Complementary Usage of Mathematica and I-DEAS in Mechanism Design

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Abstract

All mechanical engineering majors at the Naval Academy are required to take a course in Computer Aided Design during their senior year. The underlying philosophy of the course is to use the computer to solve problems that would be impractical to solve by hand. The vehicle used to illustrate this is the design of four bar mechanisms. During the first part of the course the students write programs using *Mathematica* to perform synthesis, position, velocity, acceleration, and force analyses for the complete range of motion of a four bar mechanism. Using *Mathematica* helps the students develop an understanding of the equations being solved, as well as, develop an appreciation for the progression of the solution from synthesis through force analysis. In the second part of the course the students are introduced to the SDRC *I-DEAS* solid modeling software package. Using *I-DEAS* they build physically realistic models of the mechanisms including animation of the complete range of motion. The position, velocity, acceleration, and force analyses are repeated within *I-DEAS* by means of menu picks. The only part of the design process that *I-DEAS* cannot be used for is the synthesis of the mechanism. The use of *Mathematica* enhances the students understanding of the mechanism design process, while the use of *I-DEAS* gives the students an appreciation for the ease with which physically realistic models can be generated using high end solid modeling packages.

I. Introduction

The past twenty years has seen a rapid advancement in the capability of computer-aided design tools. Commercial software is readily available to assist with all phases of the design process from ideation though synthesis and analysis, detail design and testing to prototype and production. Computer-aided design tools have become an essential part of the modern design and manufacturing environment and engineering curricula has evolved to include instruction in this field. Virtually all engineering schools include instruction in computer-aided design to some degree. Design software has become so powerful that a novice can conduct sophisticated analyses without knowing very much about the details or limitations of the analysis process.

While it is important for engineering schools to educate students about the use of computer-aided design tools, they must also ensure that the students have an understanding of the underlying mathematical models upon which these computer programs are based. It is a continuing challenge to strike a proper balance between teaching the fundamentals in sufficient depth so that the student understands the underlying principles, and teaching the technology which does most of the repetitive calculations automatically, and makes it practical to solve physically realistic problems in a short amount of time.

All Mechanical Engineering students at the Naval Academy are required to take a course in computer-aided design in the fall semester of their senior year. This course uses the topic of mechanism design as a vehicle for introducing the computer as a tool to assist in the design process. The use of a computer allows the students to solve and analyze real-world design problems that would be difficult to solve by hand in a timely manner. The objective of this paper is to illustrate how commercial software packages for mathematical analysis, *Mathematica*, and for solid modeling and design, *SDRC I-DEAS*, are used together in the curriculum to teach the students a balance of the fundamentals of mechanism design and modern design tools that assist in the process.

II. Mechanism Design

The task of mechanism design lends itself to the general design process described in many texts on design[1,2]. These steps are listed below along with some of the more specific activities that relate to mechanism design in particular.

Mechanism Design Process

1. *Recognition of need* - This step identifies the specific problem to be solved and results in a problem statement and a set of specifications that must be met. In the case of a mechanism design, the specific motion or positioning that is required of the mechanism is specified along with other requirements and constraints such as speed of operation, size of the mechanism, etc.
2. *Ideation* - Possible mechanisms types are considered as candidates for the task. Various types of multi-bar linkages or cam-follower systems may be suggested as possible solutions.
3. *Synthesis* - For each particular mechanism type selected as a candidate, the specific lengths of the links or cam profile, the position of ground points and types of joints or followers that result in the desired motion must be determined. This can be done graphically or analytically.
4. *Analysis* - The synthesized mechanism must be analyzed over its full range of motion to determine the path, velocity and acceleration of key locations on the mechanism. A force analysis must also be conducted in order to determine the reactions at the joints and the torque and power required to operate the mechanism. All aspects of this analysis can be performed analytically while some parts of the analysis can also be done graphically.
5. *Selection* - The results from the analysis stage are evaluated to determine how well the specifications are met. Candidate solutions are compared. Usually the synthesis, analysis and evaluation sequence will require several iterations to achieve a satisfactory solution.
6. *Detail Design* - Individual components are designed, materials are selected, stresses are analyzed, and joints and components are sized based on expected forces. This may require additional analysis of the mechanism kinetics if the mechanism geometry changes significantly from what was analyzed in step 4.
8. *Prototype and Testing* - Virtual or physical prototypes are developed and tested to determine whether the design meets all of the specifications.
9. *Production* - The completed design is fabricated for use.

Several commercial software packages are available to assist with almost every phase of the design process outlined above. LINCAGES[3] is an example of a program that will perform analytical synthesis of linkages and will also perform the kinematic and kinetic analysis of the mechanisms. *ADAMS* (Mechanical Dynamics, Inc.) and *Working Model* (Knowledge Revolution) are other examples of commercial software available for performing the kinematic and kinetic analysis of the mechanism. There are a variety of solid modeling packages available for the detail design of the components. Many of the packages will export the geometry from the solid modeller to the analysis package. In this case, some phases of the detail design task are accomplished prior to the analysis task. The point to be made is that there are a range of commercial programs available that will perform many of the tasks without the user having to understand the underlying modeling that is taking place. It is a topic of debate among educators exactly which aspects of the modeling the students should understand.

It is important for the student to have some understanding of the underlying mathematical modeling that is taking place in the synthesis and analysis stages of the process. Without some knowledge of this process, it can be difficult to interpret the results that the commercial packages produce. By understanding the mathematical modeling that is taking place, the student can make a more informed interpretation of the results to decide if they make sense. All too often, students and professionals alike, readily accept the results generated by the computer without casting a critical eye at them. Knowledge of the underlying modeling is also important to understand how to identify and correct problems when the commercial packages generate cryptic error messages and won't solve the problem because of problems with the input or the way in which the mechanism was assembled.

At the U. S. Naval Academy the Computer-Aided Design course is heavily oriented towards team design projects. Teams of two students select a mechanism design project from a list of suggested projects at the beginning of the semester. The lectures are organized to develop the new material the students will need to learn, in a logical manner that parallels the phases of the design process. Ample time is provided once the new material has been presented to allow the students to apply it to their projects. In addition, a few short lab exercises, quizzes and homework assignments are given to reinforce the concepts developed in the lectures.

The first part of the course introduces the kinematic synthesis and analysis of four-bar mechanisms, depicted schematically in Figure 1. Mechanism synthesis is the task of determining the lengths and orientations of the links of the mechanism so that they will achieve some desired motion. An analytical approach from Erdman and Sandor[4] that lends itself well to implementation in a computer program is used in this course. The students learn the underlying loop equations that define the configuration of the mechanism at any point in time. In order to synthesize a mechanism using the analytical approach, the lengths of the physical links are represented by vectors. Figure 2 shows a four-bar mechanism in two successive positions where the links have been replaced by vectors. The unprimed vectors represent the geometry in the initial position and the primed vectors correspond to the orientation in the second position. For a pin-jointed four-bar mechanism, the lengths of the links do not change, only the orientation of the links. The precision positions, points 1 and 2, represent the design constraints. It may further be required that the coupler change orientation by some specified amount, α_1, between points 1 and 2. The loop equation which governs the length and orientation of the vectors on the left-hand side of the mechanism is given by:

$$\overline{Z}_2' + \overline{Z}_5' - \overline{P}_{12} - \overline{Z}_5 - \overline{Z}_2 = 0$$

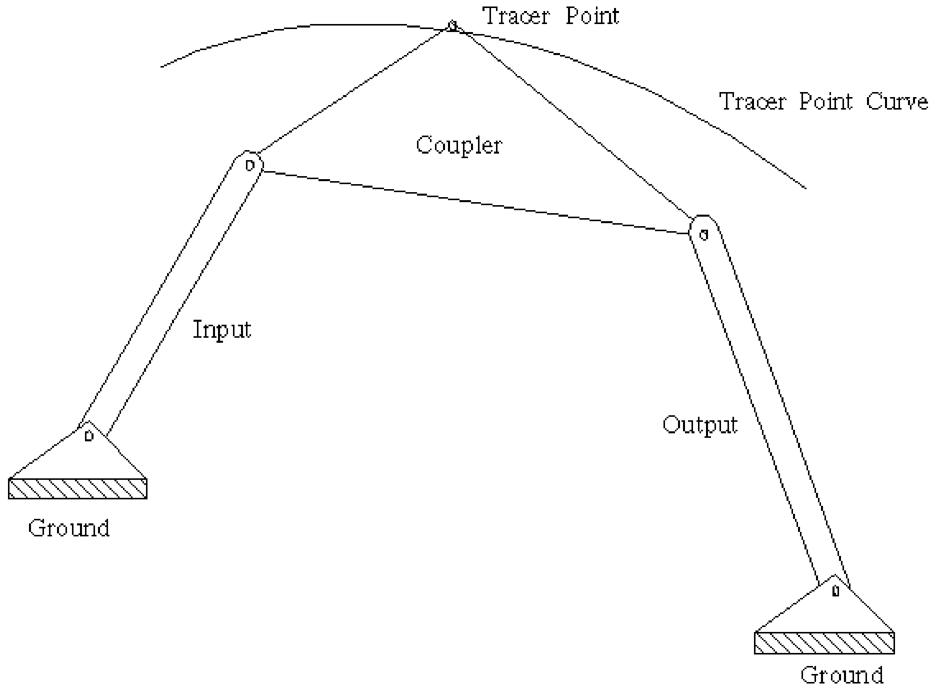

Figure 1 - Schematic representation of a pin-jointed, four-bar mechanism.

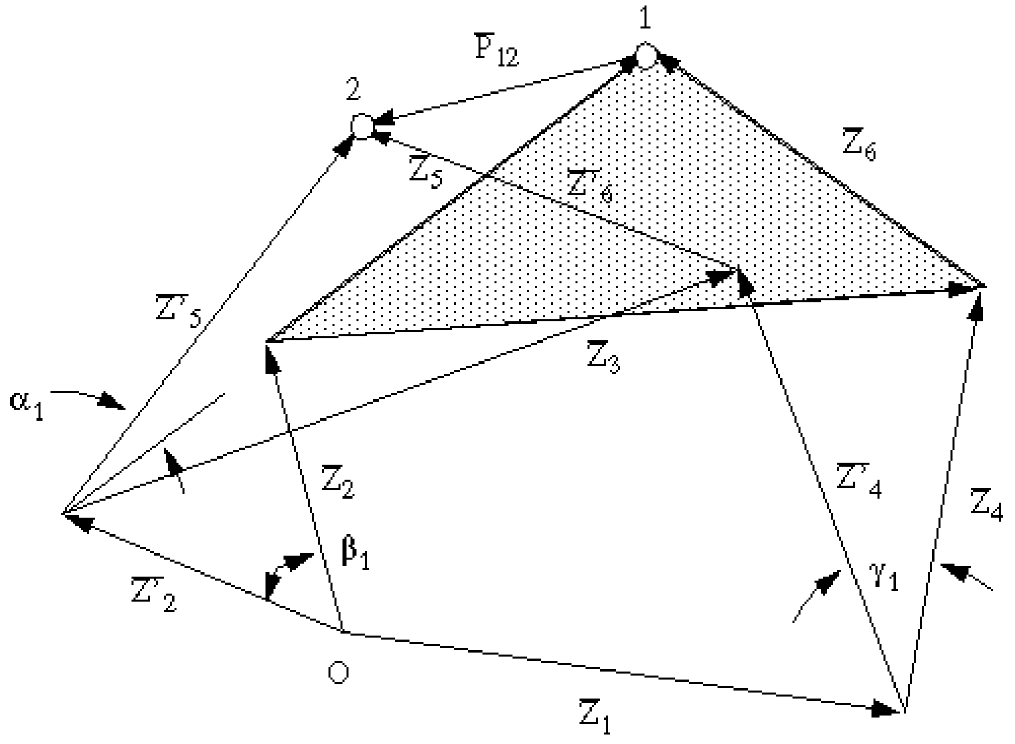

Figure 2 - Vector representation of a four-bar mechanism in two successive positions.

where $\overline{P}_{12}$ is the vector from position 1 to position 2, α_1 is the change in orientation of the coupler and β_1 is the rotation of the input link, $\overline{Z}_2$. The vector equation can be written as two scalar equations by separating the x and y-components of the vectors:

$$Z_{2x}(\cos\beta_1 - 1) - Z_{2y}\sin\beta_1 + Z_{5x}(\cos\alpha_1 - 1) - Z_{5y}\sin\alpha_1 = P_{12x}$$

$$Z_{2x}\sin\beta_1 + Z_{2y}(\cos\beta_1 - 1) + Z_{5x}\sin\alpha_1 + Z_{5y}(\cos\alpha_1 - 1) = P_{12y}$$

For the general problem of motion generation in which the position and orientation of a point on the coupler are specified (P_{12x}, P_{12y} and α_1 are given) there are five unknowns (Z_{2x}, Z_{2y}, Z_{5x}, Z_{5y} and β_1) but only two equations. A similar set of equations can be written for the right-hand side of the mechanism resulting in five more unknowns. An additional set of equations can be written for each additional precision position that is specified. In general, there are more unknowns in the problem than there are equations and the student must reduce the degrees of freedom by specifying values for some of the unknowns. A summary of the number of equations, solution variables and free choices that must be made for the problem of motion generation with a four-bar mechanism is listed in Table 1.

Table 1: Free Choices vs. Precision Positions for Motion Generation

Precision Position	Displacement Vectors	Loop Equations	Scalar Equations	Unknown Variables	Free Choices
2	1	2	4	10	6
3	2	4	8	12	4
4	3	6	12	14	2
5	4	8	16	16	0

Once the system of equations has been fully constrained, it can be solved to determine the remaining unknowns. The system of equations can be non-linear and may require an iterative root-finding algorithm. After the equations have been solved, the resulting mechanism can be plotted to show the configuration at each of the precision positions. Since there are usually several free choices made for some of the unknowns, there are an infinite number of possible solutions. It is up to the designer to decide which values for the free choices will lead to an optimum design. The student quickly learns the value of the computer with graphical output when he is faced with the task of repeatedly solving a system of equations while varying a few parameters.

The general purpose, mathematical analysis program, *Mathematica*, is used as the tool for solving the equations and plotting the resulting mechanisms. The students learn how to set up the governing system of equations in a general manner so that it is easy to iterate on various proposed solutions to select viable candidate mechanisms. *Mathematica* has powerful root-finding algorithms such as `Solve` and `FindRoot` that make it easy to solve complex systems of equations. The students must also write the equations that specify the position and orientation of each of the links for the mechanism so that the built-in plotting routines can be used to generate pictures of the resulting candidate mechanism.

Candidate mechanisms that have been synthesized must be further analyzed to determine the motion of the mechanism over its full range of operation. This analysis includes determining the position, velocity and acceleration of each link in the mechanism for many intermediate positions. This is accomplished by having the students write the general loop equations that govern the position of the mechanism at any point in time. Consider the pin-jointed, four-bar mechanism shown in Figure 3. The vector loop equation that governs the mechanism configuration is:

$$\overline{Z}_2 + \overline{Z}_3 - \overline{Z}_4 - \overline{Z}_1 = 0$$

This vector equation can be expressed as two scalar equations as follows:

$$Z_2 \cos\theta_2 + Z_3 \cos\theta_3 - Z_4 \cos\theta_4 - Z_1 \cos\theta_1 = 0$$

$$Z_2 \sin\theta_2 + Z_3 \sin\theta_3 - Z_4 \sin\theta_4 - Z_1 \sin\theta_1 = 0$$

For a given orientation of the input link, θ_2, the orientation of the other two links in the mechanism can be determined and the position of all points on the mechanism can also be computed. The position equations are solved at many intermediate positions and the mechanism can be plotted at each step. The resulting sequence of images can be easily animated within *Mathematica* to demonstrate the full range of motion of the mechanism.

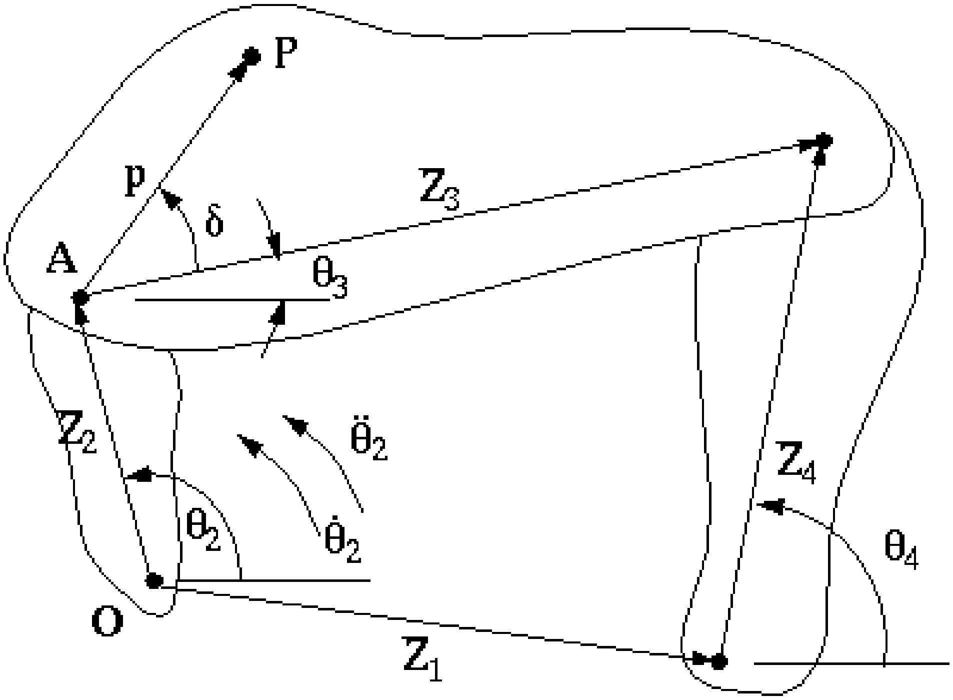

Figure 3 - Vector representation of four-bar mechanism for position analysis.

As an example, a mechanism was synthesized to move a keyboard rack from a storage position under a desktop to a deployed position in front of the desk. In addition, the mechanism must incline the keyboard 10 degrees when it is in the deployed position. A *Mathematica* program was written to solve the loop equations for synthesizing the mechanism and the motion of the mechanism was analyzed from the stored to the deployed position. A sequence of frames from the *Mathematica* animation are shown in Figure 4.

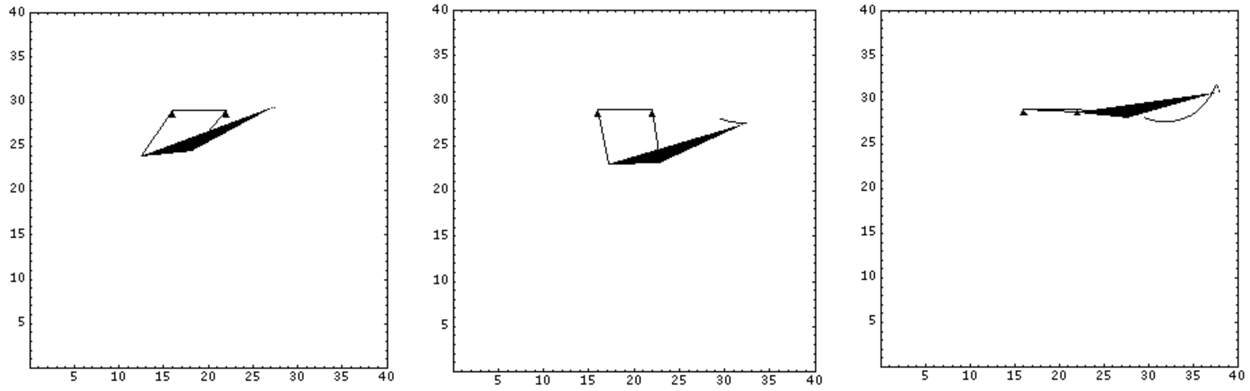

Figure 4 - Selected frames from a *Mathematica* animation showing the motion of the mechanism.

The velocity and acceleration characteristics of the mechanism can be determined from equations that are developed by differentiating the position equations with respect to time. It is not difficult to differentiate the position equations by hand, but *Mathematica* can be used to perform the differentiation symbolically and generate the necessary equations for determining the velocity and acceleration characteristics of the mechanism. The position, velocity, and acceleration results are used to solve the equilibrium equations for calculating the forces and torques developed within the mechanism. Any parameter such as the force at a particular joint or the torque required to drive the mechanism can be plotted as a function of time or the position of the input link.

Each student is encouraged to develop a *Mathematica* worksheet that includes all of the governing equations for the synthesis and analysis of a four-bar mechanism. If the student develops the equations symbolically, the worksheet becomes a general-purpose mechanism synthesis and analysis program for four-bar mechanisms.

By developing the *Mathematica* worksheets, the students learn about developing systems of equations, solving non-linear equations and some rudimentary computer graphics. Even though built-in subroutines are employed for many of the operations, the student must understand the essential elements necessary for developing a properly constrained system of equations in order to get the worksheet to solve. The graphics for animating the results are constructed from simple graphics primitives like lines and points so the student gains an appreciation for the computations that must be performed in a commercial graphics program.

The students are tested on their mastery of mechansim synthesis and analysis through the use of on-line quizzes. Each student must take a quiz on each of the following topics: mechanism synthesis, position analysis, velocity analysis, acceleration analysis and force analysis. The *Mathematica* programs that they develop to answer the quizzes can then be used for their design projects.

Once the fundamentals of mechanism design have been presented, the students are introduced to solid modeling with the *I-DEAS* computer-aided design program. This program is an integrated design and analysis package that includes solid modeling of parts, assemblies and mechanisms, drafting, finite element analysis, manufacturing and other features as well. Students use the *I-DEAS Master Series Student Guide*[5] as a workbook that guides them through the basics of part modeling, building an assembly, and performing a mechanism analysis.

I-DEAS is used for the bulk of the design and analysis of the four-bar mechanism project. Since *I-DEAS* has no facility for the mechanism synthesis, *Mathematica* is used to synthesize the mechanism parameters initially. With the link lengths in hand from the synthesis, the student can proceed to develop solid models of each piece of the mechanism in I-DEAS, build up the assembly of parts, add joints to create a mechanism, specify external forces and the required motion, and solve for the resulting mechanism motion and the joint forces. The mechanism operation can be animated within *I-DEAS* to verify that the mechanism meets the design specifications. Engineering drawings are easily created for each of the parts and a final design report is prepared.

A completed mechanism for the keyboard tray is shown in Figure 5. A sequence of images showing the successive orientations of the mechanism is shown on Figure 6. The position, velocity and acceleration of a point on the front edge of the keyboard tray were analyzed in *I-DEAS* and *Mathematica* and the results are presented in Figures 7-9. These results demonstrate that the mechanism produces the desired motion and that the results from both *I-DEAS* and *Mathematica* are in agreement.

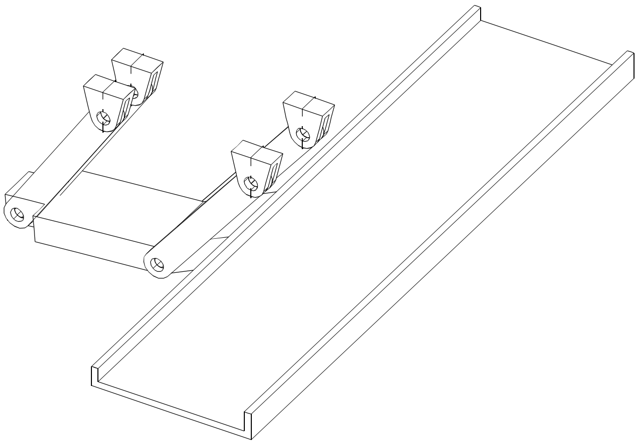

Figure 5 - Solid model of keyboard tray mechanism generated in *I-DEAS*.

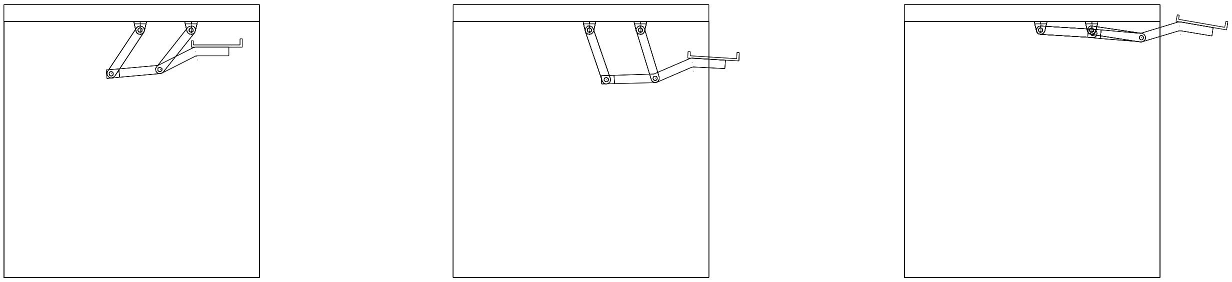

Figure 6 - Sequence of images from the animation of the mechanism operation in *I-DEAS*.

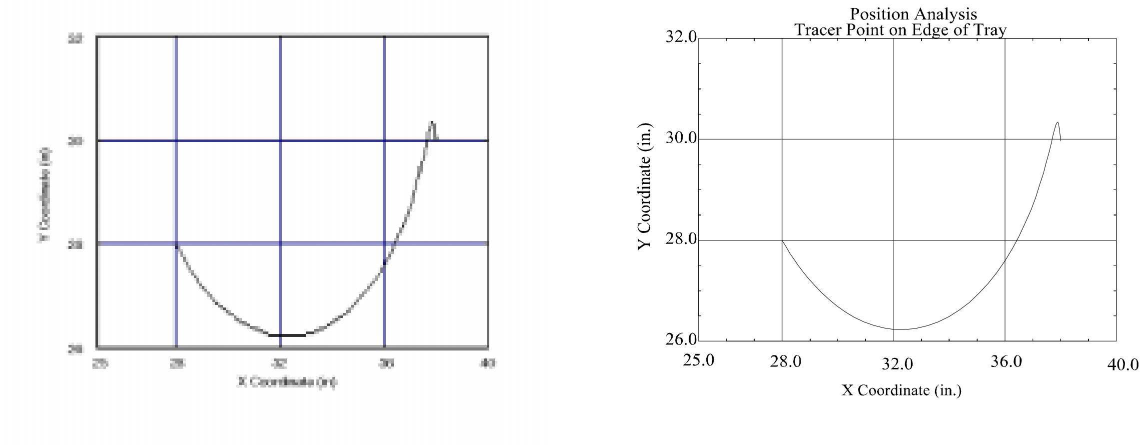

Figure 7 - Comparision of the position of a point on the front edge of the tray table determined using *Mathematica* (left) and *I-DEAS* (right).

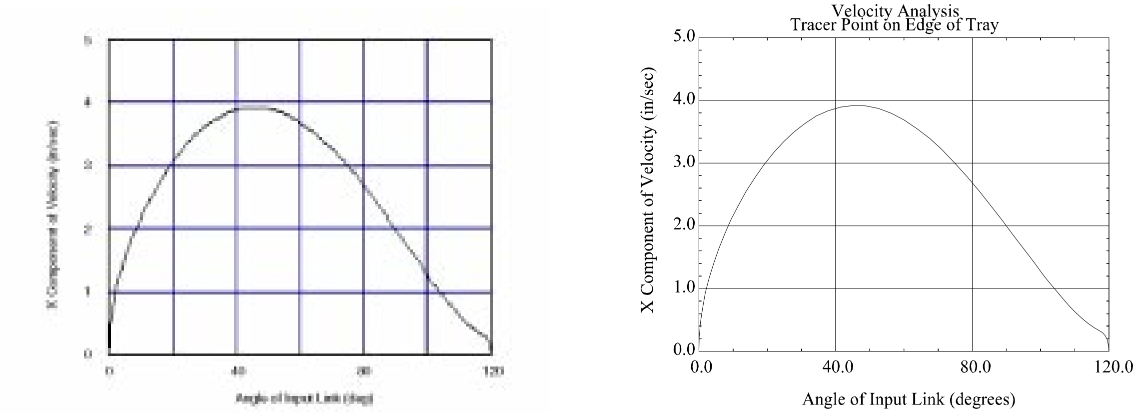

Figure 8 - Comparison of *x*-component of velocity of a point on the front edge of the tray table determined using *Mathematica* (left) and *I-DEAS* (right).

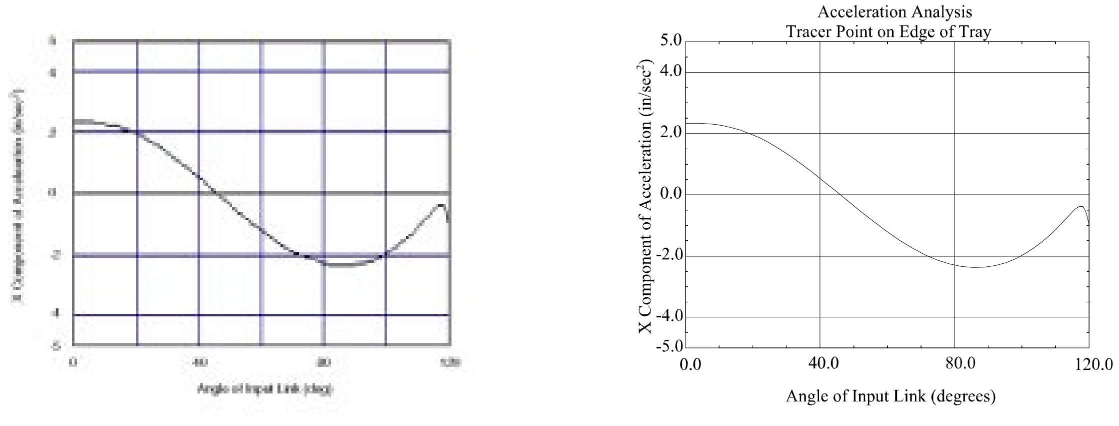

Figure 9 - Comparison of *x*-component of acceleration of a point on the front edge of the tray table determined using (a) *Mathematica* and (b) *I-DEAS.*

III. Summary

A computer-aided design course which employs *Mathematica* and *I-DEAS* to assist in mechanism design was described. The complementary usage of a mathematics analysis software program and a high-end solid modeling package gives students an understanding of the analytical procedures and algorithms used in canned design and analysis packages. *Mathematica* reinforces the understanding of the governing equations for synthesis, position, velocity, acceleration, and force analysis. This is achieved by having the students generate and analyze mechanisms using these equations. In addition, the usage of the mathematics analysis software helps the students develop an appreciation for the iterative nature of the solution as it progresses from synthesis through force analysis. The use of *I-DEAS* enhances their experience by allowing them to develop visually and physically realistic models of the mechanisms. It also demonstrates the power/risk associated with high-end software packages where the user can easily generate a set of results, but may not have a complete understanding of how the results were obtained.

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