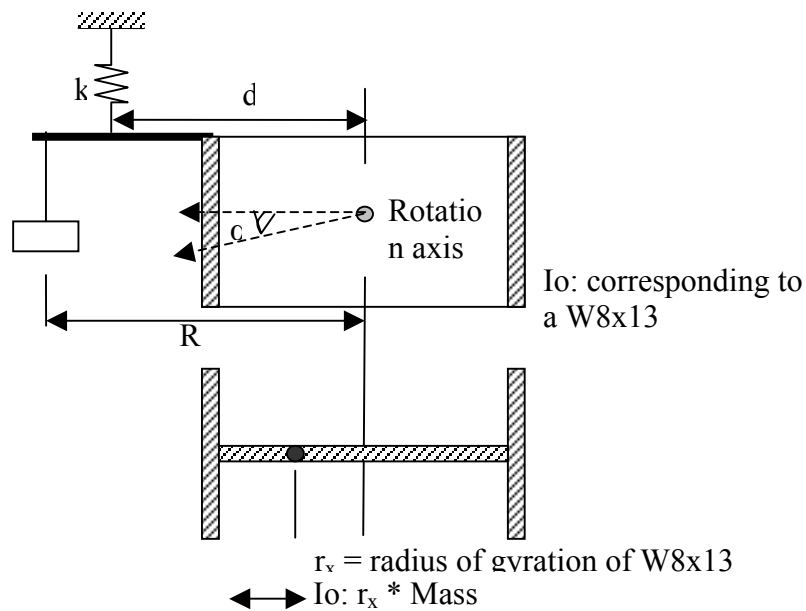
The error of this experiment is close to 5%.

Figure 2 Rotational Single Degree-of-Freedom System

Figure 3: Setup for the Rotational Single Degree-of-Freedom System

Experiment 3: Simply Supported Aluminum Beam

The beam is a square tube, 1 in. by 1 in. and 0.05 in thickness. The span center-to-center of supports is 96 in. Both supports are round bars, permitting free rotation and axial displacement of the beam which may be modeled as a pin and roller support.

The experiment started with a static test, as shown in Figure 4, designed to obtain deflection at the center of the aluminum beam to determine (and check) the modulus of elasticity of the material.

The aluminum beam was then tested dynamically in order to find its first three natural frequencies. The experimental results were compared with the mathematical model and with the results of a finite elements analysis.

A rubber bicycle wheel was used to impact the center of the aluminum beam. An oscilloscope was used to capture the vibration measurement signal from an accelerometer located at the one-fourth point of the beam span. Recorded data is voltage vs. time in second. After obtaining time domain data, Fast Fourier Transformation (FFT) analysis is used to convert the sampled data into voltage vs. frequency [2], as shown in Figures 5 and 6 where the prominent peaks corresponding to the structural frequency modes.

Only 512 readings of 1000 sampled data points acquired by the digital oscilloscope are used for FFT to compute the natural frequency. With the accelerometer located at one-fourth of the beam span, the experimental results for the first three natural frequencies are: 13.0 Hz, 48.0 Hz, and 112.4 Hz. This particular location of the accelerometer is selected to permit the capture of these natural frequencies.

The theoretical values from the closed formulation were obtained using the following equation:

$$f = \omega/(2\pi)$$

Where:

$$\omega = [(n^2\pi^2)/L^2]\sqrt{(E.I_x/m)}$$

f =   Natural frequency in cycles/sec or Hz for mode n

ω = Circular frequency, in rad/sec

n =   Mode number

L =   Beam span

E = Modulus of elasticity of aluminum

I_x = Centroidal moment of inertia of the beam respect to the horizontal axis

m = Mass per unit length of the aluminum beam

Which yields results of 12.5 Hz, 50.0 Hz, 112.4 Hz.

The computer model using Finite Element Method (FEM) found the first three natural frequencies as 13.0 Hz, 51.9 Hz, and 164 Hz respectively

The experimental results agree with the theoretical results with a difference of 3.8%, 4.1% and 0% for the first, second and third modes using the theoretical calculation, and 0%, 8.1% and 3.2% for the same modes using FEM. Figure 7 shows the theoretical results. The advantages of FEM computer models are that the student can visualize the vibration shape of the aluminum beam. Furthermore, the FEM model is more realistic, permitting the modeling of the accelerometer mass, eccentricities, etc.

Table 1 Characteristics of the Aluminum Beam

Length in.	96
Width in.	1
Thickness in.	0.052
Known Weight lbs	2.248
Experimental Vertical Displacement in.	0.146
EI = (w*L^3)/(488*d) lbs-in	284,000
I in^4	0.0296236
E = E*I/I psi	9,586,859

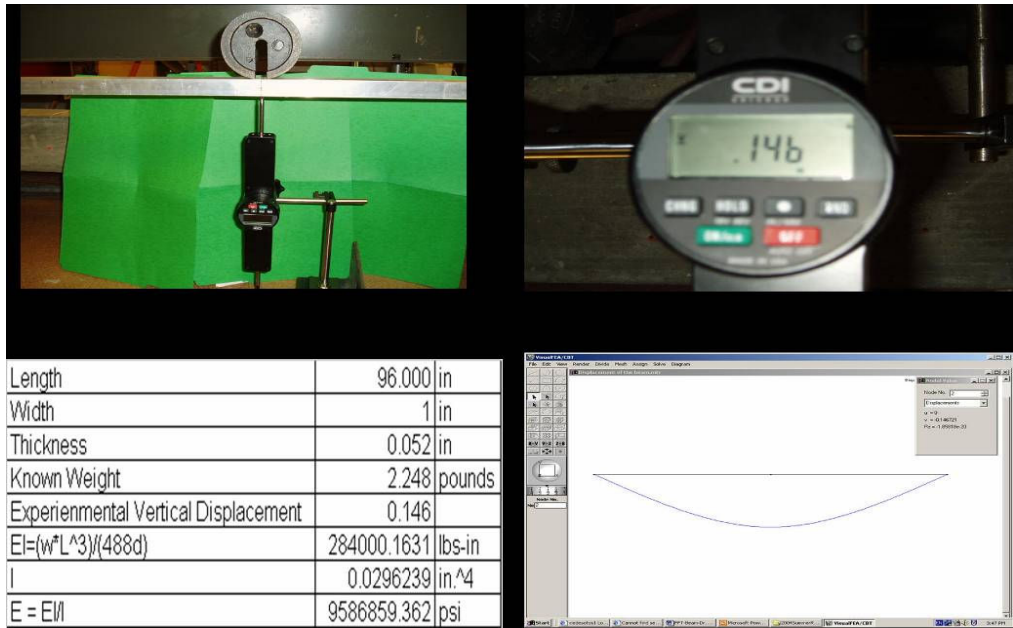| Length	96.000	in
Width	1	in
Thickness	0.052	in
Known Weight	2.248	pounds
Experienmental Vertical Displacement	0.146	
EI=(w*L^3)/(488d)	284000.1631	lbs-in
I	0.0296239	in.^4
E = EI/I	9586859.362	psi

Figure 4 Static Test of the Aluminum Beam

Figure 5 Aluminum beam, accelerometer and oscilloscope
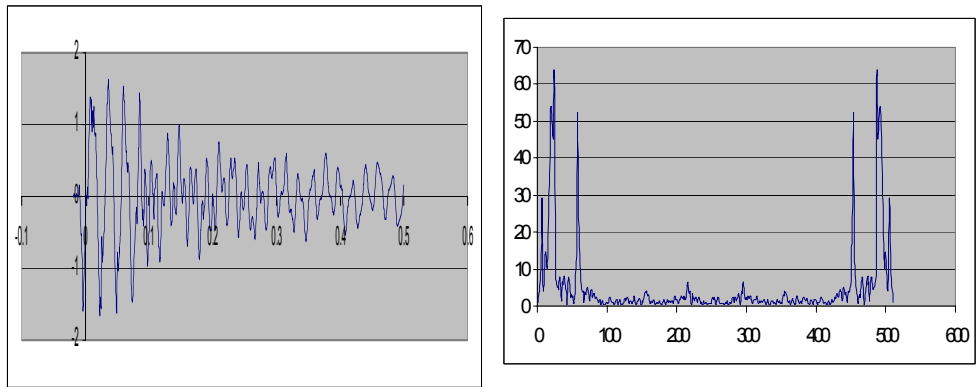
Figure 6 Aluminum beam response: Time domain vs. frequency domain

1st mode n=1	1
2nd mode n=2	2
3rd mode n=3	3
mass=lbs/in/gravity	5.28506E-05
L = Length (inches)	96
E = Elasticity (lbs/inch)	9586859.362
I=Inertia (inch^4)	0.0296239
Natural frequency = $f_{(cps)}$ = w/(2p)	
Circular frequency = w = $((p^2*n^2)/L^2)*(sqrt((EI)/m))$	
$f_{(cps)} =(n^2*p/(2L^2))*sqrt((EI)/m)$	
1st mode (Hz-cycle per second)	12.5
2nd mode (Hz-cycle per second)	50.0
3rd mode (Hz-cycle per second)	112.4

Modes calculated using a closed formulation.

First Mode using a FEM computer

Second Mode using a FEM computer

Third Mode using a FEM computer
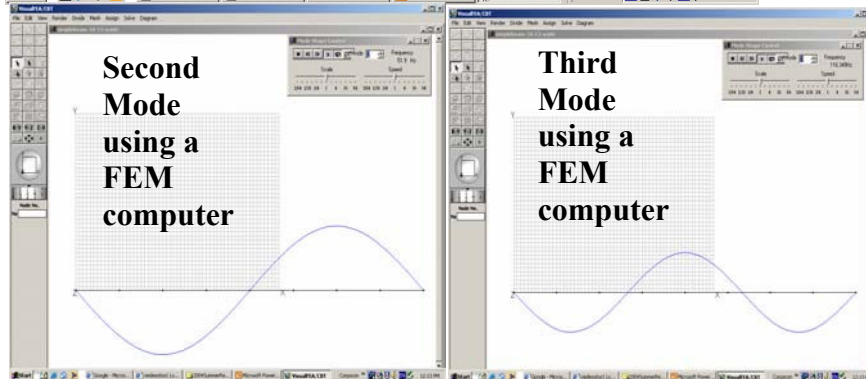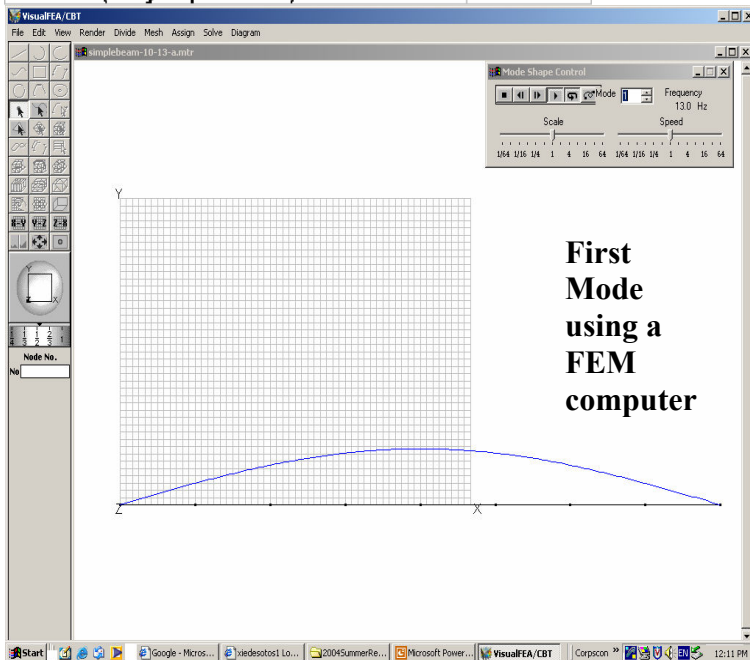
Figure 7 Theoretical results for the first three natural frequencies of the simply supported aluminum beam

Experiment 4: Reinforced Concrete T-beam, before cracking, after cracking, and after epoxy repair

A Reinforced Concrete T-Beam was designed, manufactured, and tested in order to understand its structural behavior for different levels of load. Concrete cylinders were tested on the same day of the T-Beam test. Thus, the Modulus of Elasticity and the concrete strength were well defined. The steel used was a #7 rebar. The T-beam was constructed as an assignment in the Reinforced Concrete class, but was tested during the semester by the students taking the Dynamics of Structures course. Figures 8 and 9 show the construction process and the geometry of the T-Beam.

The simply supported reinforced concrete T-Beam was tested dynamically to find the natural frequency when it was sound, that is, without cracks, obtaining a natural frequency of 52 Hz. After this test, the T-Beam was tested under a monotonic and static loading until the T-Beam showed plastic behavior, resulting in flexure cracks in the bottom of the beam and diagonal cracks in its web. After the T-Beam cracked, as shown in Figure 10, the natural frequency was obtained again, as 41 Hz, showing a variation of approximately 20% less than the frequency of the original uncracked beam. The test setup including the accelerometer and oscilloscope is shown in Figure 11. A typical curve of acceleration versus time and the corresponding frequency domain response is shown in Figure 12.

This T-beam was repaired after one year of the original tests. The repairing work consisted of sealing the larger cracks with epoxy injected under pressure. The thin cracks are not repaired because the equipment does not allow further repair. After crack sealing, the natural frequency is again obtained, showing the same value as those from the sound beam. This shows that small cracks do not change the natural frequency of the beam, only thick and large cracks are important.

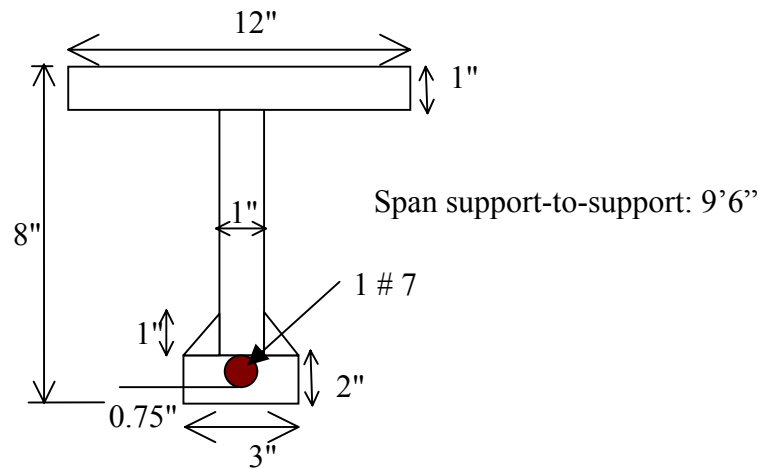**Figure 8** Construction of the Reinforced Concrete T-Beam
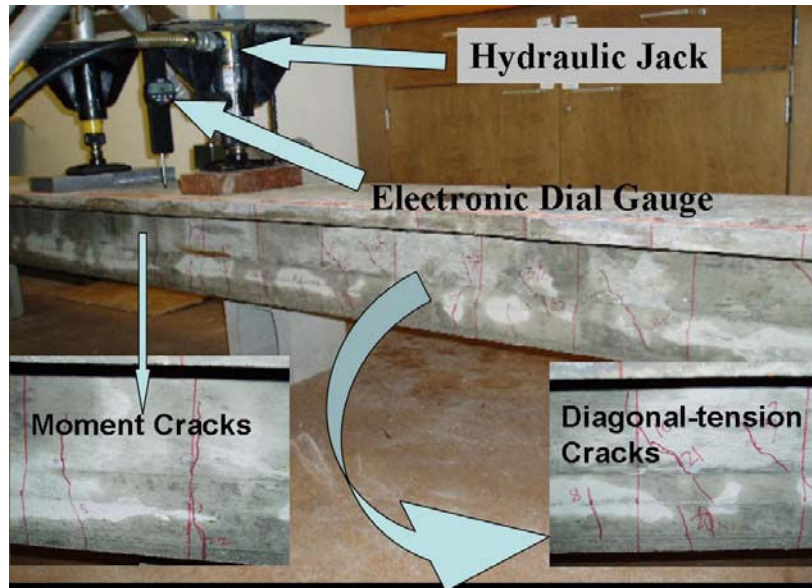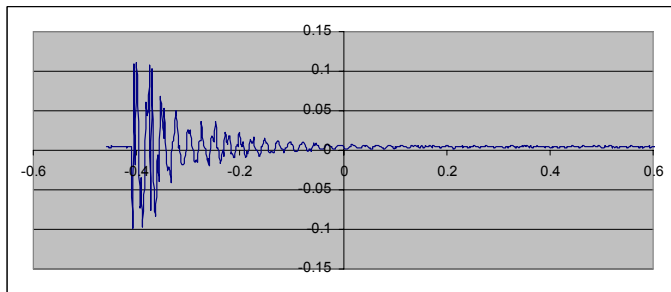
Figure 9 Geometry of Reinforced Concrete T-Beam

**Figure 10** Setup of the static test and cracks of the Reinforced Concrete T-beam

Figure 11. Accelerometer at L/4 and Oscilloscope on the Reinforced Concrete T-Beam

FFT (Fast Fourier Transformation) – is used to transfer time domain to frequency domain. The natural frequency of 52 Hz is obtained before cracking, and 41 Hz is obtained after beam failure.   There is a 20% of reduction in the natural frequency.

Time Domain Graph - After Cracking
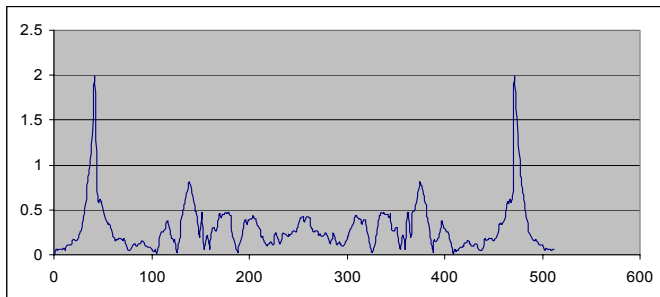
Frequency Domain Graph – After Cracking

Figure 12. Theoretical results for the first natural frequencies of the simply supported Reinforced Concrete T-Beam

Experiment 5: Post-Tensioned Concrete T-Beam before, after cracking, and after crack sealing.

The Post-tensioned Concrete T-beam was constructed as part of the objectives of the Prestensioned Concrete Design class, but was tested by the students taking the Structural Dynamics course. The T-Beam was designed, manufactured, and tested in order to understand its structural behavior for different levels of load. The concrete used was characterized using cylinders which were tested on the same day of the T-Beam test, in this way the Modulus of Elasticity and the Strength of the concrete were well defined. The high strength steel is a 3/8" strand. Figure 13 shows the geometry of the T-Beam.

The T-Beam was tested dynamically to find the natural frequency when it was sound, or without cracks, obtaining a natural frequency of 35 Hz. After this test the T-Beam was tested under a monotonic and static loading until the T-Beam showed its plastic behavior, resulting in flexure cracks in the bottom of the beam, as shown in Figure 14. The natural frequency of the cracked T-Beam was measured to be 29 Hz, showing a variation of approximately 20% less than the frequency corresponding to a sound beam. The setup of the accelerometer and oscilloscope is similar to that of Figure 10.

This T-beam was repaired after 6 months of the original tests. The repairing work consists in sealing the thick cracks with epoxy injected under pressure. The thin cracks are not repaired because the equipment does not allow sealing of thin cracks. After crack sealing, the natural frequency was obtained, showing the same value as the frequency of the sound beam. This shows that small cracks do not change the natural frequency of the beam, only thick and large crack are significant.
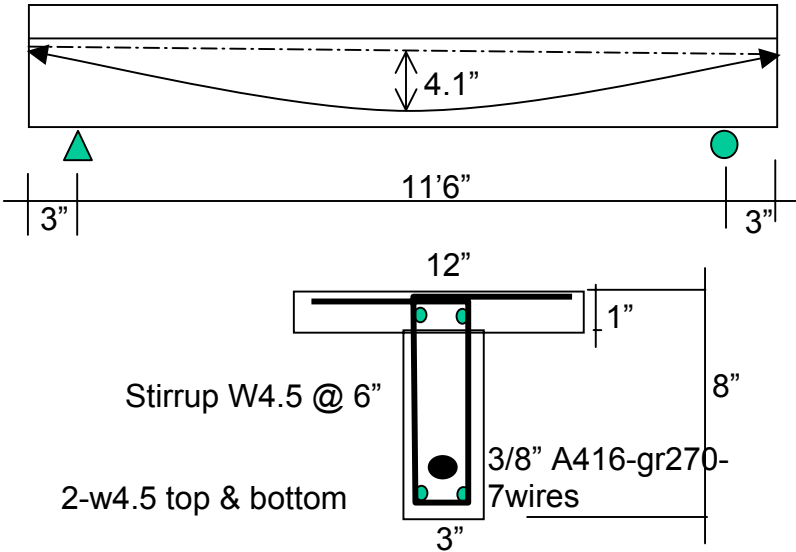

Figure 13. Geometry of Post-Tensioned Concrete T-Beam

Figure 14.   Setup of the static test and cracks of the Post-Tensioned Concrete T-beam

Experiment 6: One Bay – Two Stories Steel Frame

A steel frame model was constructed using 2 in x 0.125 in steel plates. The frame bay has approximately 24 in span, and consists of 2 stories of 12 inches in height.

The first task for the students is the measurement of the real dimension of the cross-sections, bays and heights in order to model the frame using commercial computer software.

Static horizontal loads are applied to the frame using a pulley system, as shown in Figure 15. The horizontal displacements at each story are measured using digital dial gauges, and the strain at the base of the column is measured using a strain gage and a strain indicator. These values are compared with the computer program result, shown at Figure 16, obtaining the real modulus of elasticity of the material, which is 29,900 ksi, and this value is used for the dynamic analysis.
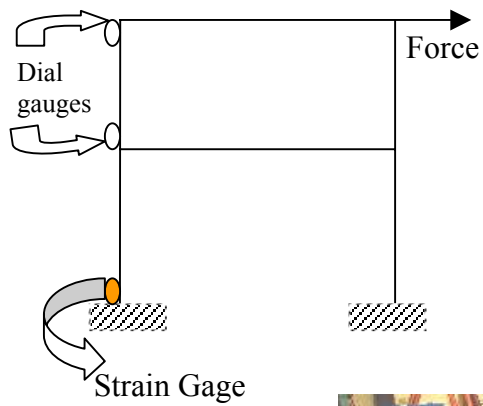
1. A strain gage is attached at the bottom of a column.
2. The strain gage was connected to a strain indicator.
3. The frame was bolted to a rigid foundation.
4. A pulley was installed on another structure which is near to the frame.
5. A digital dial gauge is placed at each story.
6. Horizontal forces were applied through the pulley.
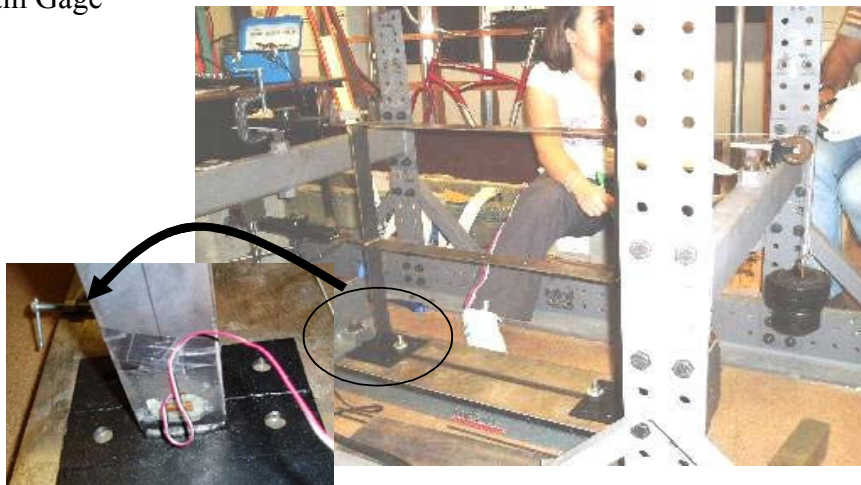7. Deformation and strain were recorded.

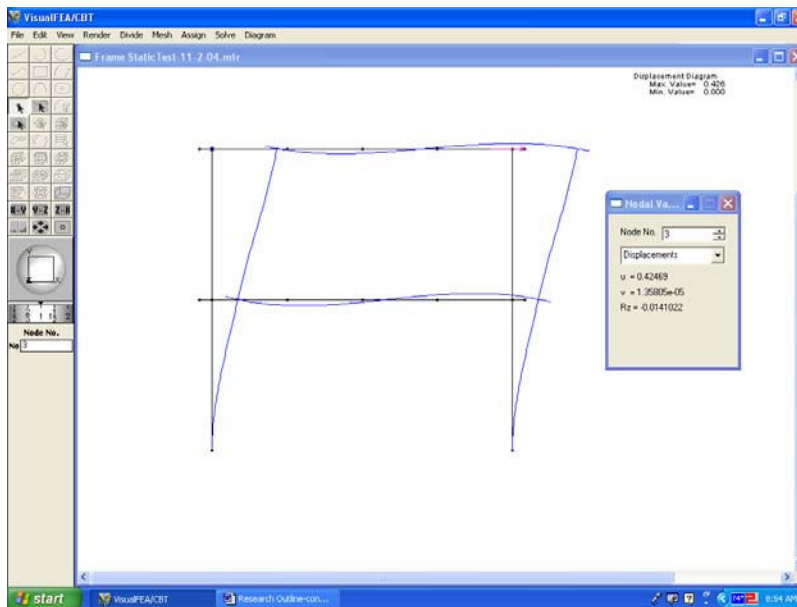**Figure 15.** Setup for the Static Test

Figure 16. Displacement of the beam from the Finite Element program

Two different tests are performed on the same frame. The first test is for the frame only, without any additional mass, and the second test consists of the frame with additional masses in the middle of the beams.

The frequencies of the frames are obtained using three experimental methods. The first one consists of using data from an accelerometer located at the second story, and the second method consists of using data obtained from the strain gage located at the column base. Figure 17 shows the setup for these methods. The frame is initially excited with a tap using a rubber hammer, and the data (voltage versus time) from the accelerometer or strain gage is recorded in the computer. A Fast Fourier Transformation (FFT) program is used to find the natural frequencies.

The third method consists of application of a dynamic force using an electronic function generator that sends sinewave to a coil carefully placed near to the frame, generating an electromagnetic force to the top of the steel frame. The frequency is varied and resonance of the frame occurs when the excitation frequency is equal to the natural frequency. This non-destructive method has the advantage that it may be repeated easily. Figure 18 shows the setup for this method.

The theoretical results are obtained using the computer program Visual Analysis v5.5. The theoretical mode shapes and frequencies are shown in Figure 19. They are obtained by modeling the frame with the real dimensions of the model, and considering the mass of the accelerometer only or from the masses added to the stories.

Table 2 shows that the theoretical and experimental results for the frame without additional masses are in very good agreement; the difference is less than 3%. When additional masses are considered, the theoretical and experimental values diverge by 17% and this may be due to the difficulty in modeling the effect of the added masses on the rigidity and total mass of the structure, and also may be due to rocking of the additional masses. However, the difference in the second mode is smaller.
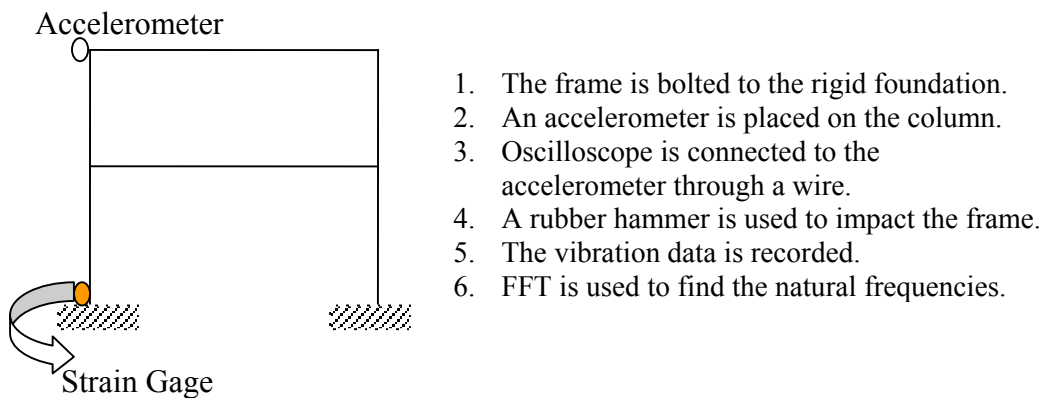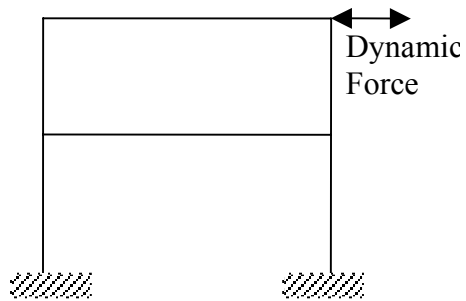
Accelerometer

1. The frame is bolted to the rigid foundation.
2. An accelerometer is placed on the column.
3. Oscilloscope is connected to the accelerometer through a wire.
4. A rubber hammer is used to impact the frame.
5. The vibration data is recorded.
6. FFT is used to find the natural frequencies.

Strain Gage

Figure 17. Setup for the Dynamic Test Using an Initial Excitation

Figure 18. Setup for the Dynamic Test Using a Dynamic Force

1. The frame is bolted to the rigid foundation.
2. A solenoid is closely secured to the frame.
3. A function generator is connected to the solenoid.
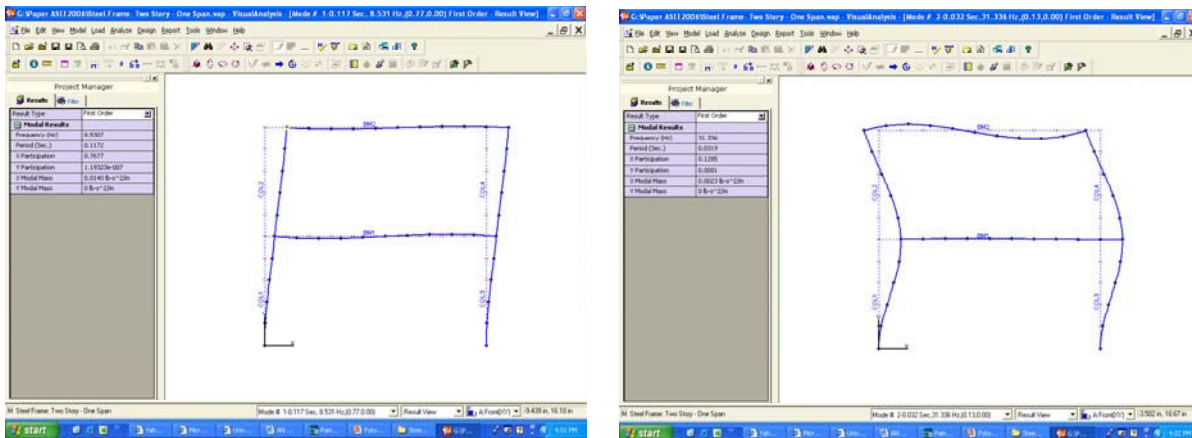4. Frequency is changed until resonance is observed.

Figure 19. Theoretical Natural Frequencies Using the Computer Program VisualAnalysis v5.5

Figure 20 Frame with Additional Masses on Beams

Table 2 Theoretical and Experimental Results

Test	Method	Natural frequency Mode 1 (Hz)	Natural frequency Mode 2 (Hz)
Self Weight only	Accelerometer	8.8	32.2
	Strain Gage	8.5	31.3
	Dynamic Force Inducing Resonance	8.5	--
	Theoretical from Visual Analysis v5.5	8.5	31.3
Adding a weight of 3.299 lb at 2nd story, and 3.054 lb at 1st story	Accelerometer	6.8	22.5
	Strain Gage	6.8	22.5
	Dynamic Force Inducing Resonance	6.8	--
	Theoretical from Visual Analysis v5.5	5.8	21.7

Conclusions

A Structural Dynamics course for undergraduate engineering technology students was developed using the basic theory of natural frequencies and a set of experimental exercises in order to permit the students to easily assimilate the theoretical concepts.

These experiments range from the basic one degree-of-freedom system to more complicated experiments using beams and frames. Test specimens constructed in other courses, such as the Reinforced Concrete and the Postensioned Concrete T-Beams were used also to demonstrate that the theory works for real structures.

The professor can develop different methods and experiments to teach this course. It is important the use of a Finite Elements Computer program to compare partial results and reach conclusions about the effectiveness of the experiment.

Reference:

[1]. Tedesco, J. W., McDougal, W. G., and Ross, C. A. *Structural Dynamics: Theory and Applications.* Addison Wesley Longman, Inc., 1999.

[2] Feng, W., Gomez-Rivas, A., and Pincus, G., "An Experimental Approach for Evaluating Harmonic Frequencies of a Flexible Beam", ASEE 2005, Portland, Oregon.

Acknowledgement