Comparing learning outcomes and student experiences in Engineering Math using virtual and physical robots

Daniel Godrick

Dan Godrick, P.E., is a Teaching Assistant Professor with the Integrated Design Engineering program at the University of Colorado, Boulder. He brings a wealth of experience to his teaching, including time spent as a buisness consutant, project manager, and as a engineering consultant in private practice.

He holds degrees in Mechanical Engineering (BS Duke University, MS CU) and in Civil Engineering (MS George Washington University). He is a licensed P.E. in Colorado.

Angela Bielefeldt

Dr. Bielefeldt, P.E., is a Professor at the University of Colorado Boulder (CU Boulder) in the Department of Civil, Environmental, & Architectural Engineering and the Director of the Integrated Design Engineering program. She has taught at CU Boulder since 1996, including a variety of courses for first-semester students. Her primary research interests in engineering education include sustainability, community engagement, social responsibility, and ethics.

Rachel Sharpe

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ABSTRACT

The Wright State Engineering Math curriculum turns math problems into engineering questions, and it includes labs where students investigate deeper engineering challenges. The University of Colorado took several of the labs created and enabled by Wright State and turned these labs into physical, hands-on exercises. The COVID-19 pandemic forced remote lab work and enabled a unique opportunity to compare the virtual and physical labs. The paper explores one of these labs and qualitatively discusses the pros and cons of each lab format. The lab in question teaches trigonometry through 2-link robots. The learning objectives of the lab are discussed. Then the three iterations of the lab, a virtual robot simulator, a "peg-board" robot, and an Arduino-controlled robot are examined in more detail.

All three labs accomplished the learning objectives surrounding the *mathematical and trigonometric* concepts necessary to manipulate 2-link robots. The current, Arduino-robot, lab enabled students to explore/play with physical 2-link robots. This fostered deeper understanding and discussions of manufacturing tolerances, programming, electronics, and ambiguity in engineering. This learning helped develop the engineering mindset that was valuable not only in future mathematics courses, but also in future engineering project and design-based courses.

BACKGROUND

The Wright State Engineering Math model [1,2] introduces students to their math curriculum through problems taken directly from engineering subjects. The University of Colorado incorporates a modified Wright State Engineering Math model in its curriculum, where this course is implemented and taught by the Integrated Design Engineering (IDE) program. The IDE program's focus is active learning, and it creates hands-on labs/exercises in its courses.

For this course, several hands-on labs were created to mimic the Wright State labs. These labs were iterated and continuously refined. Remote learning during the COVID-19 pandemic forced the use of virtual labs and allowed a unique opportunity to compare the virtual with the hands-on labs.

The focus of this paper is a lab that brings the mathematical subject of trigonometry to life. Like other math courses, the learning objectives of this lab involved introducing students to trigonometry in an engineering context. Students were expected to learn the unit circle and find the sine, cosine, and tangent of common numbers. Students were introduced to reference angles, rectangular to polar coordinate conversion, polar to rectangular coordinate conversion, and basic trigonometric identities.

What makes the Wright State engineering curriculum [3] unique is that the unit circle is not presented as a unit circle. Instead, this is presented as a one-link robot. Once students get familiar with the one-link robot, a two-link robot is introduced. The second link is directly attached to the first link. Students must shift their frame of reference and use mathematical techniques to appropriately position the second link, and therefore the robot tip.

The lab has 4 main exercises:

- The first exercise utilizes a one-link robot, and from a math perspective takes students through a polar (angle and length) to rectangular (x, y) unit conversion.
- The second exercise also uses a one-link robot and students practice translating from rectangular coordinates to polar coordinates.
- The third exercise incorporates a two-link robot. Students command the angles for each robot link and determine the final position of the robot arm.
- The fourth exercise asks students to determine the command angles that place the tip of the second link at a specific position.

More than 280 students have tried three different versions of this lab. The first version was a virtual robot simulator made available through Wright State. This virtual simulator

(Figure. 1) allowed students to place each link on a simulated grid with very high resolution. The varying length of the robot link (for one-link robots) allowed students to precisely place the links during exercises one and two.

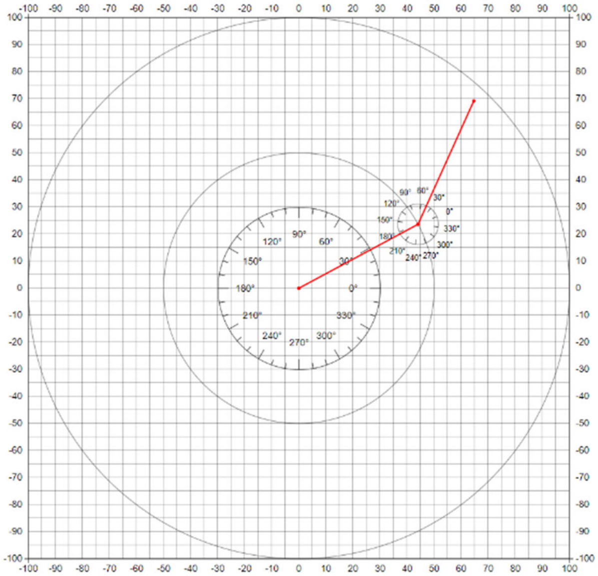

Figure 1: Virtual lab with 2-link robot [4]

For exercises three and four, the robot links were "snapped" to a specific circle, and this constrained the length of the robot links. Virtual protractors were automatically embedded on the screen, which allowed students to precisely measure and place their robot links.

The second iteration of the lab took the 360-degree virtual lab and brought this to a hands-on peg board (Figure 2). For exercises 1 and 2, students placed a peg at the origin. They positioned a second peg and placed a rubber band between the pegs to simulate a robot link. For exercises 3 and 4, students placed the first link, and then placed a third peg and rubber band to simulate the second link. Since the board mimicked the virtual lab well, the exercises were the same as the virtual lab. Said another way, few adjustments to the lab procedure from Wright State needed to be implemented.

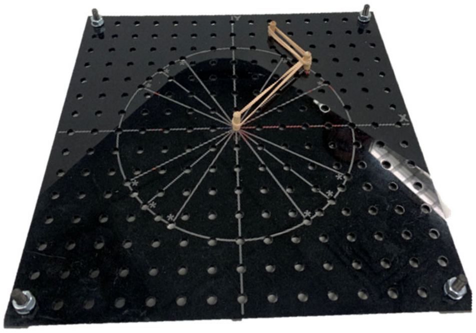

Figure 2: Peg Board Lab with 2-link robot

The third iteration of the lab created physical, 2-link, Arduino-controlled robots (Figure 3). These robots were created in CAD, 3d printed, and incorporated off the shelf servos. The chosen servos only traveled from $0-180°$ (due to cost constraints), instead of travelling from $0-360°$ like the other iterations of the lab. Therefore, the lab procedure needed to be adjusted versus the other lab setups. Students were not required to code the Arduino, but they were encouraged to walk through the provided code during the lab.

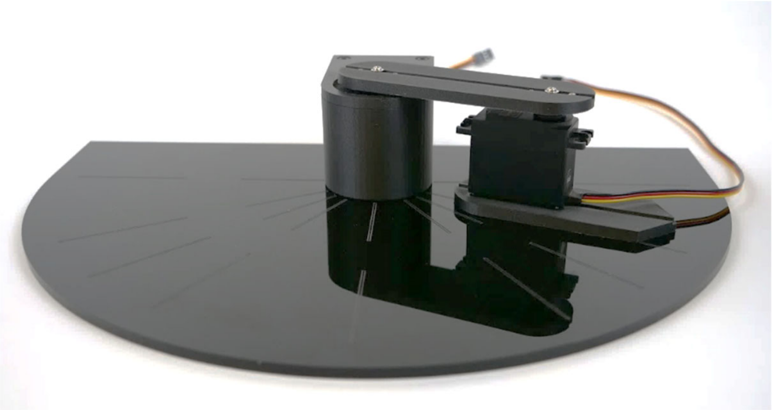

Figure 3: Arduino 2-link robot

COMPARISON OF THE LEARNING OUTCOMES AND STUDENT EXPERIENCES

The goal of the course was to have students think about Math as engineers. Wright State lists educational objectives of this lab: “After performing this experiment, students should be able to: 1. Understand the basic trigonometric functions. 2. Understand the concept of a unit circle and four quadrants. 3. Understand the concept of a reference angle. 4. Be able to perform the polar to rectangular and rectangular to polar coordinate conversion. 5. Prove a few of the basic trigonometric identities.” [5]. From this perspective, all three labs accomplished the set learning objectives. All three labs challenged students to think through the trigonometry necessary to control the robot links, and thus also how to implement the math “tool” of trigonometry to answer the engineering challenge. The three labs had differing benefits and costs regarding student experiences in how they accomplished this.

The Virtual Lab was the least expensive (Wright State allows access to this online tool for free). The website always was reliable during lab (no downtime was found) and was robust throughout the lab (no errors). This tool was the most accessible. Accessibility enabled the lab to continue during the pandemic shutdowns, when in-person classes were forced to remote learning. The lab enabled students to measure precisely. During exercises 1 and 2, students traced their simulated robot links to specific lengths. The lab placed a protractor perfectly on the end of the first robot link, and this enabled students to measure angles precisely and accurately. This lab followed the Wright State template, robots traveled through a full 360 degrees, and students gained trigonometry experience in every quadrant.

At approximately $25, the peg board solution’s cost fell between the free, Virtual lab, and the more expensive Arduino-robot. The pegboard was robust (no pegs broke). At times a peg pulled out of the board and a rubber band flew across the room (students enjoyed this, so this may not have been accidental). The peg boards allowed for different link lengths, which enabled students to generally match the requested positions. However, since the holes were positioned in 1-inch intervals, the students could not perfectly match the requested positions for exercises 2-4 (holes were laser cut in appropriate positions for exercise 1). Students used handheld protractors placed on top of the pegs to position the links and measure angles. This led to error and both precision and accuracy suffered. The board enabled 360-degrees of exploration and followed the Wright State lab template.

The Arduino-robot solution was the most expensive (~$60 per robot). At first there were issues with the durability of these robots, but future iterations solved these issues. The robot links were fixed length, and always had 2-links (the position of the second link was disregarded in exercises 1 and 2). Additionally, the servos had an approximate tolerance of $\pm 2.5°$ of the desired angle. Students used handheld protractors and rulers were placed directly on the robot, which decreased the precision and accuracy of the measurements. Finally, the robot links traveled through 180°, and therefore only the top two quadrants were explored directly with the robots. Therefore, the lab had to be adjusted from the lab provided by Wright State.

DISCUSSION: AMBIGUITY AND UNCERTAINTY BECAME A FEATURE AND NOT A BUG

Viewed in a certain light, the virtual lab was a clear winner. However qualitative student experience showed a different clear favorite: the in-person Arduino-robot. As the students used the Arduino-robots, there was a clear joy that was absent or reduced during the peg-board and virtual lab. During the semester with hybrid learning, 23 out of 35 (66%) student reflections said the in-person labs were a highlight of the course. These findings replicated findings from other implementations of the course where students indicated a preference for hands-on labs [6].

Several of the students specifically pointed to the robot lab, captured with this reflection: *"With online lectures and labs, it can all get a bit boring, however, when I am actually applying what I learned in lecture to something I can touch with my own hands, it becomes much more enjoyable. The most memorable lab for me would probably be the one where we manipulated the robot arm and calculated the point at the end of the arm. It was really cool to see math transform into reality through robot."* Said simply, there was no substitute for having a physical, moving, hands-on robot to examine engineering questions about robots.

With the right mindset, the cons of the robot lab enabled students to explore ambiguities common with engineering. The learning that students took away from the Arduino-robot lab was not only about trigonometry, but also about engineering mindset. Examining this further, the servos used in the lab were not perfect, with a tolerance of $\pm 2.5\,°$ (or worse). On the outside this was a challenge, especially when students were given robots with a bottom link misaligned with a laser engraved axis. However, students were empowered to make changes to the robots. They were given screwdrivers and could adjust the servo attachments during the lab. This provided insight and sparked discussions into manufacturing tolerances. With a "real" servo, it was not an instructor "lecturing" them about the limitation of servos. Instead, students received direct experience into these components.

There were also challenges with physical robots when measuring angles and positions (necessary to fill out tables). However, students saw that they were not alone in encountering issues. There was not a perfect robot. Teams compared their robots and developed solutions to the measuring challenges. Through these discussions, they learned about concepts like accuracy, precision, and significant figures. These informal discussions built camaraderie, which was especially useful given the lab's timing, the third week of class.

In the Arduino-robot implementation, the Wright State lab was modified because the servos only traveled from $0 - 180°$ and the link lengths were fixed. Some "impossible" positions (3rd and 4th quadrant) were included in the tables (students were directed to write NA in this case). Some locations were included where the robots could get close, but not perfectly aligned, to the requested position. These "impossible" cases sparked discussions among students and developed their engineering judgement.

The use of a physical robot should not be understated. In the University of Colorado implementation of this course, many students were First-Generation college students, URM, and/or were not directly admitted to the College of Engineering. The robots used the Arduino microcontroller, and students were required to wire the robot and view the code (they did not have to program the robots themselves). The Arduino microcontroller and servos were used extensively in future classes. This lab provided a low-stakes, initial introduction to these concepts, and later conversations with students reported that they appreciated this introduction. Student reflections spoke to the trust that they felt as students, exemplified by this student quote: *"The most memorable effects of this class were being able to work hands on during the labs which is something I was not expecting to do so fast coming into school. I enjoyed being treated as someone who was able to understand the concepts and being trusted to work on things hands on rather than sending (sic) weeks trying to get us to understand how components work before actually letting us work hands on with the labs."*

As a final note, even when the labs were done with the physical robot links, there were times when the virtual lab came into play. Throughout the pandemic, sometimes the class could meet, but a specific student was quarantined. The virtual lab enabled flexibility for makeup labs and enabled students to keep up with course content, even if they could not physically attend the labs.

STUDENT EVALUATION

Students were given an optional survey at the beginning and end of the semester (protocol reviewed and approved by the Institutional Review Board for Human Subjects Research). A set of questions asked them to rate their confidence that they could successfully solve math problems if exposed to the course materials, rated on a scale of 1 (none at all) to 5 (a great deal). Three of the questions asked about geometric concepts (e.g., vector, plane, dimensions of a 3D object); these were the same items on the pre- and post- survey (among 12 and 18 items, respectively). Results are summarized in Table 1. The results show that students gained confidence in their math abilities, which was likely due to a combination of the labs and other activities in the course.

Table 1. Confidence in solving geometry-related math problems

	Fall 2022	Fall 2021	Fall 2020	Fall 2019^
n pre / n post	38 / 38 ^	17 / 5	33 / 33 ^	83 / 148
Pre survey average	3.14	2.59	3.04	2.83
Post survey average	3.51	4.00	3.43	3.50
t-test two-tailed p-value (paired when possible)	*0.065*	*0.033*	*0.021*	*<0.001*

^ paired pre and post only

At the end of the semester, students complete confidential evaluations of teaching. Several of the write-in comments pertained to the labs. In 2020, example quotes from different students include: “loved in-person labs”, “learning felt fun”, “labs were fun”, “the labs were very good”. Another example from fall 2022: “: labs were fun to do and did a good job of applying things we learned in studio and homework”. It is significant that students found the labs both fun and impactful. The students rated the extent to which the course contributed to achievement of the ABET 1 to 7 outcomes (using a 5-point scale from 1 = lowest to 5 = highest). Students scored the course high on problem solving, as one would expect, but also very high on “experimentation” (Table 2).

Table 2. Summary of End-of-Semester Student Evaluations of Teaching

Outcome	Fall 2022	Fall 2021	Fall 2020	Fall 2019^
n evaluated / n in course	62/127	6 /15	32/38	50/101
identify, formulate, and solve complex engineering problems by applying principles of engineering, science, and mathematics	4.20	4.67	4.74	4.55
develop and conduct appropriate experimentation, analyze and interpret data, and use engineering judgement to draw conclusions	4.33	4.33	4.71	4.30

^ converted from 6-point scale

CONCLUSION

As part of the Wright State model for teaching engineering Math, students had several labs to answer mathematical questions in an engineering context. This paper explored one of these labs, a two-link robot, and compared student responses to a virtual lab with two iterations of hands-on labs.

During all three iterations of this lab, students were able to apply trigonometry concepts to a real-world engineering application. The virtual lab enabled a robust, accessible experience that enabled remote learning environments. The peg-board lab enabled a hands-on lab that introduced students not only to the trigonometry concepts, but also sparked discussions on engineering tolerances and ambiguity. The Arduino-robot lab allowed students to play with a physical 2-link robot. The "flaws" of the Arduino-robot exposed students to common issues that occurred with physical systems and enabled students to troubleshoot these issues. They developed engineering judgement as a group, which led to increased camaraderie among students in these labs.

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