

Application of Excel in Solving Heat Exchanger Problems

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Abstract

Introduction to heat exchanger analysis and design is an excellent vehicle for providing students an educational experience in practical application of heat transfer concepts in an undergraduate heat transfer course. Heat transfer textbooks commonly provide a brief coverage of heat exchangers, which contain detailed heat transfer analysis for parallel-flow and counter-flow heat exchangers. For some of the more complex flow patterns, charts are included in the textbooks to be used in solving heat exchanger problems. The accuracy of solutions is highly dependent on how precise one can read the charts, but also in problems requiring several stages of iteration, the process could become extremely tedious. Use of computer software programs increases the accuracy and eases the task of solving problems involving repeated calculations. Microsoft Excel is one of the tools that can be used in solving heat exchanger problems. It is simple to use and usually available on almost all desktop or laptop computers. This paper demonstrates how Excel can be used in solving heat exchanger problems. Formulas are provided for replacing charts in solving heat exchanger problems. Several examples are included to demonstrate the application of Excel in heat exchanger analysis and design.

Introduction

An introductory heat transfer course typically contains a brief coverage of heat exchangers. In this coverage students are introduced to the thermal analysis of a few standard heat exchangers. The coverage in most textbooks includes heat transfer analysis of heat exchangers involving simple flow patterns such as parallel or counter-flow heat exchangers. Until more recent years, many heat transfer textbooks¹⁻⁷ provided special charts for use in thermal analysis of shell-and-tube, and cross-flow heat exchangers. Although these charts were useful in solving heat exchanger problems, the accuracy of the analysis was highly dependent on how precisely one could read the applicable charts. In addition, solution of problems requiring several stages of iteration, made the process extremely tedious. Many of the current heat transfer textbooks⁸⁻¹⁵ include formulas that can effectively replace some charts used in the heat transfer analysis of heat exchangers. These formulas could be employed in computer software programs to improve the accuracy of analysis and ease the solution of problems requiring repeated use of heat exchanger charts.

Currently many publishing companies provide computer software with heat transfer textbooks.⁸⁻¹⁵ Most commonly used software tools in heat transfer courses are Interactive Heat Transfer (IHT)¹⁶ and Engineering Equation Solver (EES)¹⁷. These programs are general purpose, non-linear equation solvers with built-in property functions. They are capable of exploring and graphing the effects of change in variables on the solution to a given problem. There are also software packages available in the market that could be integrated into a heat transfer course and

used in the analysis and design of heat exchangers. These include Microsoft Excel spreadsheet, Mathcad, MATLAB, and Maple.

All available software packages are extremely useful tools which can be utilized for analysis and design of heat exchangers. The most significant advantage of these software programs is that no prior knowledge of programming language is necessary in their applications. Excel, which is available on almost all desktop or laptop computers, is an example. This paper focuses on the application of Microsoft Excel in solving heat exchanger problems.

The following sections in this paper demonstrate how Excel can be used as a tool to aid the solution process of heat exchanger problems. Formulas are provided to replace some of the charts used in solving heat exchanger problems. Several examples are included in this paper to display the application of these formulas in solving heat exchanger problems, using Excel.

Basic Heat Exchanger Analysis Formulas

In undergraduate heat transfer courses students are introduced to two different methods used in the analysis and design of heat exchangers. They are Logarithmic Mean Temperature Difference (LMTD) method and Effectiveness-NTU methods. In a heat exchanger the flow heat capacity rate, in general, is defined as

$$C = \dot{m}c_p \quad (1)$$

where, \dot{m} denotes the mass flow rate and c_p represents the specific heat of a given fluid stream. The rate of heat transfer from or to each fluid stream can be calculated from the following relation

$$q = C_h (T_{h,i} - T_{h,o}) = C_c (T_{c,o} - T_{c,i}) \quad (2)$$

where, q is the rate of heat transfer, T denotes temperature; subscripts c and h identify cold and fluids, respectively; and subscripts i and o represent inlet and outlet conditions respectively. Equation (1) is the result of an energy balance on each fluid stream.

The rate of heat transfer, based on heat transfer concepts, is expressed as

$$q = UA_s (F) LMTD \quad (3)$$

where, U is the overall heat transfer coefficient, A_s is the surface area separating the two fluid streams, $LMTD$ is the logarithmic mean temperature difference between the two fluid streams, and F is an appropriate correction factor which value depends on the type of heat exchanger and flow conditions.

In general, $LMTD$ can be expressed as

$$LMTD = \frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a / \Delta T_b)} \quad (4)$$

where, ΔT_a and ΔT_b are the temperature differences between the two fluid streams at the terminal points of the heat exchangers, as shown in Fig. 1.

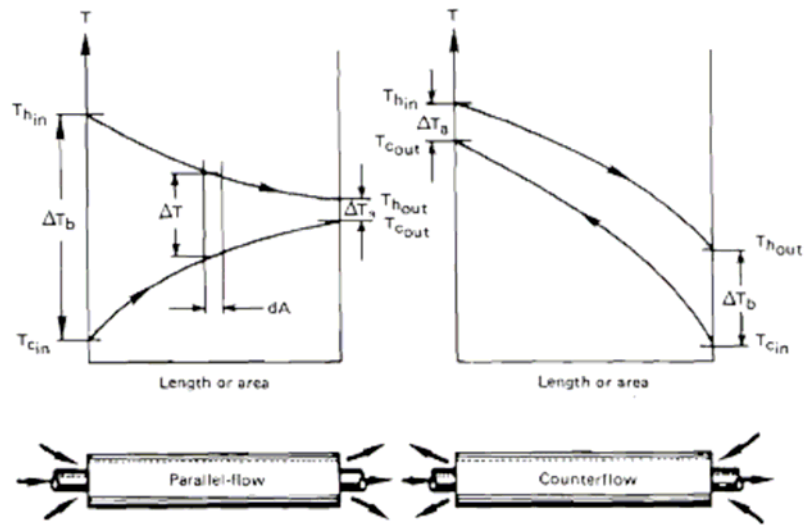


Fig. 1. Temperature variation through parallel-flow and counter flow heat exchanges⁷

The terminal point temperature differences shown for the counter-flow heat exchangers in Fig. 1 are used in Eq. (4) for the evaluation of LMTD of fluid streams in any other types of heat exchangers such as shell-and-tube or cross flow heat exchangers.

For parallel-flow and counter-flow heat exchangers, the correction factor in Eq. (3) has a value of $F=1$. For other types of heat exchangers, specific charts or equations are used for the correction factor, F . For example, Fig 2 is a correction factor chart for a one shell-pass, even number tube-pass heat exchanger.

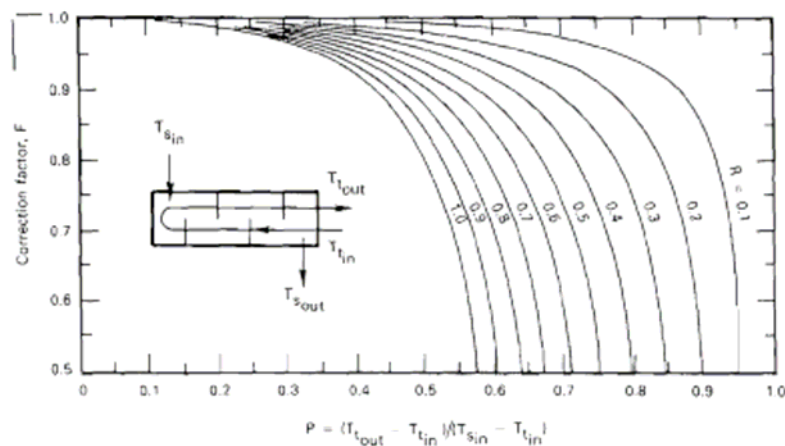


Fig. 2. LMTD correction factor, F , for a one shell-pass, even number of tube-passes heat exchanger.⁷

As shown in Fig. 2, the correction factor, F , is a function of two parameters P and R . The first parameter, P , is defined as

$$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} \quad (5)$$

Since the denominator in Eq. (5) represents the maximum temperature difference between the two fluid streams, the value of P is always less than one. The second parameter, R , is defined as

$$R = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = \frac{C_c}{C_h} \quad (6)$$

Depending on the flow heat capacity ratios (or temperature changes for the hot and cold fluids), the value of R could be less than one or greater than one. If the value of R in Eq. (6) ends up being greater than one, then R should be replaced by $1/R$ and P replaced by PR , since Fig. 1 displays curves only for R values that are less than or equal to one. In other words,

$$F = F(P, R) = F(RP, 1/R) \quad (7)$$

LMTD method is useful for sizing heat exchangers. That is when the inlet and outlet temperatures of the fluid streams are known or could be calculated directly from Eq. (2), LMTD and correction factor, F , can be easily evaluated and used in Eq. (3) for calculating the surface area or the overall heat transfer coefficient of heat exchangers. However, when two of the terminal temperatures of a heat exchanger are unknown and must be evaluated, LMTD can not be evaluated explicitly from Eq. (4). Hence, in these situations an iterative procedure is required for the evaluation of LMTD and the correction factor. In these cases the effectiveness-NTU method is employed for the analysis of heat exchangers.

The effectiveness of a heat exchanger is defined as

$$\varepsilon = \frac{\text{actual heat transfer}}{\text{maximum possible heat transfer}}$$

or

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{\min}(T_{h,i} - T_{c,i})} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})} \quad (8)$$

where, the C_{\min} is the smaller of the C_h and C_c . From Eq. (9) it follows that

$$q = \varepsilon C_{\min} (T_{h,i} - T_{c,i}) \quad (9)$$

The number of transfer units is defined as

$$NTU = \frac{UA}{C_{\min}} \quad (10)$$

The capacitance ratio is defined as

$$C_R = \frac{C_{\min}}{C_{\max}} \quad (11)$$

The derivation of heat exchanger effectiveness equation for parallel-flow is given in most heat transfer textbooks. For a parallel flow heat exchanger, the effectiveness is expressed as

$$\varepsilon = \frac{1 - \exp[-NTU(1 + C_R)]}{1 + C_R} \quad (12)$$

In this form the effectiveness is explicitly expressed as a function of NTU and C_R . Alternatively, NTU could be expressed as a function of ε and C_R . For a parallel-flow heat exchanger, NTU is given as

$$NTU = \frac{\ln[1 - \varepsilon(1 + C_R)]}{1 + C_R} \quad (13)$$

Heat transfer textbooks provide effectiveness charts for several types of heat exchangers. For example, Fig. 3 displays the effectiveness chart for a single pass cross-flow heat exchanger, one fluid unmixed.

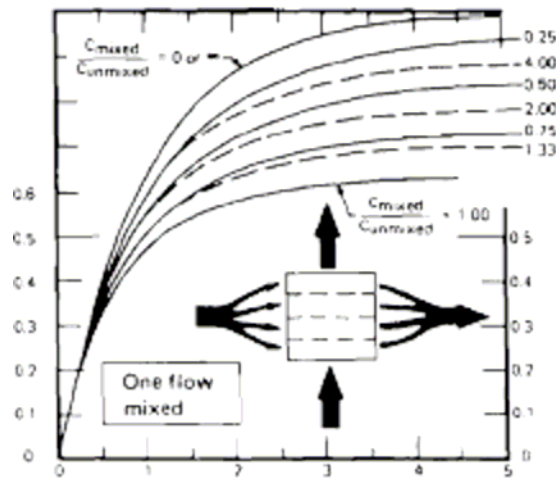


Fig. 3. Effectiveness chart for a one-pass cross-flow heat exchanger, one fluid unmixed⁷

More recent heat transfer textbooks also provide equations for $\varepsilon = \varepsilon(NTU, C_R)$ or $NTU = NTU(\varepsilon, C_R)$ for several types of heat exchanger. A list of effectiveness equations, $\varepsilon = \varepsilon(NTU, C_R)$, for common heat exchangers are included in the appendix section.

Application of Excel Spreadsheet

Excel is an effective and inexpensive tool available on all computers equipped with Microsoft Office. This software has the necessary functions for solving a large class of engineering problems, including those related to heat transfer. It has been shown^{18, 19} that Excel is an effective computational tool in solving boundary layer problems. Among the functions integrated into Excel are 39 engineering functions, as well as various math and trigonometry functions. The engineering functions include Bessel functions, error functions, and other functions used in heat transfer.

Excel is a useful computational tool in solving heat exchanger problems, when

- a) the solution requires varying one of the parameter in a given problem
- b) it is required to plot the results in a given problem
- c) when an iteration process is required to obtain a solution

For the cases listed above, it might be necessary for the user to know how to enter applicable equations into the spreadsheet cells. Otherwise, the applications for case (a) and (b) should be clear for anyone who has used Excel.

For problems requiring iterative calculations, the “Goal Seek” or “Solver” tools of Excel can be employed. By using the tool menu of Excel, and selecting either the “Goal Seek” or the “Solver” option, a dialog box appears, as shown in Fig 4. By selecting the target cell and fixing the desired value for that cell, values in the selected cells automatically change to reach the solution given for the target cell. This will be demonstrated later in an example.

The application of Excel in solving heat exchanger problems is demonstrated in the following examples.

Example 1

Water at 15 °C with a mass flow rate of 8 kg/s is available to cool hot oil from 90 °C to 30 °C. The oil mass flow rate is 4 kg/s. A shell-and-tube heat exchanger with one-shell pass and four-tube-passes is proposed for this process. Using uniform c_p values of 2.5 kJ/(kg °C) and 4.2 kJ/(kg °C) for oil and water, respectively, and assuming an overall heat transfer coefficient of 250 W/(m²·°C) for the heat exchanger

- a) determine the surface area of the heat exchanger
- b) plot the heat exchanger surface area as a function of water mass flow rate, when the mass flow rates vary between 6 and 30 kg/s.

Solution

This problem can be easily solved, using the LMTD method and the correction chart in Fig. 2. Using the specific heat and mass flow rate data given in the problem statement, Eq. (1) yields the following results:

$$C_c = 33.6 \text{ kW/}^\circ\text{C}$$

$$C_h = 10 \text{ kW/}^\circ\text{C}$$

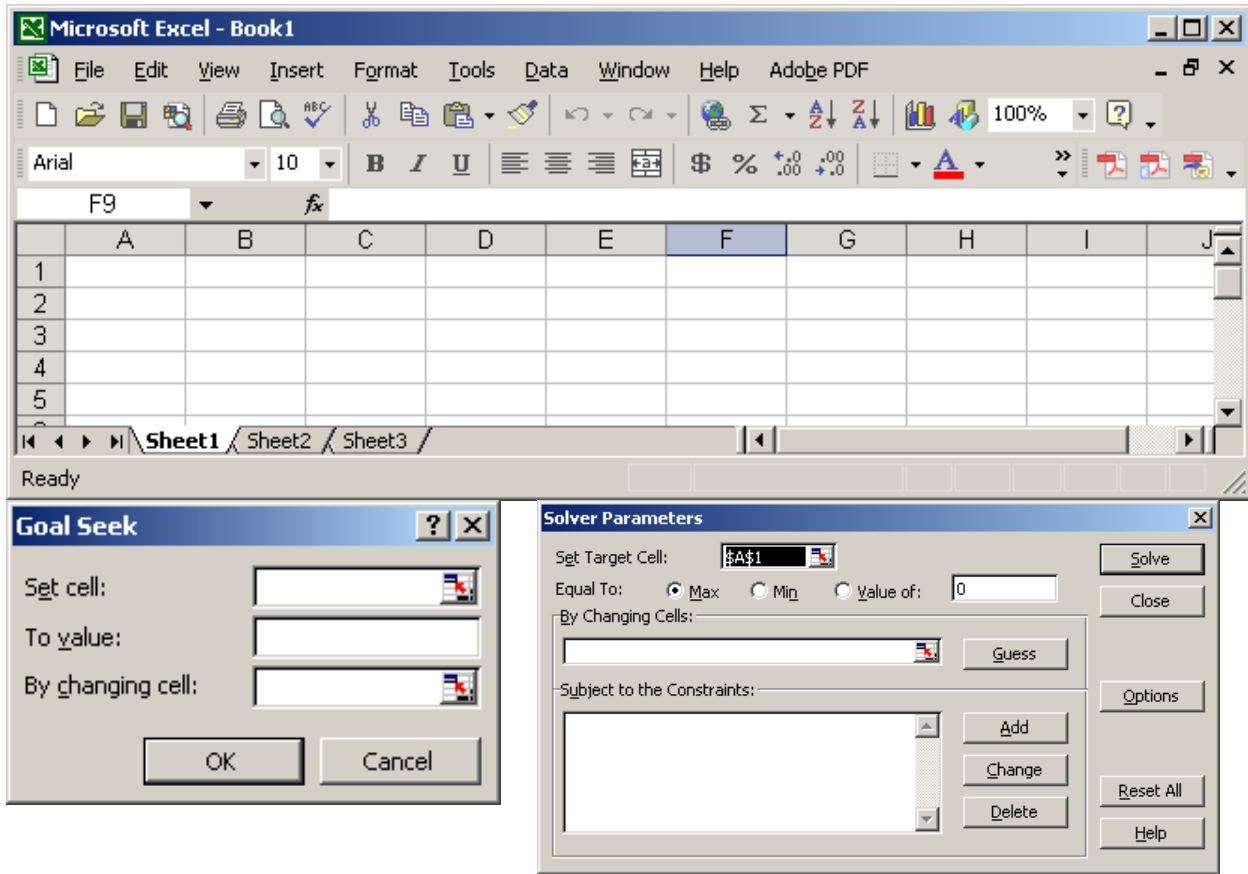


Fig. 4. Excel worksheet, Goal Seek, and Solver windows

The results are substituted into Eq. (2) to obtain $T_{c,o}$

$$q = C_h \Delta T_h = 10 \text{ kW/}^\circ\text{C} (90-30) ^\circ\text{C} = 600 \text{ kW}$$

$$q = 600 \text{ kW} = C_c (T_{c,o} - T_{c,i}) = 33.6 \text{ kW/}^\circ\text{C} (T_{c,o} - 15) ^\circ\text{C}. \text{ This gives}$$

$$T_{c,o} = 32.86 ^\circ\text{C}$$

Then,

$$\Delta T_a = 90 - 32.86 = 57.14 ^\circ\text{C}$$

$$\Delta T_b = 90 - 32.86 = 57.14 ^\circ\text{C}$$

Substituting ΔT_a and ΔT_b into Eq. (4), yields

$$\text{LMTD} = 31.51 ^\circ\text{C}$$

In order to evaluate the correction factor, the terminal temperatures of the heat exchanger are substituted into Eqs. (5) and (6) to find the values of P and R.

$$P = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} = 0.2381$$

$$R = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}} = 3.36$$

However, since $R > 1$, in order to use Fig. 2 to evaluate the correction factor, P needs to be replaced with PR , and R replaced with $1/R$

$$PR = (0.2381)(3.36) = 0.8$$

$$1/R = 1/3.36 = 0.2978$$

Then from Fig.2, the correction factor is approximated as

$$F = 0.74$$

Substituting the known values into Eq. (3) the heat exchanger area is calculated

$$q = UA_s (F) LMTD$$

$$600 \text{ kW} = 0.250 \text{ kW}/(\text{m}^2 \cdot ^\circ\text{C}) A_s (0.74) (31.51 ^\circ\text{C})$$

$$A_s = 102.9 \text{ m}^2$$

The same procedure can be used to solve part (b) of this example, by varying the mass flow rate of water. However, it is clear that the manual solution of part (b) will consume a great deal of time without adding much to the learning process. Employing Excel will ease and speed up the calculation process.

In order to use Excel to solve part (b) of this example, the correction factor chart, Fig. 2, must be replaced by an appropriate equation. For a one-shell-pass and even number of tube passes, the equation for the correction factor is given as⁹

$$F = \frac{\sqrt{1+R^2}}{1-R} \ln\left(\frac{1-RP}{1-P}\right) \left[\ln\left(\frac{2-P(1+R-\sqrt{1+R^2})}{2-P(1+R+\sqrt{1+R^2})}\right) \right]^{-1} \quad (14)$$

where, R and P are defined by Eqs. (4) and (6).

Equation (14), along with equations for C_c , LMTD, P, R, were used in an Excel spreadsheet to determine the surface area of the heat exchanger by varying the cooling water mass flow rates. Table 1 represents the results of the heat exchanger area calculations. Excel was used to plot the heat exchanger area as a function of water mass flow rate, as shown Fig. 5.

Example 2

Hot water at $90 ^\circ\text{C}$ ($c_p = 4178 \text{ J/kg} \cdot ^\circ\text{C}$) enters the tubes of a cross-flow heat exchanger with both fluids unmixed to heat air ($c_p = 1006 \text{ J/kg} \cdot ^\circ\text{C}$) from $15 ^\circ\text{C}$ to $30 ^\circ\text{C}$. The air flows across the tubes at a rate of 3 kg/s . The overall heat transfer coefficient of the heat exchanger is $300 \text{ W/m}^2 \cdot ^\circ\text{C}$ and the total surface area of is 9.5 m^2 . Calculate the exit temperature and mass flow rate for water.

Table 1. Excel spreadsheet calculation of heat exchanger area for example 1.

m_w , kg/s	C_c kW/°C	C_h , kW/°C	ΔT_b , °C	$T_{c,o}$, °C	ΔT_a , °C	LMTD °C	P	R	F	A_2 m ²
6	25.2	10	15	38.81	51.19	29.48	0.3175	2.52	0.500	162.81
7	29.4	10	15	35.41	54.59	30.65	0.2721	2.94	0.675	115.99
8	33.6	10	15	32.86	57.14	31.51	0.2381	3.36	0.750	101.51
9	37.8	10	15	30.87	59.13	32.17	0.2116	3.78	0.796	93.73
10	42.0	10	15	29.29	60.71	32.70	0.1905	4.2	0.827	88.76
11	46.2	10	15	27.99	62.01	33.12	0.1732	4.62	0.850	85.28
12	50.4	10	15	26.90	63.10	33.48	0.1587	5.04	0.867	82.69
13	54.6	10	15	25.99	64.01	33.78	0.1465	5.46	0.881	80.68
14	58.8	10	15	25.20	64.80	34.03	0.1361	5.88	0.892	79.08
15	63.0	10	15	24.52	65.48	34.25	0.1270	6.3	0.901	77.77
16	67.2	10	15	23.93	66.07	34.45	0.1190	6.72	0.909	76.67
17	71.4	10	15	23.40	66.60	34.61	0.1120	7.14	0.915	75.74
18	75.6	10	15	22.94	67.06	34.76	0.1058	7.56	0.921	74.95
19	79.8	10	15	22.52	67.48	34.90	0.1003	7.98	0.926	74.26
20	84.0	10	15	22.14	67.86	35.02	0.0952	8.4	0.931	73.65
21	88.2	10	15	21.80	68.20	35.13	0.0907	8.82	0.934	73.11
22	92.4	10	15	21.49	68.51	35.23	0.0866	9.24	0.938	72.64
23	96.6	10	15	21.21	68.79	35.32	0.0828	9.66	0.941	72.21
24	100.8	10	15	20.95	69.05	35.40	0.0794	10.08	0.944	71.83
25	105.0	10	15	20.71	69.29	35.48	0.0762	10.5	0.946	71.48
26	109.2	10	15	20.49	69.51	35.55	0.0733	10.92	0.949	71.16
27	113.4	10	15	20.29	69.71	35.61	0.0705	11.34	0.951	70.87
28	117.6	10	15	20.10	69.90	35.67	0.0680	11.76	0.953	70.60
29	121.8	10	15	19.93	70.07	35.73	0.0657	12.18	0.955	70.36
30	126.0	10	15	19.76	70.24	35.78	0.0635	12.6	0.956	70.13

Solution

In this problem air represents the cold fluid and water is the hot fluid. Using the specific heat and mass flow rate data given in the problem statement, C_c is calculated from Eq. (1)

$$C_c = 3018.0 \text{ W/}^\circ\text{C}$$

Since the mass flow rate of water is unknown, C_h cannot be evaluated from Eq. (1). Therefore, there are too many unknowns in order to calculate $T_{c,o}$ from Eq. (2) (energy balance). As a result, not all heat exchanger terminal temperatures are known or can be calculated. Solution of this problem, using the LMTD method is challenging, since the calculation of LMTD and the correction factor, F, require the knowledge of all four terminal temperatures. Instead, we can attempt to solve the problem using the effectiveness-NTU method.

The rate of heat transfer can be calculated from Eq. (2).

$$q = C_c \Delta T_c = 3018.0 \text{ W/}^\circ\text{C} (30-15) ^\circ\text{C} = 45,270 \text{ W}$$

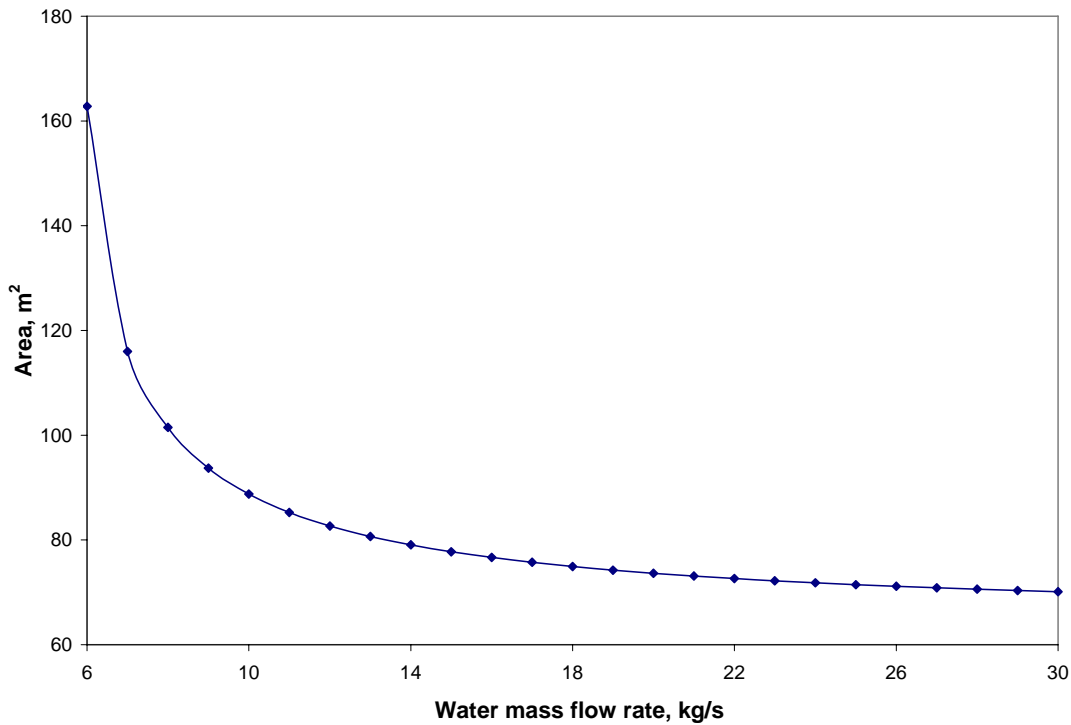


Fig. 5. Example 1, Variation of heat exchanger area with the mass flow rate of cooling water

Since C_h is unknown, we don't know which fluid represents C_{\min} . Therefore, we can assume one of the fluids represent the C_{\min} , proceed with the solution, and use the results to see if the assumption is valid. In this case let's assume that air represents the C_{\min} . Based on this assumption, the effectiveness, ε , and NTU can be calculated directly from Eqs. (8) and (10), respectively.

$$\varepsilon = \frac{C_c (T_{c,o} - T_{c,i})}{C_{\min} (T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})} = 0.20$$

$$NTU = \frac{UA}{C_{\min}} = 0.9443$$

As seen from Fig 6, these values fall outside of the range of C_R curves. Therefore, the assumption that C_c represents the C_{\min} is not valid.

Now let's choose C_h to represent the C_{\min} . Since C_h is not known in this case, NTU cannot be calculated directly from Equation 10. Therefore, the solution requires an iterative process.

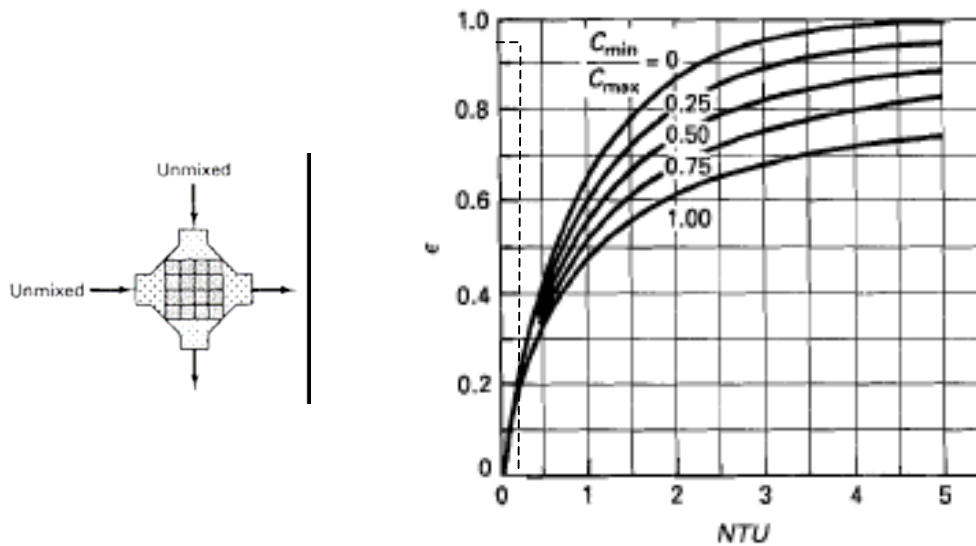


Fig. 6. Effectiveness for cross-flow heat exchanger with both fluids unmixed

The following procedure could be used to determine the exit temperature and mass flow rate for water:

1. Assume a value for $C_R = C_{\min}/C_{\max}$
2. Evaluate $C_h = C_{\min} = (C_R)(C_{\max}) = (C_R)(C_c)$
3. Use the calculated value of C_{\min} in Eq. (10) to evaluate $NTU = UA/C_{\min}$
4. Use the values of NTU and C_R to evaluate effectiveness, ε , from Figure 2. Alternatively, the effectiveness can be calculated from the effectiveness equations given in the textbooks.⁸⁻¹¹

For a cross-flow heat exchanger with both fluids unmixed, the effectiveness is expressed as

$$\varepsilon = 1 - \exp\left[\left(\frac{1}{C_R}\right)(NTU)^{0.22} \left\{\exp[-C_R(NTU)^{0.78}] - 1\right\}\right] \quad (15)$$

5. Use the value of ε , evaluated at step 3, in Eq. (8), to calculate $(T_{h,i} - T_{h,o})$

$$\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{\min}(T_{h,i} - T_{c,i})} = \frac{(T_{h,i} - T_{h,o})}{(T_{h,i} - T_{c,i})}, \text{ or } (T_{h,i} - T_{h,o}) = \varepsilon(T_{h,i} - T_{c,i})$$

6. Calculate $(T_{c,o} - T_{c,i})$ from Eq. (2)

$$q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i}), \text{ or}$$

$$(T_{c,o} - T_{c,i}) = (C_h / C_c)(T_{h,i} - T_{h,o}) = C_R(T_{h,i} - T_{h,o})$$

7. If the value of $(T_{c,o} - T_{c,i})_{\text{cal}}$ calculated in step 6 is the same (or approximately) as the actual value of $(T_{c,o} - T_{c,i})_{\text{act}}$ from the problem statement, stop the process and use the last values of C_h and $(T_{h,i} - T_{h,o})$ to evaluate the mass flow rate and exit temperature for water. Otherwise, assume a new value for C_R , go to step 2 and repeat the iteration process.

The procedure described above is a time consuming process when a computer program is not used to conduct the calculations. Excel can be employed to implement the procedure and speed up the calculation process. The formulas used in the procedure, including the effectiveness equation, can be inputted into the cells of the spreadsheet worksheet. Table 2 shows the results of the iteration procedure, using Excel spreadsheet. The columns represent the results of each step in the procedure described above and each row represents an iteration step.

Table 2. Excel results of iteration process used to solve example 2.

Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
C_R	C_h	NTU	ϵ	$\Delta T_h, ^\circ C$	$(\Delta T_c)_{calc}$
0.5	1509.00	1.889	0.6985	52.389	26.194
0.6	1810.80	1.574	0.6171	46.280	27.768
0.4	1207.20	2.361	0.7928	59.462	23.785
0.3	905.40	3.148	0.8931	66.984	20.095
0.29	875.22	3.256	0.9028	67.706	19.635
0.28	845.04	3.373	0.9122	68.413	19.156
0.27	814.86	3.498	0.9214	69.102	18.657
0.26	784.68	3.632	0.9302	69.768	18.140
0.25	754.50	3.777	0.9388	70.408	17.602
0.24	724.32	3.935	0.9469	71.018	17.044
0.23	694.14	4.106	0.9546	71.595	16.467
0.22	663.96	4.292	0.9618	72.134	15.869
0.21	633.78	4.497	0.9684	72.631	15.253
0.2	603.60	4.722	0.9744	73.084	14.617
0.205	618.69	4.607	0.9715	72.863	14.937
0.206	621.71	4.584	0.9709	72.818	15.000
0.2055	620.20	4.595	0.9712	72.841	14.969
0.2058	621.10	4.589	0.9710	72.827	14.988
0.2059	621.41	4.586	0.9710	72.822	14.994
0.205994	621.69	4.584	0.9709	72.818	15.000

Table 2, shows that when $C_R = 0.205994$, the iteration process converges to a solution, where the calculated ΔT_c is equal to the actual value of $\Delta T_c = 15 ^\circ C$. The mass flow rate and the exit temperature of water can be calculated from C_h and ΔT_h values in the last row of Table 2.

$$\dot{m}_h = C_h / c_{p,h} = 621.69 / 4178 = 0.149 \text{ kg/s}$$

$$T_{h,o} = T_{h,i} - \Delta T_h = 90 - 72.82 = 17.18 ^\circ C$$

The iteration steps of the iteration process presented in Table 2 can be reduced to a single step if we take advantage of either the “Solver” tool or “Goal Seek” tools in Excel.

Figure 7 shows the first iteration step used in Table 2. By using the tool menu, and selecting the Goal Seek option a dialog box appears, as shown in Fig 8. By selecting the target cell (calculated

ΔT_c in this case) and setting it to the desired value for that cell (the actual value of $\Delta T_c = 15^\circ \text{C}$), and identifying the cell that needs to be changed (C_R in this case). By clicking on the OK button, the values in all cells automatically change to reach the solution given for the target cell. Now the value of C_R in Fig. 7 changes so that ΔT_c is equal to 15. The final solution is shown in Fig. 9.

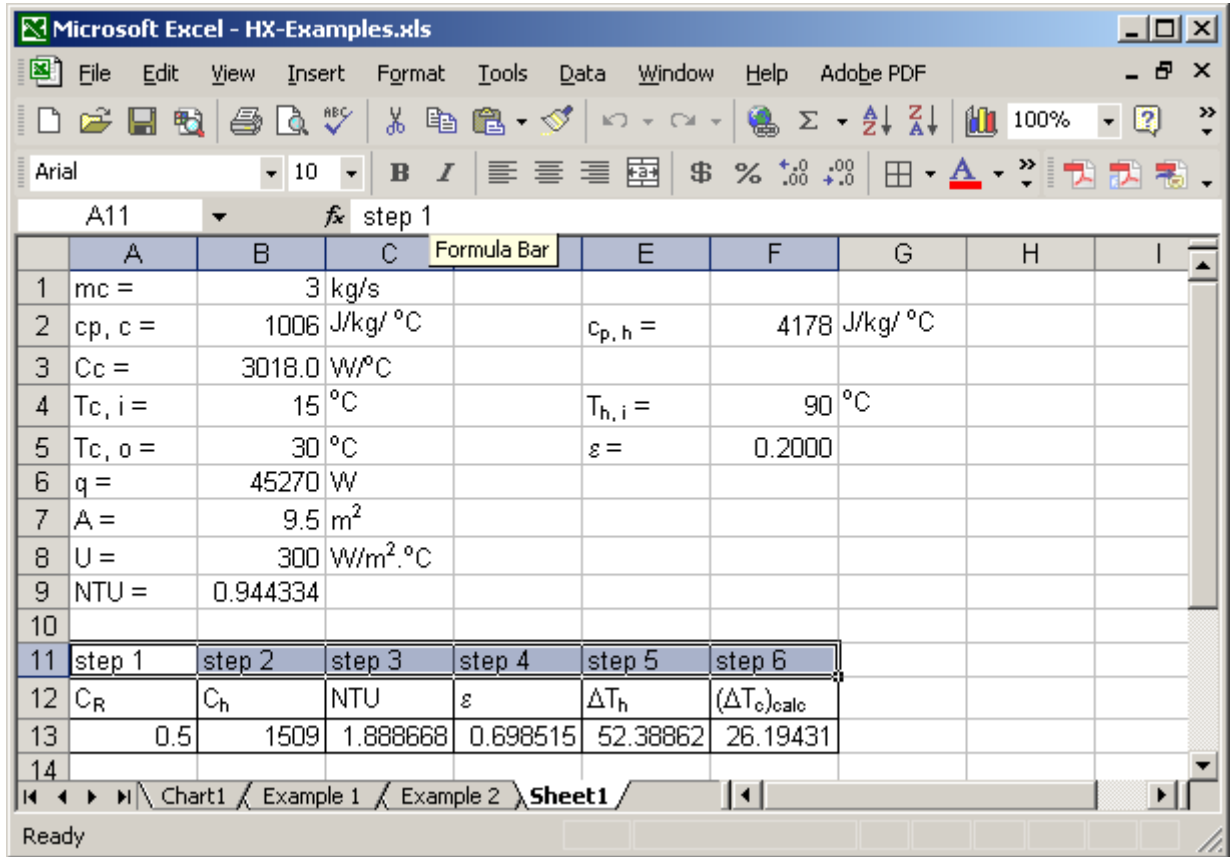


Fig 7. The first step of the iteration process, using Excel's Goal Seek tool

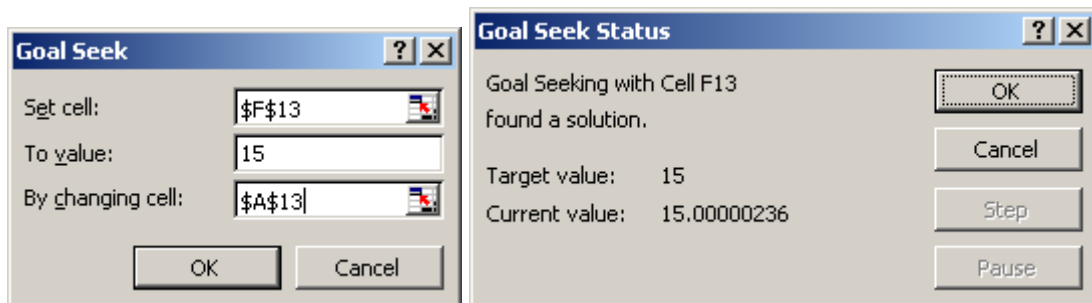


Fig. 8. Goal Seek tool-identifying the target cell, target value, and the cell that its value changes

In solving the problems in the last two examples, it was assumed constant overall heat transfer coefficient, U . However, since the mass flow rate of one of the fluid streams was changing during the solution process,

	A	B	C	D	E	F	G	H
1	$m_c =$	3	kg/s					
2	$c_{p,c} =$	1006	J/kg/°C		$c_{p,h} =$	4178	J/kg/°C	
3	$C_c =$	3018.0	W/°C					
4	$T_{c,i} =$	15	°C		$T_{h,i} =$	90	°C	
5	$T_{c,o} =$	30	°C		$\varepsilon =$	0.2000		
6	$q =$	45270	W					
7	$A =$	9.5	m ²					
8	$U =$	300	W/m ² .°C					
9	$NTU =$	0.944334						
10								
11	step 1	step 2	step 3	step 4	step 5	step 6		
12	C_R	C_h	NTU	ε	ΔT_h	$(\Delta T_c)_{calc}$		
13	0.205992	621.6853	4.584313	0.970909	72.8182	15		
14								
15								

Fig. 9. Solution of Example 2, using Excel's Goal Seek tool

Example 3

Consider a cross-flow heat exchanger containing a tube bank that consists of a square array of 100 thin-walled tubes (10x10), each 2.5 cm in diameter and 5 meter long. The tubes are aligned with a transverse pitch of 5 cm. Water is used in this heat exchanger to cool hot air from 800 K, to 500 K. Water makes a single pass through each tube entering at 12 °C. Hot air enters the heat exchanger with a velocity of 5.0 m/s in a cross flow over tubes with a mass flow rate of 2.25 kg/s. Determine the water mass flow rate and the exit temperature.

Solution

The following property values are given for air at an average temperature of 650 K:

$c_{p,a} = 1063$ J/kg.K, $\mu_a = 322.5 \times 10^{-7}$ N.s/m, $\nu_a = 60.21 \times 10^{-6}$ m²/s, $k_a = 0.0497$ W/m.K, and $Pr_a = 0.69$

Assuming an average temperature of 340 K for water, the following property values are obtained $c_{p,w} = 4188$ J/kg.K, $\mu_w = 420 \times 10^{-6}$ N.s/m, $\nu_w = 5.35 \times 10^{-7}$ m²/s, $k_w = 0.660$ W/m.K, and $Pr_w = 2.66$

For the external flow over an aligned tube bundle, as shown in Fig 10, the maximum velocity is given by

$$u_{\max} = \frac{S_T}{S_T - D} u_{\infty} \quad (16)$$

where u_{∞} is the free stream velocity, D denotes the tube diameter, S_T represents the transverse pitch. Using the data given in the problem statement

$$u_{\max} = 10 \text{ m/s}$$

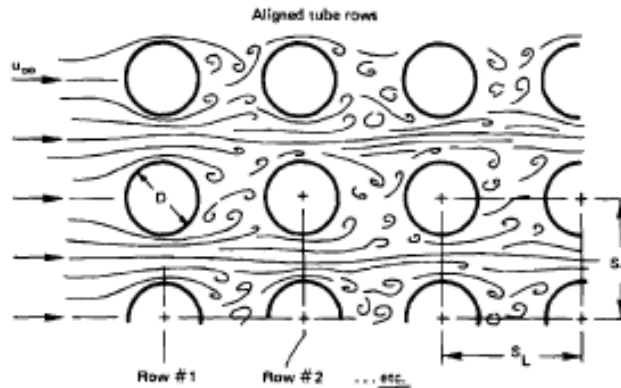


Fig. 10. External flow over an aligned tube bundle⁷

The Reynolds number based on the maximum flow velocity is expressed as

$$Re_{D,\max} = \frac{u_{\max} D}{\nu} \quad (17)$$

where, ν is the kinematic viscosity. Using the kinematic viscosity value given for air

$$Re_{D,\max} = 4152.1$$

For air flow across a tube bundles consisting of 10 or more rows, Grimson²⁰ gave the following correlation for the average Nusselt number

$$Nu_{D,\max} = \frac{h_o D}{k} = C Re_{D,\max}^m \quad \left[\begin{array}{l} n_L \geq 10 \\ 2000 < Re_{D,\max} < 40,000 \\ Pr \approx 0.7 \end{array} \right] \quad (18)$$

where n_L represents the number of rows, and h_o is the external heat transfer coefficient. The values of C and m depend on the ratios of S_T/D and S_L/D . For the case when $S_T/D = S_L/D = 2$ (this example), the values of C and m are given as²⁰ 0.229 and 0.632, respectively. Based on these values, Eq. (18) reduces to

$$Nu_{D,\max} = 0.229 Re_{D,\max}^{0.632} \quad (19)$$

Substituting the value of $Re_{D,\max} = 4152.1$, into Eq. (19)

$$Nu_{D,\max} = 44.318$$

Then,

$$h_o = k Nu_{D,\max} / D = 88.11 \text{ W/m}^2 \cdot \text{K}$$

For the internal flow, the Reynolds number is defined as

$$Re_D = \frac{u D}{\nu} = \frac{4(\dot{m}_w)_i}{\pi D \mu_w} \quad (20)$$

where $(\dot{m}_w)_i$ represents the mass flow rate in each tube. If Re_D indicates a fully developed laminar flow, the Nusselt number, assuming constant surface temperature, is given as

$$Nu_D = \frac{h_i D}{k} = 3.66 \quad (21)$$

where, h_i represents the heat transfer coefficient inside the tubes. When Re_D indicates a fully developed turbulent flow, the Nusselt number, can be approximated by Dittus-Boelter equation²¹

$$Nu_D = \frac{h_i D}{k} = 0.023 Re_D^{0.8} Pr^{0.4} \quad (22)$$

Ignoring the thermal resistance of the tube wall, the overall heat transfer coefficient is expressed as

$$U = \left[\frac{1}{h_i} + \frac{1}{h_o} \right]^{-1} \quad (23)$$

The surface area of the heat exchanger is evaluated, using the following relationship

$$A = N \pi DL = 100 \pi (0.025 \text{ m}) (4 \text{ m}) = 31.42 \text{ m}^2$$

NTU was defined earlier as

$$NTU = \frac{UA}{C_{\min}} \quad (10)$$

Using the specific heat and mass flow rate data, C_h is calculated from Eq. (1)

$$C_h = C_a = 2364.8 \text{ W/K}$$

The rate of heat transfer is calculated from Eq. (2)

$$q = C_h \Delta T_h = 2364.8 \text{ W/K} (800-500) \text{ K} = 709,425 \text{ W}$$

At this point there exist too many unknowns to solve the heat exchanger problem directly either by the LMTD method or the effectiveness method. For the LMTD, the exit temperature of water is unknown and cannot be calculated directly. For the effectiveness method NTU, C_R , and ε cannot be calculated directly without the knowledge of the mass flow rate of water. Therefore, an iterative procedure, similar to the one used in Example 2, is required to solve this heat exchanger problem.

We will employ the effectiveness-NTU method in the iterative procedure described below. Some steps in the procedure depend on which fluid is assumed to represent the C_{\min} . When $C_c = C_w$ is chosen as the C_{\min} , the steps operation is presented in [*brackets and italic*]. If the one assumption does not converge to an answer, then the other assumption can be implemented in iteration process).

1. Assume $C_h = C_a$ represents C_{\min} [*Assume $C_c = C_w$ represents C_{\min}*]
2. Assume a value for $C_R = C_{\min}/C_{\max}$
3. Evaluate $C_c = C_w = C_{\max} = C_{\min}/C_R = C_h/C_R$ [*evaluate $C_c = C_w = C_{\min} = C_{\max} C_R = C_h C_R$*]
4. Calculate the total mass flow rate of water, $\dot{m}_w = C_w/c_{p,w}$; $(\dot{m}_w)_i = \dot{m}_w/100$
5. Use the calculated value of $(\dot{m}_w)_i$ in Eq. (20) to evaluate Re_D
6. If the flow is laminar use Eq. (21) to evaluate Nu_D . Otherwise use Eq. (22)
7. Calculate the internal heat transfer coefficient from the results in step 6
8. Evaluate the overall heat transfer coefficient from Eq. (23)
9. Evaluate NTU from Eq. (10)
10. Substitute the values of NTU and C_R in an appropriate effectiveness equation. For a cross-flow heat exchanger when fluid representing C_{\min} is mixed and fluid representing C_{\max} is unmixed, the effectiveness, ε , is expressed as

$$\varepsilon = 1 - \exp\left\{-C_R^{-1} \left[1 - \exp(-C_R NTU)\right]\right\} \quad (24)$$

[*or a cross-flow heat exchanger when fluid representing C_{\max} is mixed and fluid representing C_{\min} is unmixed, the effectiveness, ε , is expressed as*

$$\varepsilon = \left(\frac{1}{C_R}\right) \left[1 - \exp\left\{-C_R \left[1 - \exp(-NTU)\right]\right\}\right] \quad (25)$$

11. Use the value of C_c , evaluated at step 3, in Eq. (8), to calculate $(T_{c,o} - T_{c,i})$

$$\varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - T_{c,i})}{C_R(T_{h,i} - T_{c,i})}, \text{ or } (T_{c,o} - T_{c,i}) = \varepsilon C_R(T_{h,i} - T_{c,i})$$

[*Use the value of C_c , evaluated at step 3, in Eq. (8), to calculate $(T_{c,o} - T_{c,i})$*

$$\varepsilon = \frac{C_c(T_{c,o} - T_{c,i})}{C_{\min}(T_{h,i} - T_{c,i})} = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})}, \text{ or } (T_{c,o} - T_{c,i}) = \varepsilon(T_{h,i} - T_{c,i})]$$

12. Calculate $(T_{h,i} - T_{h,o})$ from Eq. (2)

$$q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i}), \text{ or } (T_{h,i} - T_{h,o}) = (C_c/C_h)(T_{c,o} - T_{c,i}) = (T_{h,i} - T_{h,o})/C_R$$

[*Calculate $(T_{h,i} - T_{h,o})$ from Eq. (2)*

$$q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i}), \text{ or } (T_{h,i} - T_{h,o}) = (C_c/C_h)(T_{c,o} - T_{c,i}) = C_R(T_{h,i} - T_{h,o})]$$

13. If the value of $(T_{h,i} - T_{h,o})_{\text{cal}}$ evaluated in step 12 is the same (or approximately the same) as the actual value of $(T_{h,i} - T_{h,o})_{\text{act}}$ (determined from the values given in the problem

statement), stop the process and use the last values of C_c and $(T_{c,o} - T_{c,i})$ to evaluate the mass flow rate and exit temperature for water. Otherwise, assume a new value for C_R , go to step 3 and repeat the iteration process.

Excel was employed to implement the procedure described above. In this process it was assumed that $C_h = C_a$ represents C_{min} . Table 3 shows the results of the iteration process. It shows that when $C_R = 0.26105$, the calculated value of ΔT_h converges to the actual value of $\Delta T_h = 300$. Then water exit temperature can be calculated from

$$T_{w,o} = T_{w,i} + \Delta T_c = 12 + 78.31 = 90.31 \text{ } ^\circ\text{C}$$

Table 3 shows that the water mass flow rate in each tube is 0.0216 kg/s or the total mass flow rate of water is 2.16 kg/s.

Table 3. Iteration process results from Excel spreadsheet for example 3

step 2	step 3	Step 4	step 5	step 6	Step 7	step 8	step 9	step10	step 11	step 12
C_R	$C_c = C_{max}$	$(m_w)_i$	Re_D	Nu_D	h_i	U	NTU	ϵ	ΔT_c	$(\Delta T_h)_{calc}$
0.1	23648	0.0565	6847.0	39.82	1051.2	81.29	1.0800	0.6407	33.00	329.98
0.2	11824	0.0282	3423.5	22.87	603.8	76.89	1.0214	0.6030	62.11	310.55
0.3	7883	0.0188	2282.3	16.53	436.5	73.31	0.9739	0.5702	88.10	293.67
0.21	11261	0.0269	3260.5	21.99	580.6	76.50	1.0163	0.5995	64.84	308.76
0.22	10749	0.0257	3112.3	21.19	559.4	76.12	1.0112	0.5961	67.54	307.00
0.23	10282	0.0245	2977.0	20.45	539.9	75.74	1.0063	0.5927	70.21	305.26
0.24	9853	0.0235	2852.9	19.77	521.8	75.38	1.0014	0.5894	72.85	303.54
0.25	9459	0.0226	2738.8	19.13	505.0	75.02	0.9966	0.5861	75.46	301.85
0.26	9095	0.0217	2633.5	18.54	489.4	74.66	0.9919	0.5829	78.05	300.17
0.27	8758	0.0209	2535.9	17.99	474.9	74.32	0.9873	0.5796	80.60	298.52
0.261	9060	0.0216	2623.4	18.48	487.9	74.63	0.9915	0.5825	78.30	300.01
0.262	9026	0.0216	2613.4	18.43	486.5	74.59	0.9910	0.5822	78.56	299.84
0.2611	9057	0.0216	2622.4	18.48	487.8	74.63	0.9914	0.5825	78.33	299.99
0.26105	9059	0.0216	2622.9	18.48	487.9	74.63	0.9914	0.5825	78.31	300.00

The Goal Seek or the Solver tools of Excel can be employed to speed up the iteration process for solving the problem in example 3. Fig. 11 shows the assumed (C_R) and calculated values for each step of the procedure described earlier for the iteration process. It shows that for an assumed value of $C_R = 0.4$ the procedure calculates $(\Delta T_h)_{calc} = 278.66 \text{ } ^\circ\text{C}$, which is different from the actual value of $\Delta T_h = 300 \text{ } ^\circ\text{C}$ given in the problem statement.

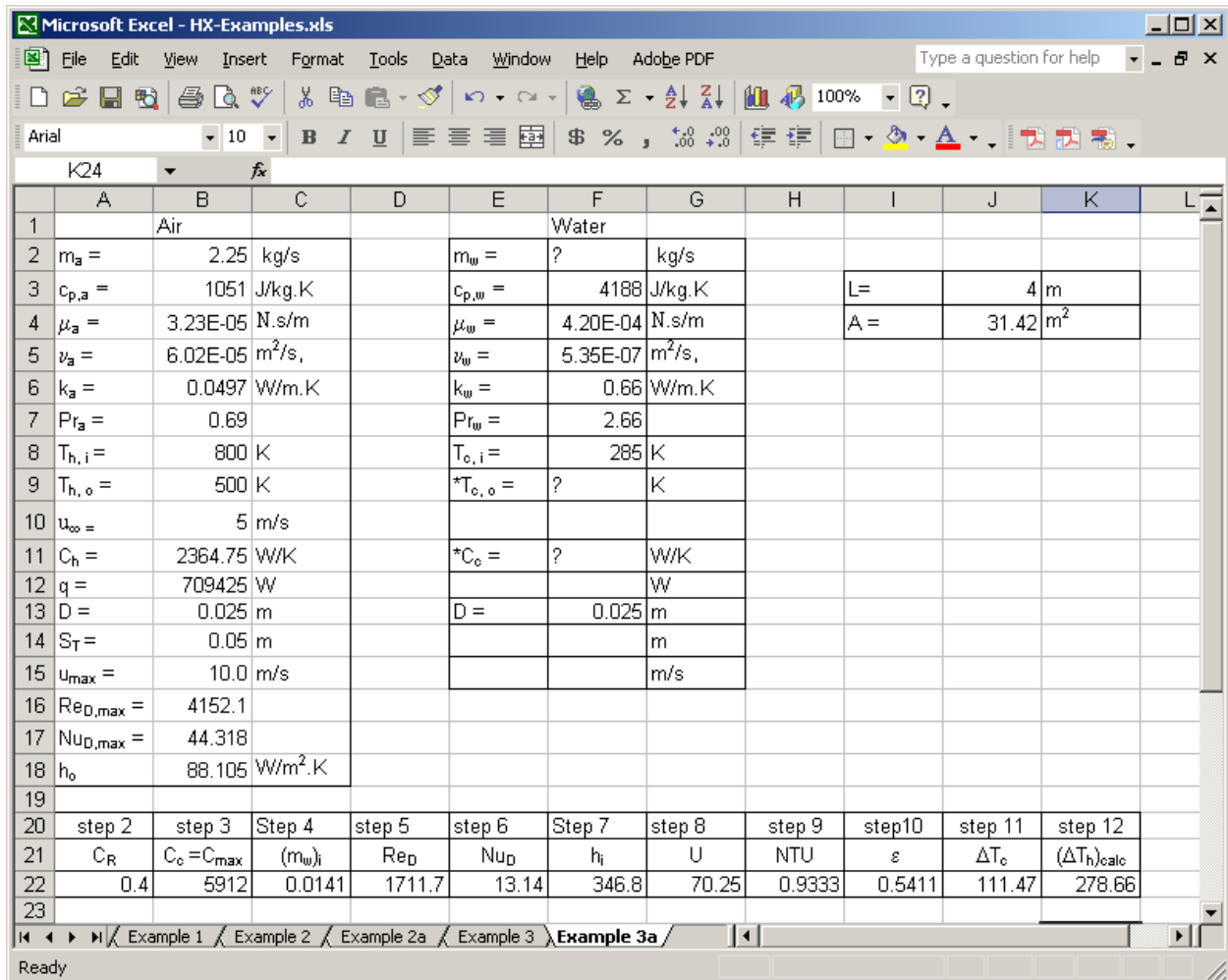


Fig. 11. First step in using Solver for the iteration process in Example 3

By using the tool menu on the Excel worksheet and selecting the Solver option a menu appears as shown in Figure 12. In this menu we can set the target cell (K22) equal to a value of 300 [$(\Delta T_h)_{calc} = 300$ °C]. We also identify the cell (A22, C_R) that its value needs to be changed during the iteration process. By clicking on the Solve button, the Solver will search for a value of C_R that results in a value of $(\Delta T_h)_{calc} = 300$ °C. The values of all other cells will be changed to correspond to the final value of C_R . Figure 13 displays the final results obtained from Excel's Solver tool for example 3.

Summary

The application of Excel spreadsheet in solving heat exchanger problems was briefly discussed. It was shown that Excel is a useful computational tool when the solution to problems requires (a) varying one of the parameters, (b) plotting the results of calculations, and (c) an iteration process. The effectiveness of Excel in solving heat exchanger problems was demonstrated through several examples.

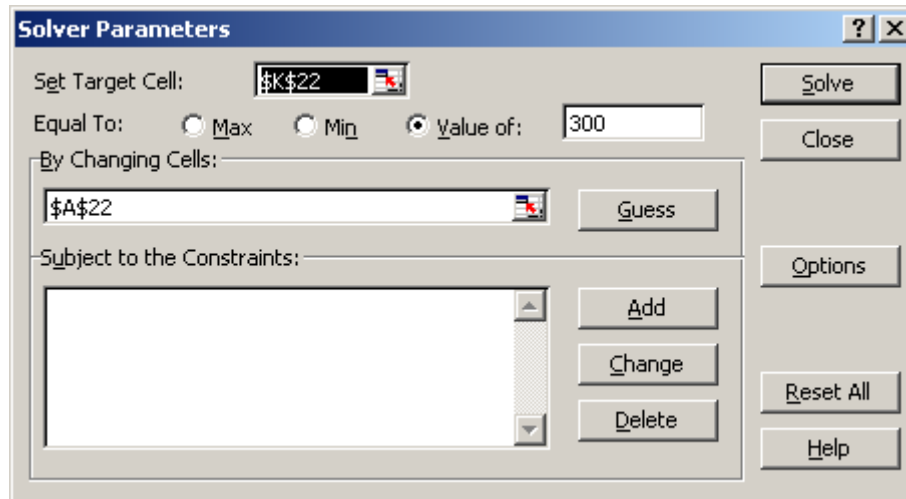


Fig. 12. Solver Parameter menu

	A	B	C	D	E	F	G	H	I	J	K
17	$NU_{D,max} =$	44.318									
18	h_o	88.105	$W/m^2.K$								
19											
20	step 2	step 3	Step 4	step 5	step 6	Step 7	step 8	step 9	step10	step 11	step 12
21	C_R	$C_o = C_{max}$	$(m_w)_i$	Re_D	NU_D	h_i	U	NTU	ϵ	ΔT_c	$(\Delta T_h)_{calc}$
22	0.261045	9059	0.0216	2622.9	18.48	487.9	74.63	0.9914	0.5825	78.31	300.00
23											
24											

Fig. 13. The iteration results obtained by Excel's Solver tool for example 3

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Nomenclature

A =	surface area or cross-sectional area, m ²
C=	Fluid capacitance, W/K
C _R =	capacitance ratio, C _{min} /C _{max}
c _p	specific heat, J/kg.K
D	diameter, m
F=	correction factor
h =	heat transfer coefficient, W/m ² ·K
k =	thermal conductivity, W/m·K
L=	length, m
\dot{m} =	mass flow rate, kg/s
LMTD=	logarithmic mean temperature difference, K
NTU=	number of transfer units, Eq. 10
Nu	Nusselt number
P=	correction factor parameter, Eq. 5
q =	heat transfer rate, W
R =	correction factor parameter, Eq. 6
Re =	Reynolds number
S	pitch
T =	temperature, °C or K
U =	overall heat transfer coefficient, W/m ² ·K

Greek letters

Δ	difference
ε	heat exchanger effectiveness
μ =	viscosity, N.s/m
ν	kinematic viscosity, m ² /s

Subscripts

c=	cold fluid stream
h =	hot fluid stream
i =	inlet condition
L	linear
min=	minimum value
max	maximum value
o =	outlet condition
T	transverse

$\infty =$ ambient condition

Biography

AMIR KARIMI

Amir Karimi is a Professor of Mechanical Engineering and an Associate Dean of Undergraduate Studies at The University of Texas at San Antonio (UTSA). He received his Ph.D. degree in Mechanical Engineering from the University of Kentucky in 1982. His teaching and research interests are in thermal sciences. He has served as the Chair of Mechanical Engineering (1987 to 1992 and September 1998 to January of 2003), College of Engineering Associate Dean of Academic Affairs (Jan. 2003-April 2006), and the Associate Dean of Undergraduate Studies (April 2006-present). Dr. Karimi is a Fellow of ASME, senior member of AIAA, and holds membership in ASEE, ASHRAE, and Sigma Xi. He is the ASEE Campus Representative at UTSA, ASEE-GSW Section Campus Representative, and served as the Chair of ASEE Zone III (2005-07). He chaired the ASEE-GSW section during the 1996-97 academic year.

Appendix

Table 1. Heat Exchanger Effectiveness Equations

Heat Exchanger Type	Flow arrangements	Effectiveness relation
Double-tube	Parallel-Flow	$\varepsilon = \frac{1 - \exp[-NTU(1 + C_R)]}{1 + C_R}$
	Counter Flow, $C_R < 1$	$\varepsilon = \frac{1 - \exp[-NTU(1 + C_R)]}{1 - C_R \exp[-NTU(1 - C_R)]}$
	Counter Flow, $C_R = 1$	$\varepsilon = \frac{NTU}{1 + NTU}$
Shell-and-tube	One shell pass, even number of tube passes	$\varepsilon_1 = 2 \left\{ 1 + C_R + (1 + C_R^2)^{0.5} \left[\frac{1 + \exp\left[-NTU(1 + C_R^2)^{0.5}\right]}{1 - \exp\left[-NTU(1 + C_R^2)^{0.5}\right]} \right] \right\}^{-1}$
	n shell pass, even number of tube passes	$\varepsilon = \left[\left(\frac{1 - \varepsilon_1 C_R}{1 - \varepsilon_1} \right)^n - 1 \right] \left[\left(\frac{1 - \varepsilon_1 C_R}{1 - \varepsilon_1} \right)^n - C_R \right]^{-1}$
Single pass-cross flow	Both fluids unmixed	$\varepsilon = 1 - \exp\left[\left(\frac{1}{C_R} \right) (NTU)^{0.22} \left\{ \exp[-C_R (NTU)^{0.78}] - 1 \right\} \right]$
	C_{\max} mixed, C_{\min} unmixed	$\varepsilon = \left(\frac{1}{C_R} \right) (1 - \exp\{-C_R [1 - \exp(-NTU)]\})$
	C_{\min} mixed, C_{\max} unmixed	$\varepsilon = 1 - \exp\{-C_R^{-1} \{ [1 - \exp(-C_R NTU)] \}\}$
All	$C_R = 0$	$\varepsilon = [1 - \exp(-NTU)]$