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# **AC 2011-1184: BASIC CLASS MATERIALS AND LABORATORY PROJECTS WITH DC MOTORS IN AN INTRODUCTORY UNDERGRADUATE ECE CLASS FOR NON-MAJORS**

## **Sergey N. Makarov, Worcester Polytechnic Institute**

Submitting author: Sergey N. Makarov earned his B.S./M.S./Ph.D./Dr. Sci. degrees at the State University St. Petersburg (Leningrad), Russian Federation Faculty of Mathematics and Mechanics. Dr. Makarov joined Institute of Mathematics and Mechanics at State St. Petersburg University in 1986 as a researcher and then joined the Faculty of State St. Petersburg University where he became a full professor in 1996. In 2000 he joined the Faculty of Department of Electrical and Computer Engineering at Worcester Polytechnic Institute, MA. His current teaching interests include fundamental ECE classes.

## **Reinhold Ludwig, Worcester Polytechnic Institute**

Reinhold Ludwig is a Professor of Electrical and Computer Engineering with joint appointments in Mechanical and Biomedical Engineering. His research and teaching is related to high frequency circuit design and electromagnetics.

## **Kaung Myat Win, WPI ECE Dept.**

Kaung Myat Win. Sophomore Electrical and Computer Engineering student at Worcester Polytechnic Institute. Undergraduate assistant in ECE Department at WPI.

## Basic class materials and laboratory projects with DC motors in an introductory undergraduate ECE class for non-majors

Most electrical and computer engineering departments in the United States and abroad typically offer a fundamental one or two-semester course in ECE for non-major students. Sometimes, this course is offered to both majors and non-majors. In general, it is a very difficult task to teach complex electrical engineering concepts, including circuit theory, semiconductor fundamentals, and digital fundamentals in a single course. Therefore, the intro class for non-majors (or for both majors and non-majors) is frequently devoted to circuit (and digital) fundamentals only. This scenario creates a visible dissatisfaction, especially among ME and CE majors, who wish to be exposed to power electronics - motors, even in their first ECE class. The same trend is observed for ECE majors, when both classes are combined together. The exposure to power electronics is not only beneficial for the intro class, but it may also stimulate further long-term interest in ECE and Robotics. On the other hand, the systematic study of motor fundamentals (including AC motors<sup>1-4</sup>) in the intro class may not be possible, as it would require a significant extension of the already tight syllabus. What can be done to resolve this dilemma?

This paper reports on our pedagogic approach of introducing the motor-related materials into the class syllabus (3-4 lectures) and into one or two class laboratories in an engaging yet informative manner that is particularly appealing to non-major students.

The course is taught to both majors and non-majors. Course objectives include studying basic DC circuits, circuit equivalents, circuit laws, the basics of power transfer, the DC model of operational amplifiers, simple switching circuits, and the basics of A to D conversion. Throughout their studies in this course, students will be introduced to the commercially popular mathematical computation program known as MATLAB. Major course outcomes include students' confidence and fluency with DC circuit analysis, as well as the ability to immediately apply their knowledge to real-world scenarios.

The supporting class materials on DC motors are prepared based on the first principles. They are reasonably different from some common popular guidelines<sup>5</sup>. At the same time, these materials have the following (simplifying) features:

1. Only an introductory analysis of DC motors and DC generators operating in *steady state* is given, starting with the Lorentz force concept and linear motors.
2. DC motor resistance, the torque constant, and the voltage constant are then introduced and studied together.
3. Next, it is suggested that the class learn how to find and interpret motor constants from the datasheet is suggested next.
4. Afterwards, the torque-speed curves are studied using four measurable motor quantities:  $\omega_{no\ load}$ ,  $I_{no\ load}$ ,  $T_{stall}$ , and  $I_{stall}$  are studied afterwards.
5. Finally, the analysis leads to an example - a selection of a DC motor for a particular application, including both steady-state operation and acceleration.

The supporting laboratory syllabus currently has the following features:

1. In the first laboratory, a motor (a 12V surplus Mabuchi model for ~\$5) is introduced as a load device. Students measure  $R_M$ ,  $I_{no\ load}$ , and  $I_{stall}$ .
2. In the following (or the same) laboratory, students connect the motor to a half H-bridge and subsequently to the full H-bridge and observe four different motor states: (i) forward

mode; (ii) reverse mode; (iii) free run to a stop and; (iv) brake. Students have the option of applying PWM to this circuit to vary motor speed.

We shall describe our experience with this scenario in an introductory course offered during Fall 2010/Spring 2011 (about 250 students total), and the corresponding assessment results.

**1. Lorentz force - linear motor/generator/brake concept (one lecture)**

The lecture sequence is outlined using the three figures that follow. We only consider linear motors since understanding the Lorentz force effect requires care. Corresponding examples are presented in class.

**1.1 First step - Motor action:**  $F = Nf = l \cdot Anqv \cdot B = I_l B$

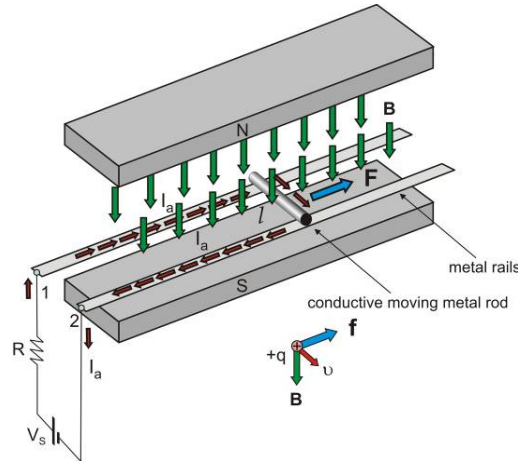


Fig. 1. Electromagnetic accelerator or a linear motor based on the Lorentz force effect.

**1.2 Second step - Generator action:**  $E = W / q = lvB$  [V]

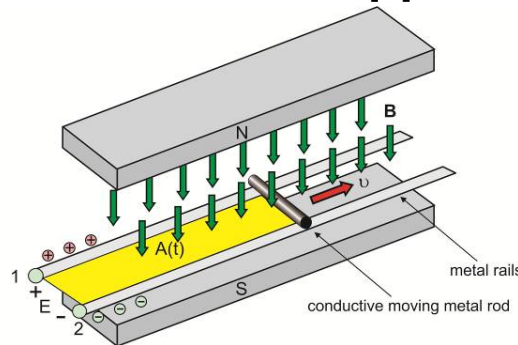


Fig. 2. Linear generator (linear dynamo) based on the Lorentz force effect.

**1.3 Third step - Brake action:**  $F_b = I_l B = l^2 v B^2 / R$

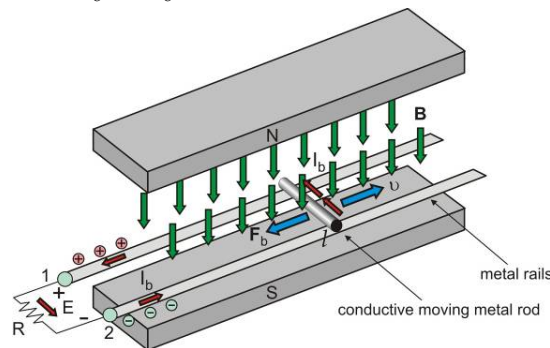


Fig. 3. Braking effect of the Lorentz force - motor (or generator) setup.

## 2. Rotating DC machine. Torque constant and voltage constant (one lecture)

The lecture sequence is outlined using four figures that follow. We first derive the torque constant, then introduce the stall torque and the stall current. Finally, we proceed to the equivalent circuit model of a motor and to the voltage constant. This lecture ends with the set of three motor equations at steady state.

**2.1 Torque constant. Idea of a DC coil rotor:**  $T = \frac{2AB}{\pi} I_a \Rightarrow T = K_T I_a$

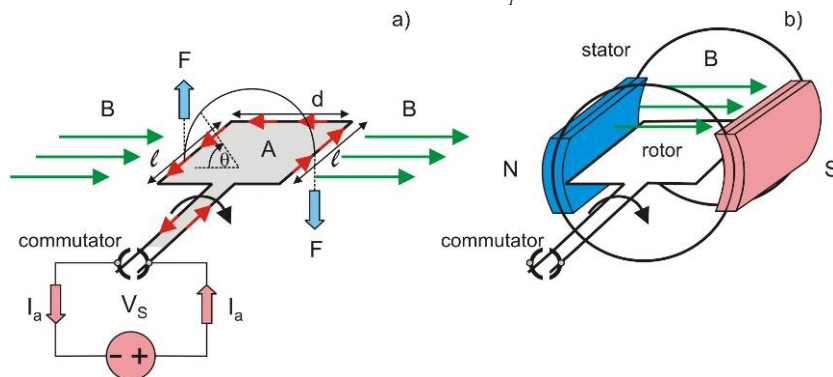


Fig. 4. Two-pole DC machine with the permanent magnets and one-turn armature.

**2.2 Model of a DC motor at stall:**  $I_a = I_{\text{stall}} = \frac{V_s}{R_M}$ ,  $K_T = \frac{T_{\text{stall}}}{I_{\text{stall}}}$

Here we introduce motor (armature/brush) resistance and stall conditions. Finding torque constant from stall conditions is explained in class and illustrated by real-life examples.

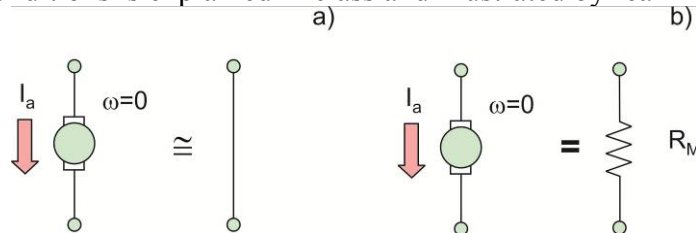


Fig. 5. Motor model at stall (no rotation).

**2.3 Model of a DC motor at steady state and power conservation law:**

Power conservation law leads to:  $E = K_T \omega$

“Iron losses” lead to:  $E = K_V \omega$ ,  $\omega T \leq EI_a \Rightarrow \omega K_T I_a \leq K_V \omega I_a \Rightarrow K_T \leq K_V$

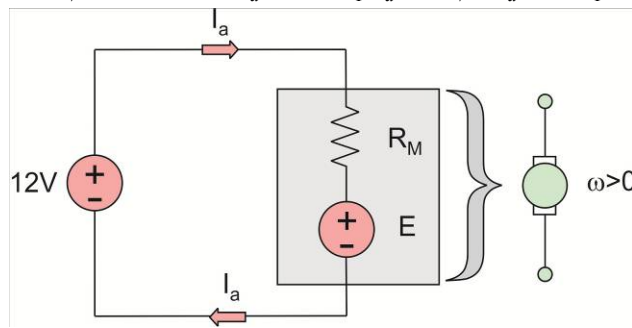


Fig. 6. Motor model for a 12V DC motor at steady-state rotation with angular speed  $\omega$ .

**2.4 Finding voltage constant from no-load conditions:**  $K_V = \frac{E_{\text{no load}}}{\omega_{\text{no load}}} = \frac{V_S - R_M I_{\text{no load}}}{\omega_{\text{no load}}}$

Examples may include any of the DC motors from Table 1:

Table 1. Motor constants and their respective ratios for five DC motors.

| Motor part #                                         | $K_T$ from stall conditions, N·m/A | $K_V$ from no-load conditions, V/(rad/sec) | Ratio $K_T / K_V$ |
|------------------------------------------------------|------------------------------------|--------------------------------------------|-------------------|
| 3V 1.4W Mabuchi DC motor FC-130RA-2270               | 0.0010                             | 0.0014                                     | 0.73              |
| 6V 5.6W Mabuchi DC motor RC-280SA-2865               | 0.0033                             | 0.0040                                     | 0.83              |
| 9V 4W Mabuchi DC motor RC-280SA-20120                | 0.0063                             | 0.0073                                     | 0.87              |
| 12V 19W Mabuchi DC motor RS-380PH-3270*              | 0.0067                             | 0.0068                                     | 0.98              |
| 12V 96W higher-power Mabuchi DC motor RS-550VC-7525* | 0.0065                             | 0.0064                                     | 1.01              |

**3. Torque-speed curves. Motor selection (one or two lectures):**

The lecture sequence is outlined using the three figures that follow. First, we define the load torque, and then we show how to create the torque-speed curves from the motor datasheet. Finally, we select the motor operating point, and discuss the motor selection based on the power criterion.

**3.1 Load torque:**  $T = T_{\text{load}} + B_f \omega$

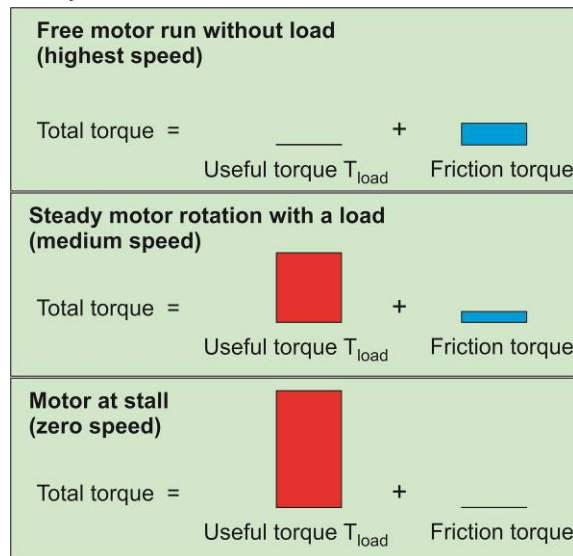


Fig. 7. Balance between the load torque and the friction torque at different motor speeds.

**3.2 Torque-speed curves from motor datasheet: use**

no-load speed  $\omega_{\text{no load}}$  in rpm (revolutions per minute) or in rad/sec at rated voltage;

no-load motor current  $I_{\text{no load}}$  in amperes at rated voltage;

stall current  $I_{\text{stall}}$  in amperes at rated voltage; and  
 stall torque of the motor  $T_{\text{stall}}$  in N·m at rated voltage.

Torque-current line: 
$$I_a = \frac{I_{\text{stall}} - I_{\text{no load}}}{T_{\text{stall}}} T_{\text{load}} + I_{\text{no load}}$$

Torque-speed line: 
$$\omega = \omega_{\text{no load}} \left[ 1 - \frac{T_{\text{load}}}{T_{\text{stall}}} \right]$$

Torque-power curve: 
$$p_M = T_{\text{load}} \omega = \omega_{\text{no load}} T_{\text{load}} \left[ 1 - \frac{T_{\text{load}}}{T_{\text{stall}}} \right]$$

Torque-efficiency curve: 
$$\eta = \frac{\omega_{\text{no load}} T_{\text{load}} [T_{\text{stall}} - T_{\text{load}}]}{V_S (I_{\text{stall}} T_{\text{load}} + I_{\text{no load}} [T_{\text{stall}} - T_{\text{load}}])}$$

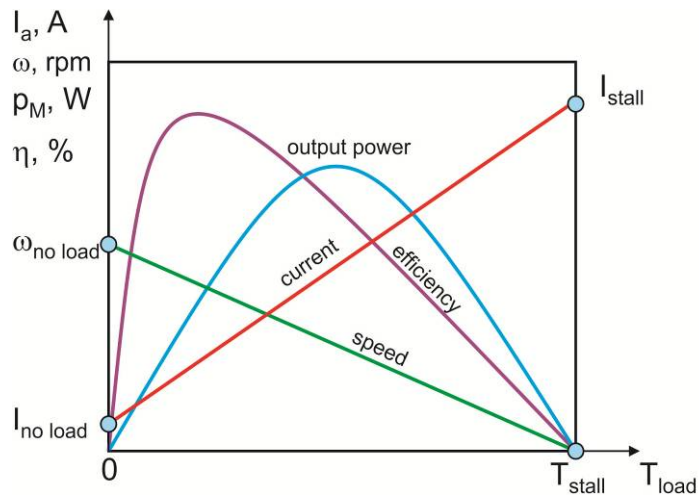


Fig. 8. Generic torque-speed curves.

### 3.3 Selecting motor operating point:

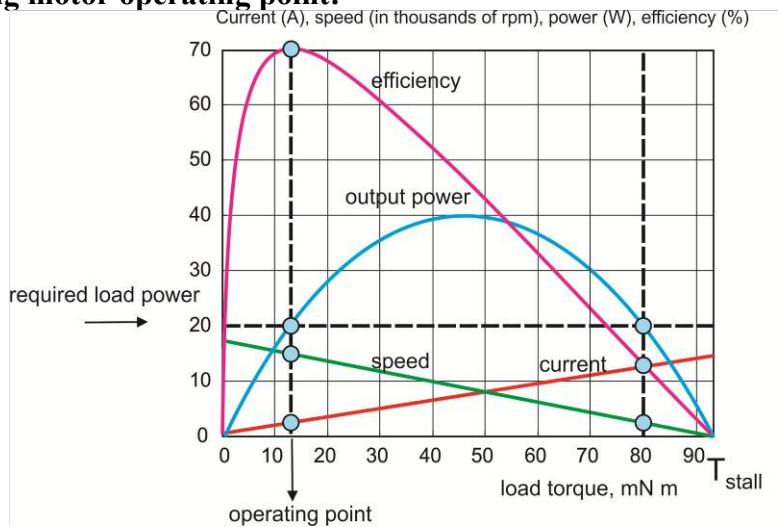


Fig. 9. Selecting proper operating point either for cruiser (left) or for acceleration (right) operation.

## 4. Laboratory

One suggested laboratory syllabus is explained below. This is a 3-hour long laboratory.

### 4.1 Measuring motor parameters

#### 4.1.1 No-load speed ( $\omega_{no\_load}$ ) measured with the strobe light illuminating the shaft notch:

- Ask a TA for a clear power LED and connect the LED directly to the function generator, then place the LED with your hand perpendicular to the notch of the motor shaft, at the distance of approximately 0.5".
- Slowly vary the frequency of the function generator in the 70Hz - 100Hz range by rotating the frequency knob of the function generator. You should be able to see the light from the LED reflecting back from the flat side of the motor shaft periodically, forming a "delayed" rotation figure. As you change the frequency, you will see the notch rotating slower or faster. This is called the strobe effect.
- At one specific frequency, you will see the notch of the motor appear to stall. Record this value. This is exactly the frequency of the motor shaft rotation in Hz.

#### 4.1.2 Motor Resistance ( $R_M$ ) measured with the calibrated DMM:

Table. 2. Set of measurements for motor resistance ("warm" motor).

| Facing direction | $R_{total}$ ( $\Omega$ )   | $R_M$ ( $\Omega$ ) |
|------------------|----------------------------|--------------------|
| (South)          |                            |                    |
| (West)           |                            |                    |
| (North)          |                            |                    |
| (East)           |                            |                    |
|                  | Average $R_M$ ( $\Omega$ ) |                    |

Then, the stall current  $I_{stall}$  is found using Ohm's law at stall.

#### 4.1.3 No-load current ( $I_{no\_load}$ ) measured with the DMM:

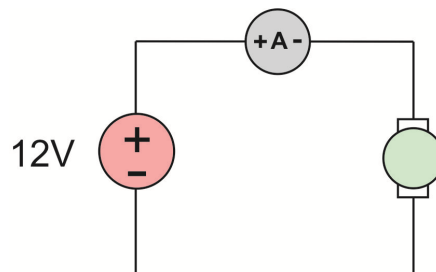


Fig. 10. No-load current measurements.

#### 4.1.4 Stall torque

The stall torque is found using the motor torque constant value given to students and the (indirectly) measured stall current.

#### 4.2 Generating torque-speed curves and finding maximum motor efficiency:

1. Open MATLAB from the desktop or from Start  $\Rightarrow$  All Programs  $\Rightarrow$  MATLAB  $\Rightarrow$  R2010a  $\Rightarrow$  MATLAB R2010a.
2. Then click File $\Rightarrow$ New $\Rightarrow$ Script or click the New Script button to open a new MATLAB m-file.
3. Copy and paste the sample MATLAB script, substitute your measured values.
4. Put your name and your partner's name at the beginning of the script
5. Save the script as lab2.m. Run the script.
6. Observe the figure and keep it on the screen.
7. Using the textbox option of the figure window, put a label near each curve (current, speed, power, efficiency).
8. Using the cursor option of the figure, determine (approximately) the maximum possible efficiency of the motor, and all of the other related motor parameters (torque, current, speed, power) at its maximum efficiency.
9. Using the cursor option of the figure, determine the maximum possible output power of the motor and all of the other related motor parameters (torque, current, speed) at its maximum power.

After completion of this assignment, students will have created a complete datasheet for the “no-name” motor.

#### 4.3 Practice question: determining unknown torque for a motor load

As a model for a low-torque motor load, we use a white plastic disk (shown in Fig. 11 that follows). The disk rotates at a rated load voltage.

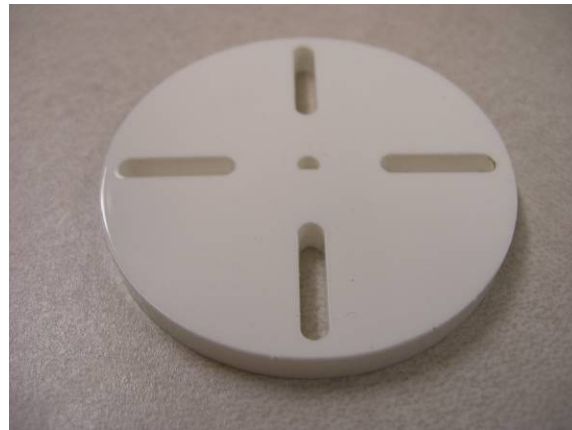
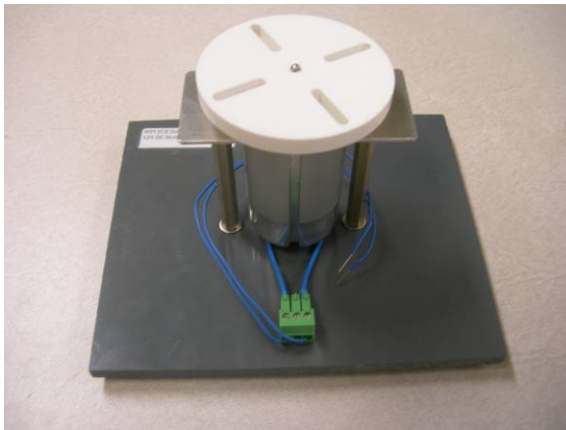


Fig. 11. An artificial low-torque load with an unknown torque.

When rotating fast, the disk experiences significant air friction, which is additionally enhanced by the four inner slots. The *load torque* is the torque that is necessary to overcome this friction, similar to the load torque of a propeller. The students' goal is to find the load torque at the rated voltage. They have all of the prerequisites to do so, and are required to do only one extra measurement.



## 5. Conclusions and assessment

Table 3 lists preliminary data: student assessment (two cases in point - *stimulation of interest in ECE* and *amount of learned material*) for two identical ECE intro classes with non-major participation: one with and one without the specific motor content taught in the Fall of 2010. Each class is one-quarter long. Classes were taught by two different instructors. About 65% students have responded in every case. The students have taken the course only once.

Table 3. Assessment results for two nearly identical non-major classes (Fall of 2010). Grade from 0 to 5.00. About 70 students per class have responded.

|                                                                                         | Without motor content | With motor content |
|-----------------------------------------------------------------------------------------|-----------------------|--------------------|
| The instructor stimulated my interest in the subject matter (compared to other classes) | 4.21                  | 4.65               |
| The amount I learned from the course was (compared to other classes)                    | 4.17                  | 4.30               |

More assessment results will be reported based on the ongoing classes.

Certain elements of our classroom teaching strategy can easily be adopted by professors and instructors who are assigned to teach introductory ECE classes for non-majors.

## References

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