2006-975: AN INNOVATIVE INQUIRY-BASED EXPERIMENT ON THE TEMPERATURE DEPENDENCE OF THE RESISTANCE OF A FILAMENT LAMP

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An Innovative Inquiry-Based Experiment on the Temperature Dependence of the Resistance of a Filament Lamp

Abstract

We have developed an innovative inquiry-based student laboratory activity dealing with the temperature dependence of the resistance of a filament lamp. This introductory experiment is appropriate for a second semester general physics laboratory. The hands-on, active learning laboratory experiment utilizes simple, inexpensive equipment to facilitate student learning of various direct current circuit concepts. The activity builds upon the results of previously published physics education research.

Students perform a transient measurement of the current-voltage characteristic of a lamp in series with a charging (or discharging) capacitor. From the data students calculate the resistance and power dissipated in the lamp. Under normal operating conditions, i.e. the filament glowing brightly, the resistance-temperature characteristic of the filament is seen to exhibit power-law behavior.

I. Introduction

We have redesigned our general physics laboratory to incorporate inquiry-based experimentation into the laboratory experience of our students.[1] We believe that this approach leads to improved student outcomes. We have recently compared the performance of a broad cross-section of our students with those published in a national study.[2] Our results indicate that our students perform slightly better than the national average.[3]

In one laboratory experiment, students measure the current-voltage characteristic, and thereby the resistance, of an incandescent lamp. They are provided with the usual definition of resistance, $R \equiv \Delta V / i$, and also with Ohm's Law $\Delta V = iR$, where ΔV is the voltage difference across the resistor and i is the current. In the first activity, students design a circuit to measure the resistance of a #40 lamp and determine if the lamp obeys Ohm's Law. The #40 lamp is a miniature threaded-base incandescent flashlight lamp rated at 125mA at 6.3V. By using from one to six lamps in series, students are able to generate a current-voltage characteristic of a single lamp. They are generally surprised to find that the graph is nonlinear and that the resistance of the lamp is a function of the current.

In a subsequent experiment, students obtain quantitative data for the charging and discharging of a capacitor through a #40 lamp. The lamp is placed in series with a capacitor and an experiment is conducted to study the time-characteristic of the current through the capacitor, as well as the current-voltage characteristic of the lamp in a dynamic context. In a previous paper, we presented a numerical model that accurately described the time-varying characteristics of the current through the capacitor.[4] The model was implemented in the introductory laboratory.

In this paper, we study the temperature dependence of the resistance of the lamp using data from the *RC* circuit experiment. The paper is arranged as follows: In Section II, we give a brief introduction to the theory of *RC* circuits with varying resistance. We also present a model for the temperature dependence of the resistance of a lamp filament. In Section III, we describe the experimental results and compare them with previously published research. We summarize our findings in Section IV.

II. Theory

The treatment of *RC* circuits in introductory physics textbooks typically involves an uncharged capacitor placed in series with a battery and constant resistance *R*.[5,6] The differential equation for the circuit is written in terms of the time-varying current *i(t)*, as

$$\frac{di}{dt}+\frac{i}{RC}=0. \tag{1}$$

The equation leads to the well-known exponential solution for *i(t)* as

$$i(t)=i_0 e^{-\frac{t}{RC}}, \tag{2}$$

where $i_o = \varepsilon/R$ is the initial current in the circuit and ε is the battery emf. This result can be reproduced in a traditional laboratory experiment by plotting the current as a function of time using data obtained from a large time-constant circuit.[7] The exponential decay of the current leads to a well-defined time-constant $\tau = RC$ that can be measured.

From a pedagogical perspective, however, this experiment raises a significant problem in that it does not resonate well with the inquiry-based laboratory experimentation pioneered by Arons[8] and McDermott.[9] In the inquiry-based approach, students construct rules for the behavior of circuits containing batteries and lamps by observing the brightness of lamps connected in various combinations with a battery. The brightness of a lamp is understood to provide a qualitative visual measure of the amount of current flowing through that lamp. A simple extension of this approach allows for a quantitative study of the behavior of an *RC* circuit with a capacitor in series with a lamp. However, the varying resistance of the lamp invalidates the solution presented above for the time-varying current *i(t)* and the definition of the time-constant. Hence a different approach is needed to solve for the circuit characteristics.

Kirchoff's Voltage Law for a capacitor in series with a resistance *R* leads to the equation

$$\varepsilon - iR - \frac{Q}{C} = 0, \tag{3}$$

where the symbols have their usual meaning. If we use the chain rule and take the time derivative of Eq. (3) we have,

$$R\frac{di}{dt}+i\frac{dR}{dt}+\frac{1}{C}\frac{dQ}{dt}=0, \tag{4}$$

assuming the value of the capacitance is constant. The last term can be rewritten by noting that $i=\frac{dQ}{dt}$. Hence, for the case of an *RC* circuit with varying resistance, the governing differential equation can be written as

$$\frac{di}{dt}+\frac{i}{R}\frac{dR}{dt}+\frac{i}{RC}=0. \tag{5}$$

The second term represents the changing resistance of the lamp in time. It is possible to model the solution in two stages. First, the resistance of the lamp, as a function of the current, is tabulated from a plot of the measured current-voltage characteristic of the lamp. Equation 5 is then solved numerically by converting it into a difference equation and using the values of the current and resistance at successive time-steps. The resulting solution for *i(t)* is found to accurately reproduce the experimental data.[4]

Additionally, one can study the temperature dependence of the resistance of the lamp using data obtained from the same experiment. This is easily accomplished by placing the lamp in series with a charging capacitor. The temperature of the filament is varied as a result of the varying current through the lamp. Following Brizuela and Juan[10], we assume that the resistance obeys a power law of the form $T \propto R^{\gamma}$. Consequently, the power dissipated by the lamp can be written as

$$P = i\,\Delta V = A\sigma T^4 = \text{const}\; R^{4\gamma}, \qquad (6)$$

where A is the surface area of the filament and σ is Stefan's constant. Previous research[11] demonstrates that the assumption in Eq. (6) is justified at high current values, where the filament operating temperature is over 500°C ensuring that the dominant mechanism of heat loss is radiation. In addition, as long as the thermal relaxation time of the lamp filament occurs on a time-scale much shorter than the rate at which the current in the filament changes, the filament behaves as if it is going through a succession of equilibrium states with well defined temperatures. In this situation, a log P – log R plot would be a straight line with a slope given by 4γ.

III. Experimental Results and Discussion

The circuit used in the experiment is shown below in Fig. 1. We use a PASPORT Xplorer Datalogger (Pasco PS-2000) with a current-voltage probe (Pasco PASPORT Voltage-Current Sensor PS-2115) to measure the current through the capacitor and the voltage across the lamp. The voltage sensor has a ±10V range with 0.005V resolution while the current probe has a ±1A range with 500μA resolution. The probe takes 1000 samples per second and the data is stored in the datalogger in 0.10s intervals. Pasco's DataStudio Software is used to extract the data from the datalogger and export it into Excel.

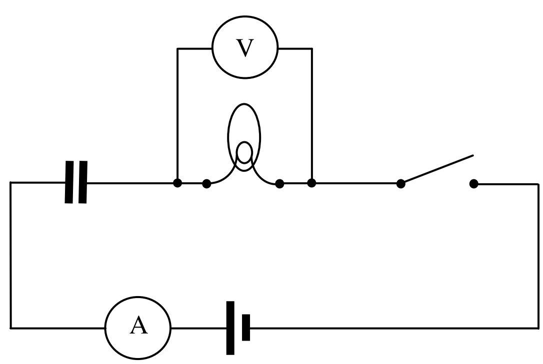

Fig.1. Schematic diagram of the circuit used to measure the current through the capacitor and the voltage difference across the lamp.

The resistance of the lamp is extracted from the current-voltage data at each time-step of the charging process. In Fig. 2, the current through the lamp as a function of voltage is shown for the static and the dynamic cases. The data for the static case in Fig. 2 was obtained by replacing the capacitor with a sequence of resistors and measuring the current and voltage. The data demonstrates that during the transient measurement the filament is indeed passing through a sequence of equilibrium states with well defined temperatures. It is readily apparent that the lamp does not obey Ohm's Law. In addition, it can be seen that more data points are located at the lower end of the current scale because the probe records data every 0.10s and as the capacitor charges, the current varies more slowly.

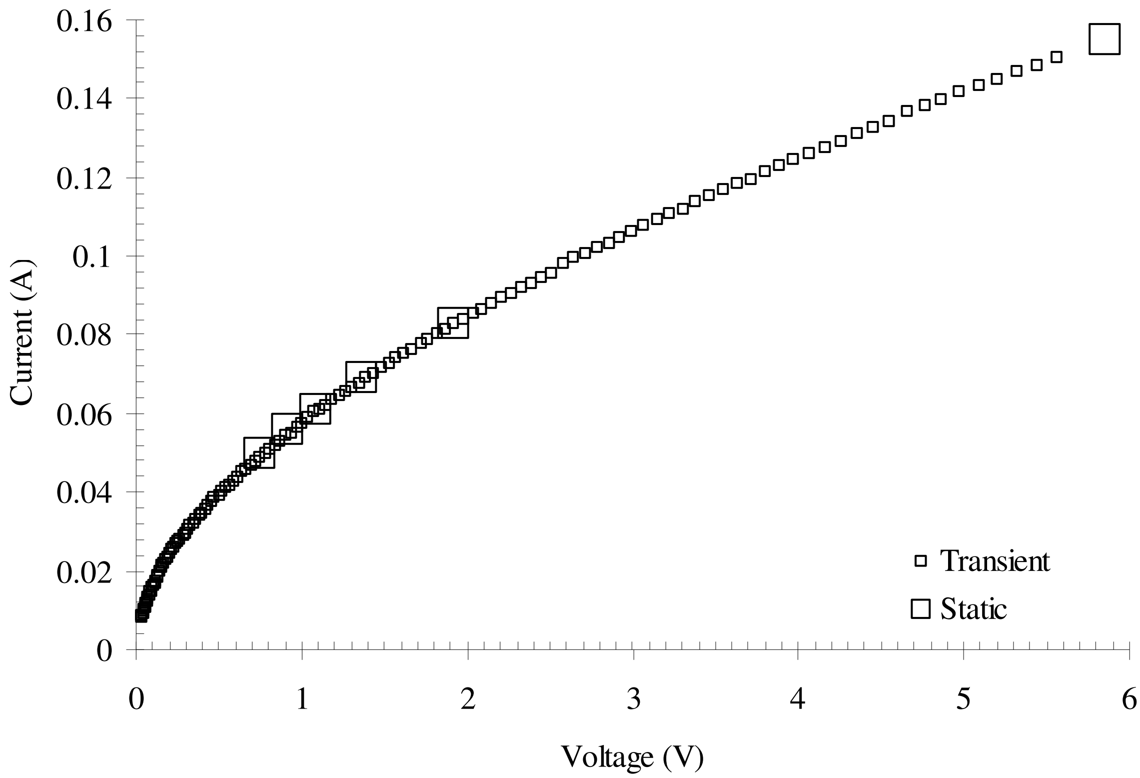

Fig. 2. The current as a function of voltage for a #40 lamp. Data points from the static case are shown as large squares.

Armed with this information, the power dissipated in the lamp filament can be calculated using the values of the current and voltage. Figure 3 below shows a typical logarithmic plot of power and resistance. Based on the resolution of the current and voltage probes, we estimate the measurement error in the power and resistance to each be less than 1.5% over the entire range of values plotted in Fig. 3. While the data was generated over the entire range of values of the current, the plot has been truncated at a lower limit of 50mA of current. This ensures that the graph reflects only the temperature range in which the radiative mechanism of power dissipation is dominant.

The linearity of the plot is strong evidence that the temperature and resistance are related via a power-law of the form $T \propto R^{\gamma}$. From the slope of the line in Fig. 3 the value of γ is determined to be 0.89. The results that we obtain are similar to those found in previously published work.[12]

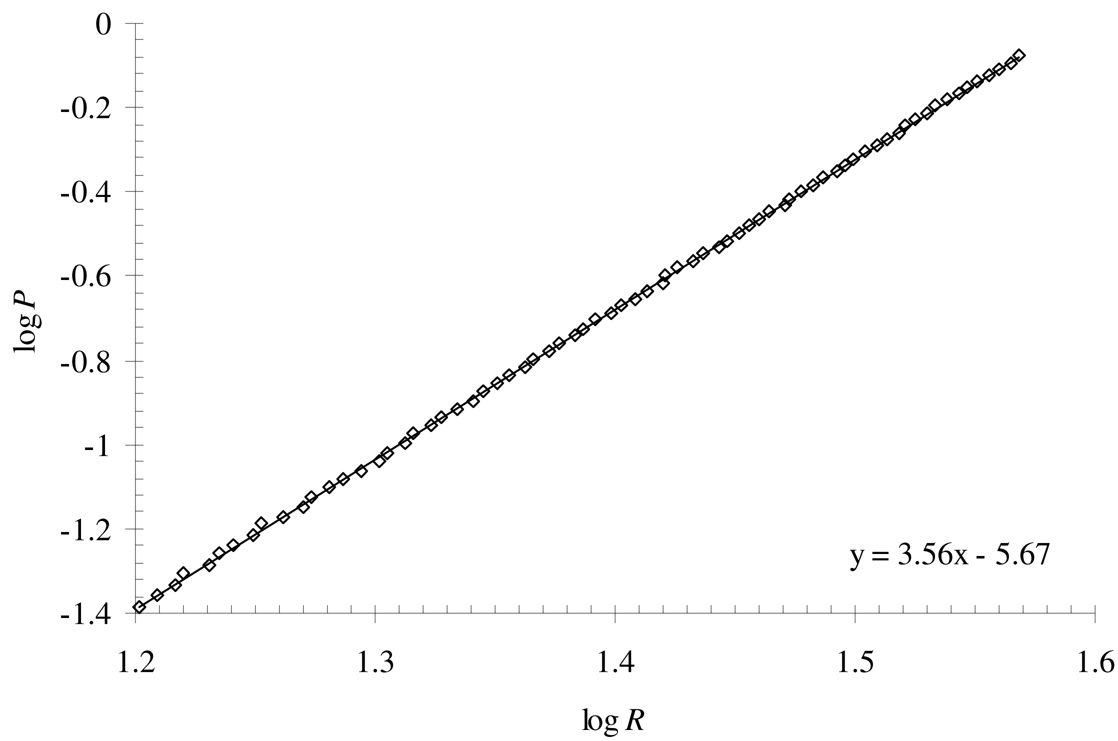

Fig. 3. Power dissipated as a function of the resistance for a #40 lamp in a transient circuit, shown on a log-log plot.

IV. Conclusion

We have developed an experiment that bridges the gap between the inquiry-based approach and introductory textbook treatments of *RC* circuits. This experiment can easily be implemented in an undergraduate physics laboratory. We are able to model the temperature dependence of the resistance of a lamp and show that it obeys a power law behavior at high current values when the lamp is glowing brightly. The model provides a good fit to the experimental data.

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