

FOURIER ANALYSIS OF A SINGLE-PHASE FULL BRIDGE RECTIFIER USING MATLAB

Bruno Osorno

California Sate University Northridge
18111 Nordhoff St
Northridge CA 91330
Email: bruno@ecs.csun.edu
Phone: (818)677-3956

Abstract

The use of trigonometric Fourier series is applied when repetitive waveforms are found. This situation occurs at the output of full bridge single-phase rectifiers. As it will be seen, the application of a very powerful mathematical technique has given the right solution of a typical electrical engineering circuit. The waveforms analyzed are in their steady-state mode with a repetitive period “T” that depends on the physical electric circuit. [1]. We will indicate that the use of the fundamental signal of a distorted waveform becomes the most important piece of information. This is because in power electronics the calculation of “Power factor”, “Displacement power factor” and “Total Harmonic Distortion” make use of the fundamental value.

Theory and simulation

Non-sinusoidal waveforms, $f(t)$, that have angular frequencies “ ω ” can be obtained as:

$$f(t) = F_0 + \sum_{h=1}^{\infty} f_h(t) = \frac{1}{2}a_0 + \sum_{h=1}^{\infty} \{a_h \cos(h\omega t) + b_h \sin(h\omega t)\} \quad 1$$

Where:

The average value is:

$$F_0 = \frac{1}{2}a_0 \quad 2$$

The average value of a periodic function is called the “DC value” of that function
The MATLAB statement for the average value of a waveform “v” is:

$$vdc = \text{mean}(v)$$

Now we will generate a waveform in order to obtain its dc value. The MATLAB code is given below.

```
% Waverform period divided in 512 points
Ts=1;dt=Ts/512
t1=0:dt:Ts/4-dt;
t2=Ts/4:dt:Ts/2-dt;
t3=Ts/2:dt:Ts;
t=[t1,t2,t3];
%Amplitudes for the waveform
v=[10*ones(size(t1)),-5*ones(size(t2)),zeros(size(t3))];
%Plotting the waveform
plot(t,v)
%Axis limits
axis([0,Ts,-12,12])
%Setting the grid on
grid
%Setting the labels
xlabel('time in seconds')
%Calculation of the average DC value
Vdc=mean(v)
```

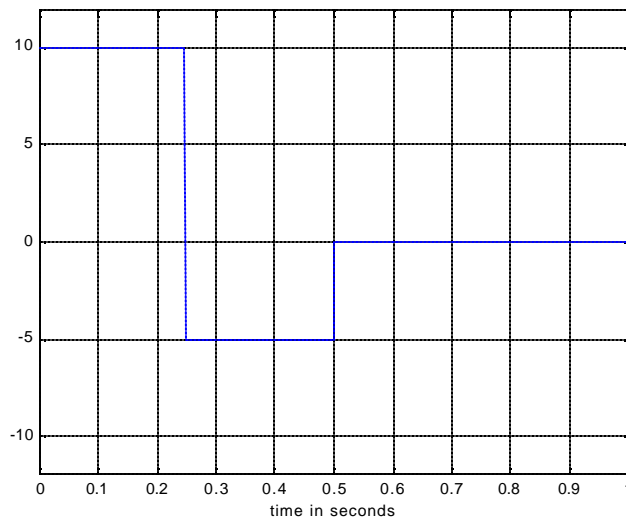


Figure1. Waveform created with MATLAB.

Vdc =

1.2476

The coefficient a_h becomes:

$$a_h = \int_0^{2p} f(t) \cos(hwt) d(wt) \quad h = 0, \dots, \infty \quad 3$$

The coefficient b_h becomes:

$$b_h = \frac{1}{p} \int_0^{2p} f(t) \sin(hwt) d(wt) \quad h = 1, \dots, \infty \quad 4$$

It is a well known fact that [2] the average value of $f(t)$ is:

$$F_0 = \frac{1}{2} a_0 = \frac{1}{2p} \int_0^{2p} f(t) d(wt) = \frac{1}{T} \int_0^T f(t) dt \quad 5$$

Where the radian frequency is:

$$w = \frac{2\pi}{T} \quad 6$$

This average was obtained using MATLAB for the waveform generated. The value was 1.2476 Volts. This can be easily proved using equation 5.

Now, moving on to the frequency domain, we can obtain the RMS (root mean square) of $f(t)$ as F_h . This notation usually is described as the “phasor” form of $f(t)$. Then:

$$F_h = F_h e^{jq_h} \quad 7$$

Another phasor form, using a_h and b_h coefficients, is given as follows:

$$F_h = \frac{\sqrt{a_h^2 + b_h^2}}{\sqrt{2}} \quad 8$$

Where the angle q_h is obtained as follows:

$$\tan(q_h) = \frac{-b_h}{a_h} \quad 9$$

Finally the RMS value of $f(t)$ is F , then we have the following equation:

$$F = (F_0^2 + \sum_{h=1}^{\infty} F_h^2)^{\frac{1}{2}} \quad 10$$

The MATLAB RMS of a waveform “v” can be obtained with the following statement:

$$V_{rms} = \text{sqrt}(\text{mean}(v^2))$$

The RMS value of the waveform “v” is obtained with the following program. The only addition to the previous code (Vdc) is the statement $V_{rms} = \text{sqrt}(\text{mean}(v^2))$.

```

Ts=1;dt=Ts/512
    t1=0:dt:Ts/4-dt;
        t2=Ts/4:dt:Ts/2-dt;
            t3=Ts/2:dt:Ts;
t=[t1,t2,t3];
% Amplitudes for the waveform
v=[10*ones(size(t1)),-5*ones(size(t2)),zeros(size(t3))];
% Plotting the waveform
plot(t,v)
% Axis limits
axis([0,Ts,-12,12])
% Setting the grid on
grid
% Setting the labels
xlabel('time in seconds')
% Calculation of the average DC value
Vdc=mean(v)
% Calculation of the RMS value.
Vrms=sqrt(mean(v.^2))

Vrms =

    5.5847

```

»

The MATLAB harmonic value of a waveform “v” is obtained as follows:

```

» Ts=1;dt=Ts/512
    t1=0:dt:Ts/4-dt;
        t2=Ts/4:dt:Ts/2-dt;
            t3=Ts/2:dt:Ts;
t=[t1,t2,t3];
% Amplitudes for the waveform
v=[10*ones(size(t1)),-5*ones(size(t2)),zeros(size(t3))];
% Plotting the waveform
plot(t,v)
% Axis limits
axis([0,Ts,-12,12])
% Setting the grid on
grid
% Setting the labels
xlabel('time in seconds')
%
% Calculation of the spectrum

```

```

%
[f,y,ph]=harmonic(v,16);
title('Spectrum of "v" waveform')
xlabel('harmonic number')
ylabel('amplitude')
ph*180/pi

ans =

    72.0387

»

```

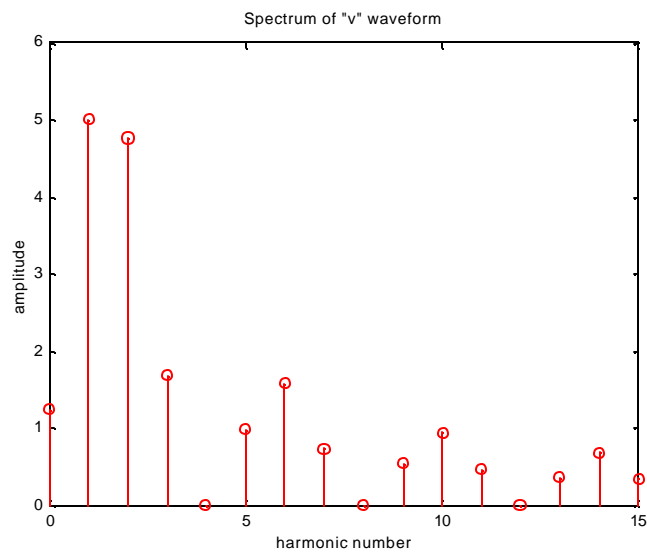


Figure2. Spectrum of figure 1 using MATLAB.

To obtain the spectrum we created a function that is stored in the work file of MATLAB. This function is listed below:

```

%Creation of a function called "harmonic"
%
function[f,y,ph]=harmonic(x,n)
N=size(x,2);
f=0:1:N-1;
y1=fft(x);
y=2*(abs(y1)/N);
y(1)=y1(1)/N;
ph=angle(y1(2))+pi/2;
stem(f,y,'r')
axis([0,n-1,0,1.2*max(y)])
%

```

Fourier analysis of voltage and current

As stated earlier, the trigonometric Fourier analysis of a repetitive waveform can be obtained using equations 1 through 10. If we let $v_s(t) = \sqrt{2}v_s \sin(\omega t)$ be the input voltage for a typical circuit (see figure 1) the input current can be obtained as:

$$i_s(t) = i_{s_1}(t) + \sum_{h \neq 1} i_{sh}(t) \quad 11$$

Where:

$i_{s_1}(t)$ is the fundamental component (at line frequency f_1).

$i_{sh}(t)$ is the component at the “h” harmonic frequency (f_h).

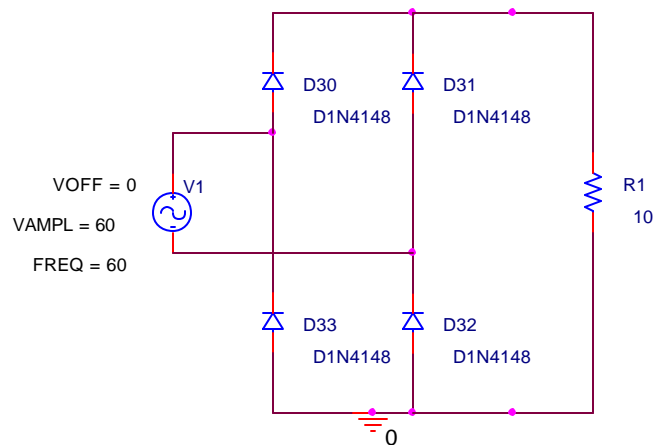


Figure3. PSPICE circuit of a single-phase rectifier

```
* source FOURIER2
V_V1      N03719 N03754
+SIN 0 60 60 0 0 0
R_R1      0 N02751 10
D_D30     N03719 N02751 D1N4148
D_D31     N03754 N02751 D1N4148
D_D32     0 N03754 D1N4148
D_D33     0 N03719 D1N4148
```

Then we have [1]:

$$f_h = hf_1 \quad 12$$

Then equation 11 can be expanded as follows:

$$i_s(t) = \sqrt{2}I_{s_1} \sin(\omega t - q_1) + \sum_{h \neq 1} \sqrt{2}I_{sh} \sin(h\omega t - q_h) \quad 13$$

Typical distortion occurs at the current level, voltages remain, for the most part, undistorted. Figure 4 shows a very typical voltage-current waveform where the distortion of currents is apparent and the process of the Fourier trigonometric analysis can be seen.

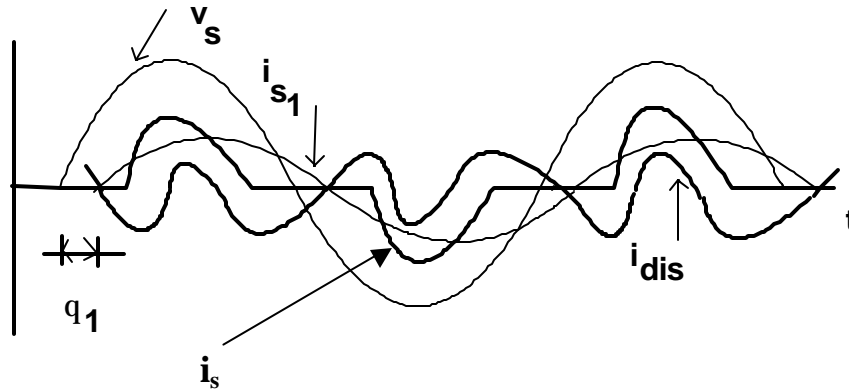


Figure 4. Typical voltage-current output and its Fourier components

The RMS current is:

$$I_s = \left(\frac{1}{T_1} \int_0^{T_1} (i_{s_1}(t) + \sum_{h \neq 1} i_{sh}(t))^2 dt \right)^{\frac{1}{2}} = (I_{s_1}^2 + \sum_{h \neq 1} I_{sh}^2)^{\frac{1}{2}} \quad 14$$

The IEEE [10] standard recommends the maximum allowed harmonic content in power electronics depending on the type of circuit. This value is calculated using equation 15. “THD” stands for “total harmonic distortion” and it is usually given in %. PSPICE determines automatically this value. In MATLAB we have to calculate it. In the next section we show a PSPICE output for a simple single-phase rectifier.

$$\%THD = 100 \times \frac{\sqrt{I_s^2 - I_{s1}^2}}{I_{s1}} \quad 15$$

Full bridge single-phase rectifier

Using PSPICE we obtained the output of a single-phase full bridge rectifier. Figure 2, shows the circuit and figure 5 shows the output. The trigonometric Fourier analysis is performed with PSPICE.

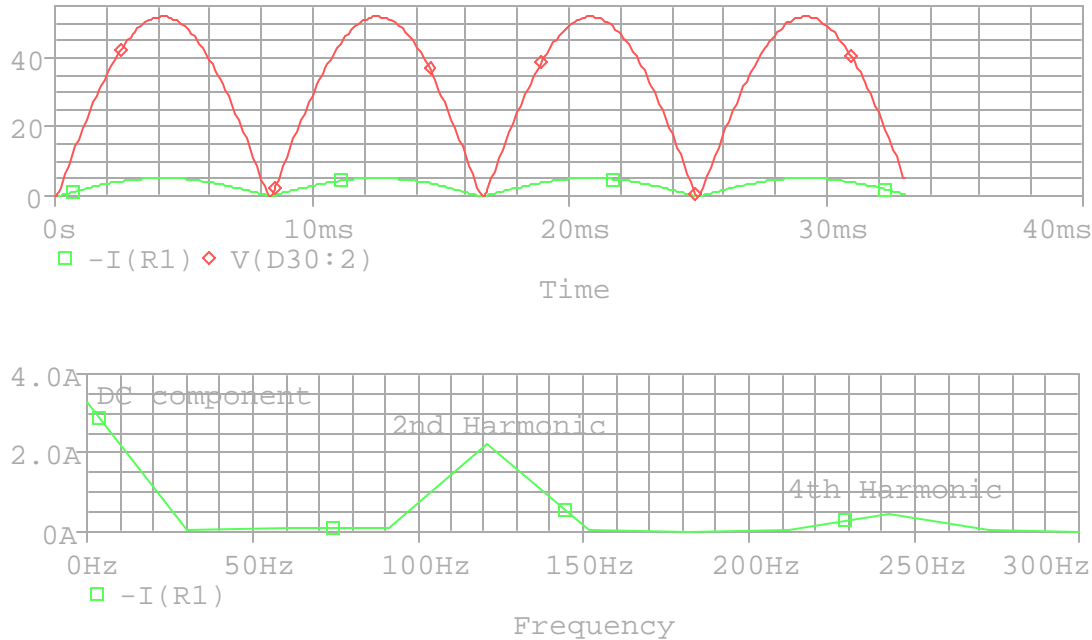


Figure5. PSPICE output of the single-phase rectifier shown in figure 3
 Upper graph indicates the voltage and current output.
 Lower graph indicates the Harmonic analysis.

Conclusions

Power electronics and Power quality are one of the major fields in electrical engineering that require the understanding of “trigonometric Fourier series” and its applications. It is of no surprise that this technique works very well in obtaining the necessary information from input/output voltage/current signals. By doing so, we go a step further and determine the %THD that gives us an indication of the “goodness” of our electrical design. Again, by using one of the most powerful mathematical techniques we arrive to a simple, yet, very important solution of a problem that in the past was tedious and cumbersome in its solution. Recently, with the powerful use of Personal Computers and, perhaps the most widely used, software simulator and mathematical packages the application and solution of trigonometric Fourier series has become a lot simpler. Consequently the design of electrical circuits in general is getting better and better.

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BRUNO OSORNO, is a professor of electrical and computer engineering at California State University Northridge. He is the lead faculty member in the Power Systems and Power Electronics program. Professor Osorno has written over 20 technical papers. His current interest in research is Fuzzy Logic applications in power electronics and electric motor drives.