AN INSTRUMENTED EGG DROP EXPERIMENT IN SUPPORT OF COURSES IN MECHANICAL CONTROL AND EXPERIMENTAL ANALYSIS

David Walters, Vince Wilczynski, Kirk Hiles
United States Coast Guard Academy
New London, Connecticut

ABSTRACT

During the Fall Semester of 1996, digitized video and real time force data were used to determine the system characteristics necessary to model a bungee cord egg drop experiment as a damped second order system. This effort was unique, in that, video tape of student "drops" were digitized and analyzed to determine position, velocity, and acceleration, as a function of time, while simultaneous in-line force data was acquired. Both sets of were used to determine the system damping factor and damped natural period. These parameters were then used by the students to calculate the system response with excellent results.

An attempt was made to use the force data to infer acceleration, velocity, and final position during the first half cycle of motion. However, this effort was unsuccessful since the mass of the elastic cord was not negligible. The experience with the force data did highlight the subtleties associated with such an experiment, in terms of the overall goal of modeling the system as a damped second order system.

INTRODUCTION

One of the goals of the USCGA courses "Mechanical Control of Dynamic Systems" and "Experimental Methods in Thermal and Fluids Sciences" is to use design projects to demonstrate phenomena and solve problems. A unique project, involving having students design a bungee cord to minimize the distance from the ground that an attached egg reaches when released from a distance of 35 feet above the ground. In the mechanical controls class, the students design the bungee cord system and in the experimentation course, the bungee cord's characteristics are determined and the experiments is conducted. Since the project includes mathematical modeling, numerical methods solutions of differential equations, design, system response, and the collection and analysis of experimental data, the exercise is a capstone experience for the mechanical engineering and naval architecture/marine engineering students.

Through this project, the students are exposed the dependence of academic courses, for they see how material covered in one discipline is used in another discipline. The project is design driven, for the students have to design their own system, and experience the joys and/or frustrations of transferring a design done on paper into a working system. The necessity for and results of experimentation are highlighted in the project, for it is only through experimentation that the system's performance characteristics can be identified. Similarly, it is through experimentation that the designs are verified and compared to theory. The value of concomitant methods of data collection is demonstrated, as the position, velocity, acceleration of the falling eggs are determined using two separate techniques, namely video recording and force measurements.

MECHANICAL CONTROL OF DYNAMIC SYSTEMS: MODELING AND SOLVING THE EQUATION OF MOTION

The course "Mechanical Control of Dynamic Systems" at the USCGA has three purposes, to introduce system modeling, solving equations of motion to predict system response, and controlling system response. In the egg drop project, the students first model the mass-spring-dash pot system of an arbitrary mass that is attached to an elastic cord and falls from a given height. In this portion of the project, the students begin with a free body diagram of the system, and create two equations of motion: one for free-fall and a second when the motion is restrained by the elastic cord. The body is restricted not only by the damping and spring forces in the attached cord, but also by the drag force on the body (which is a function of the square of the velocity).

The resulting set of second order differential equations, one for free fall that involves the mass, acceleration, and velocity of the object, and the other that involves these same parameters plus the frictional and damping forces of the attached bungee cord, are solved with a Runge-Kutta algorithm using MatLab. Once the students verify that their modeling is correct and the computer solution of the displacement versus time is correct for this specified case of a given mass, spring constant, and damping coefficient, they use this method to design their own mass-spring-dash pot system.

For the actual design, the mass is an uncooked egg and the height that the body may fall is the height of the engineering building. Their design task is to optimize the cord length and the values of the cord's spring constant and damping coefficient such that the falling egg comes as near to the ground as possible, but does not touch the ground. The problem is further complicated by having them maximize the total time the system oscillates. With these ideal values for their system determined, the project is now ready to be completed in the experimentation course.

EXPERIMENTAL METHODS IN FLUIDS AND THERMAL SCIENCES: DETERMINING SYSTEM PARAMETERS AND VERIFYING THE DESIGN

Armed with the ideal values for the cord length, and its damping coefficient and spring constant, the students complete the project in the USCGA experimentation course. During this phase of the project, each student team is given five feet of bungee cord to experiment with, and they must experimentally determine the cord's material properties. Once the material properties are determined, they scale their results for multiple strands and greater cord lengths, and then conduct the experiment where the egg-cord system is dropped from a height of 35 feet. The results of the experiment are collected on video and via a force block that is part of a CPU based data acquisition system. Each of these components of the project will be described in this section.

The first stage of the experimentation component of the project was to determine values for the bungee cord's spring constant and damping coefficient. Once these values were determined for the material at hand, the MatLab computer algorithm developed in the first phase of the project was used to determine the optimal cord length via a CPU based trail and error method. To determine the cord's spring constant and damping coefficient, the students used their sample cords with a data acquisition system that measured displacement and acceleration. Here, an ultrasonic wave was used to measure the position of a suspended body at any time, and a commercial accelerometer was used to record accelerations. During this part of the experiment, the students were able to evaluate the performance of each of these measuring devices, and experience the limitations of each device. Ultimately, most found the ultrasonic positioning system more useful for the project at hand thanks to its increased resolution over the accelerometer.

With the mass of an egg suspended on a single strand of bungee cord, the mass was displaced and released. The resulting oscillatory motion (both velocity versus time, and acceleration versus time) was recorded by the data acquisition system. Also, the lab set up allowed the students to easily determine the displacement of the cord when the mass was suspended from it (thereby leading to the cord's spring constant). From these data files, the exponentially decaying system response could be displayed, and the damped natural frequency, un-damped natural frequency, spring constant, and damping coefficient could be determined. Given this information for a single length of cord measuring a specific length, the students could extrapolate their results to calculate the spring constant and damping coefficient for any length of multiple cords. This part of the project was done outside of the normal lab time for the course. For the actual egg-drops themselves, each student team entered the lab and had to declare the length of the cord and the number of strands they wished to use in their design. They were supplied with this amount of

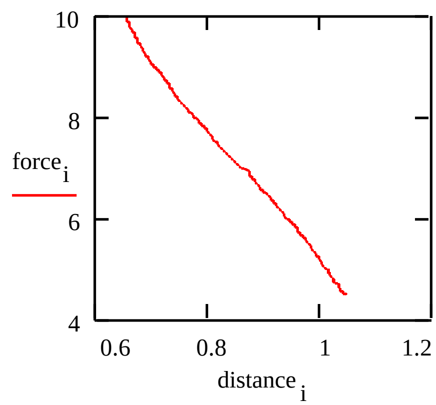
cord, and then attached snap-hooks to each end of their cord so it could be mounted to the release mechanism and to the bag holding the uncooked egg. At the given signal, their egg was released from the roof and experienced free fall and restrained flight as it fell (almost) 35 feet to the ground. The flights were simultaneously recorded on video and via a force block attached to the restrained end of the cord.

A standard video camera, capable of taking video at the rate of one frame every 1/30 of a second, was used to record the twenty seconds of oscillations for each team. This video was then digitized on a multimedia computer (using Adobe Illustrator software) and then played back on the computer. To determine distances, the wall was marked in one foot intervals, and the students had to calibrate the marking system to determine distances on the digitized video. As an example, it was helpful to know the height of an individual brick on the wall, the height of the tape used to mark the distances, and the distance between the release mechanism and the first wall marking and the release mechanism.

With this scaling information, the video was played back, and the record of position versus time was manually created by the students. While commercial particle coordinate tracking software is available, it was not used in this application. It is noted that each 20 second analog video occupied at least 50 MB of disk spaced when digitized, and therefor only the analog tapes were treated as historical files that were preserved (i.e. the digitized files were discarded after being used). Since digitizing the video was as simple and fast as showing the video through the software, this approach demonstrated the effective use of archival methods (i.e. preserve the analog and treat the digital files as disposable).

While the maximum sampling rate of the system at 1/30 second due to the camera's frame rate, most students selected a slower sampling speed to compute the path of the egg during its fall. Typically, rates as slow as 0.5 samples/second could be used to determine the system response from the digitized video. While the instructors had hoped for better temporal resolution of the digitized video, the students had other priorities.

There were two points in the egg drop experiment at which a force gauge was used. First, a laboratory set-up was used by the students to acquire force and distance data so that they might be able to deduce the proper stiffness and damping coefficients, on a per unit length basis, for use in their predictive models. The laboratory set-up consisted of a Vernier Software Incorporated force gauge. This force gauge, uses a small copper beam to which the load is attached and a magnetic proximity detector. The central deflection of the beam as measured by the proximity detector is converted to force. A simple calibration with a known mass can be conducted prior to data acquisition. In order to accurately measure distance, a Vernier Software Inc. Ultrasonic Distance transducer was used. This transducer was placed on the floor directly below the 'bungee' cord which was suspended from a laboratory stand atop a lab bench. This arrangement allowed for approximately one and a half meters of total separation between the end of the cord and the distance monitor. Students attached the force gauge to the bottom of the short length of 'bungee' cord and then simply pulled the cord downward toward the distance transducer. The distance transducer used the force gauge as a 'target' and provided a known distance measurement simultaneous to the force experienced by the 'bungee' cord. This data was saved as an ASCII file and plotted as shown.



The slope of the resultant graph provided the students with a solid estimate of the bungee cord's spring constant. Students also obtained an estimate of per unit length damping factor by suspending a mass equivalent to their egg from the 'bungee' cord, which in turn was attached to the force gauge. The mass was deflected and the force oscillations recorded. Students used the resulting time records to determine the log decrement, and thus, the damping factor.

During the actual egg drop data acquisition, in-line force data was acquired using the Vernier Software Incorporated force gauge. This force gauge was mounted horizontally on the egg launcher and the students' 'bungee' cord was attached by means of a small brass swivel at the end of a short piece of mono-filament fishing line which passed over a sheave. The swivel hung vertically so that the force measurements would not be affected by the stick and slip of the rubber 'bungee' cord over the sheave.

The data from the force gauge was acquired through a serial port interface to Vernier Software Inc.'s MacMotion software with a sampling rate of 100 Hertz. The data was stored as an ASCII file on a Macintosh Powerbook portable computer. The initial start time of an egg drop was difficult to record. However, for a given data run, data acquisition was begun just prior to launch. Then, the launcher cord, which was attached to a small sliding trap door, was pulled abruptly in order to launch the egg and create a force spike which might be used in the data analysis as an initial time marker.

ANALYSIS

The data collected in the experiment allowed the students to compare the theoretical performance of their second order system to the actual performance. While secondary to the task of learning about second order response and measurement systems, it is noted that the winning design came within six inches of the pavement, and oscillated for about 25 seconds.

The lab exercise allowed the chance for the students to apply mean and standard deviations to their results when using outlier rejection methods. Also, some of the better teams reported their results using uncertainty analysis, and showed how the uncertainty analysis explained some of the differences between the actual and theoretical values.

During the actual drops, an instructor acquired force data simultaneous to the video data, and used this data as a check of the students' analysis in terms of the damping factor, damped natural frequency and consequently, the natural frequency. The data was exported as an ASCII file from the MacMotion acquisition software and graphed for verification. A total of 2400 data points were acquired per run. The following steps in Mathcad, read the data, strip the columns dedicated to force and time respectively, zero the data relative to the weight of the egg and bungee cord, and display the results.

The two dotted vertical lines represent time markers which were used to identify peaks and subsequently to pick particular data points for calculation of the log decrement and damped natural period. As shown above, there are five cycles between the time markers.

$$s := 5 \qquad \tau_d := \frac{TIME_{EE} - TIME_E}{s}$$

$$\tau_d = 2.16$$

These same indices which were used to identify the peaks for the damped period can be used to calculate the log decrement.

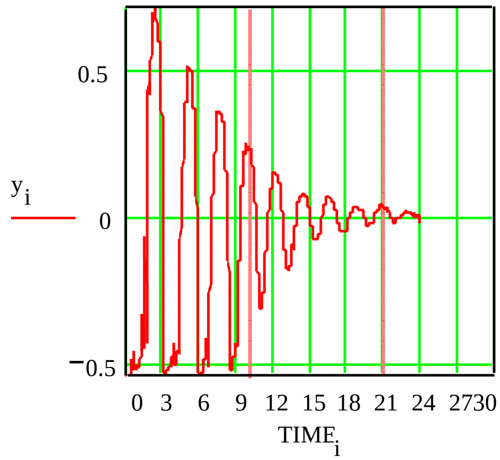
$$\delta := \ln\left(\frac{y_E}{y_{EE}}\right) \qquad \zeta := \frac{\delta}{\sqrt{\delta^2 + 4 \cdot \pi^2 \cdot s^2}}$$

$$\zeta = 0.05559$$

$$i := 0..\,2399 \qquad FORCE := READPRN(force)$$

$$TIME_i := \left(FORCE^{\langle 0 \rangle}\right)_i \qquad F_i := \left(FORCE^{\langle 1 \rangle}\right)_i$$

$$y_i := F_i - F_{2390}$$



The resultant damping factor and damped period were used to calculate the system's natural frequency and its time response. The system damped and free natural frequency were calculated.

$$\omega_d := \frac{2 \cdot \pi}{\tau_d} \qquad \omega_d = 2.90888$$

$$\omega_n := \frac{\omega_d}{\sqrt{1 - \zeta^2}} \qquad \omega_n = 2.91339$$

Using the mass of the egg in kilograms as reported by the student, an estimate of the system's stiffness was made from the definition of natural frequency and damping coefficient from the definitions for second order damped systems.

$$K_{sys} := m \cdot \omega_n^{\,2} \qquad K_{sys} = 0.509$$

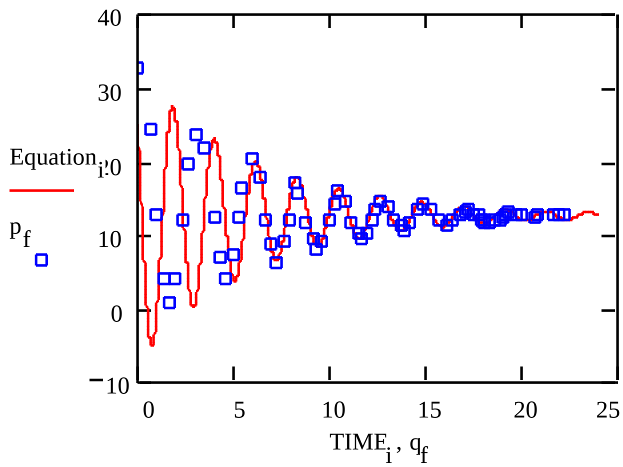
$$c := 2 \cdot \sqrt{K_{sys} \cdot m} \cdot \zeta \qquad c = 0.01944$$

Finally, the calculated damped natural period and damping factor were used to generate a theoretical system response to an initial displacement. The amplitude and equilibrium position of the real egg drop were used in the equation for comparison with the video data. Student data from video analysis was input by hand into two vectors, a distance vector and a time vector which are plotted above along with the theoretical response. Note, that the period of the data does not match the response until such time as the 'bounces' cease and the system is truly oscillatory. The response curve is phase shifted artificially to that portion of the curve from which the system parameters were taken.

$p := \mathrm{READPRN}(\mathrm{DATA1})$

$q := \mathrm{READPRN}(\mathrm{seconds})$

$$\mathrm{Equation}_i := e^{-\omega_n \cdot \zeta \cdot (\mathrm{TIME}_i)} \cdot \cos\left(\omega_d \cdot \mathrm{TIME}_i + \frac{\pi}{3.5}\right) \cdot 20 \; ... \; + 12.7$$



CONCLUSION

The chance to design an entire experiment, including selecting the best measurement devices and techniques to collect data has been welcomed by the students. They enjoyed seeing how a math model could be solved on the computer and tested as a physical system. A lab project such as this that bridges courses and disciplines is a great demonstration to the students that the real world is not as discrete and separate as their academic courses may suggest.

From an experimentation view, the lab forced students to make decisions about measurement hardware. On their own, they had to discover why the accelerometer readings had more noise than the ultrasonic position measurements (due to the +/- 25 g range of the accelerometers), what were the impacts of a measurement that fell outside the measurement range of an instrument, and how small oscillations in a position plot were amplified when the velocity and acceleration were calculated from that position file.

Using the digitized video in this project, though feared to be problem-prone by the instructors because this was the first application of this technology, was very easy and error free. With only a 5 minute demonstration of how to digitize video and play back the digitized files, the students were able to complete this phase of the project with no other instruction. It is noted that a high end video cassette recorder could have been used to replace the digitized video (if the VCR could play back the images frame by frame and display the time each frame was taken), but such equipment was not available for the project at hand. Some of the students effectively used still pictures from the video in their final reports for the project by saving individual frames as PICT files.

The force data provided additional insight for the students into measurements and provided instructors with a useful tool for analysis and evaluation of student results. Attempts to use the force data to augment distance, velocity, and acceleration data proved interesting but too sensitive to the added mass of the bungee cord to be of use.

The bungee omelet project was fun, for both the instructors and the students. This project allowed the students to apply numerous skills to solving one problem. Dropping eggs from the engineering building itself is a fun activity that soon became a high profile event on campus. Such activities are important to not only show the serious side of engineering, but also demonstrate the interesting and enjoyable aspects of our profession as well.

ACKNOWLEDGMENTS

The support and assistance of our colleagues at the U. S. Military Academy and at Arizona State University is gratefully acknowledged. The assistance of cadets at the U.S. Coast Guard Academy in collecting the experimental used in this paper is noted and appreciated.

REFERENCES

Panitz, B., "The Bungee Omelet Project," ASEE PRISM, Vol. 6, Number 1, p. 29.

Beasley, D.F., Figliola, R.S., Theory and Design for Mechanical Measurements, John Wiley & Sons Inc., 1995, New York, NY.

Close, C.M., and Frederick D.K., Modeling and Analysis of Dynamic Systems, John Wiley and Sons Inc., 1995, New York, NY.