Computational Electromagnetics in Electrical Engineering at NDSU

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Introduction

A course discussing Computational Electromagnetics has been taught several times by Electrical Engineering faculty at North Dakota State University. The course is open to both seniors and graduate students and examines various topics related to electromagnetic theory, with particular emphasis given to computational electromagnetics. Students with a wide range of interests have found the course helpful, including those interested in electronics and high-speed circuit design, bio-medical engineering, antenna theory, microwave engineering and electromagnetic interference/compatibility. The course typically begins with a review of fundamental concepts in electromagnetics, and then various analytical solution methods are examined. The majority of the semester is then devoted to examining various computational methods, including both frequency- and time-domain methods. A major revision of the course is presently underway, which will include expansion into a two-semester sequence. Selected details of the current course offering are described below, and plans for the new two-semester course sequence are briefly described.

NDSU ECE ENGR 780

Course Details

The present offering is a 3-credit class, offered alternate fall semesters. The course format has been traditional lectures, supplemented with in-class or in-lab computer demonstrations. The main text has been *Numerical Techniques in Electromagnetics*, (2nd Ed.), by M.N.O. Sadiku, (CRC Press, 2001), but material is taken from a variety of sources. There are typically 5 to 7 extensive homework assignments (which often include writing computer code to solve a given problem), one “paper project” and one “computer project”. In the “paper project” students are asked to select, read, and write a report on a recent journal article dealing with computational electromagnetics. These are typically selected from a variety of *IEEE Transactions* (e.g. *IEEE Transactions on Antennas and Propagation* or *IEEE Transactions on Electromagnetic Compatibility*). The intent of this assignment is twofold: (i) if they have not yet done a literature search for their research, this assignment helps them get familiar with how to search for and locate journal articles of interest, and (ii) as the students survey the recent literature, it often peeks their interest to see the wide variety of interesting work, and also tends to bolster their confidence that they can understand much of what the articles are discussing. In the “computer project” students have been given the option of creating their own code to solve a stated problem or to use existing software to investigate a more intricate problem of their own choosing. In the first case, students will typically base the code on the moment method, the finite element method, or the finite difference time domain methods. In the second option, they might choose to do things like investigate the electromagnetic performance of an electrical circuit (such as coupling, crosstalk, "ringing", filter performance, etc.); or to examine the performance of a given antenna; or to investigate what happens when an electromagnetic wave impinges on a given material (perhaps a human body), with typical "outputs" being things like the electric and/or magnetic fields, voltages, current distribution, field patterns, etc. The course typically involves two semester exams (often one in-class and one take-home) and a comprehensive take-home final.

Course Content

As it presently stands, the course examines both analytical and computational methods commonly used in electromagnetics. Although analytical methods are included, the primary focus entails the computational methods. Nine major aspects or topics are included in the course. Each aspect is described briefly.

A.) Review of Maxwell's Equations

Students who take this class may have taken their undergraduate coursework in electromagnetics a year or two earlier. As such, we begin the course with a review of the four fundamental equations (e.g., Maxwell's equations) and a few constitutive relations. Maxwell's four equations are:

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \tag{1}$$

$$\nabla \bullet \overline{D} = \rho_V \tag{2}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \tag{3}$$

$$\nabla \bullet \overline{B} = 0 \tag{4}$$

where $\overline{E}$ is the electric field intensity (in V/m), $\overline{B}$ is the magnetic flux density (in T), $\overline{D}$ is the electric flux density (in C/m^2), ρ_V is the volume charge density (in C/m^3), $\overline{H}$ is the magnetic field intensity (in A/m) and $\overline{J}$ is the volume current density, which has units of A/m^2. Some of these terms are related to each other via the constitutive relations (assuming homogeneous, linear, isotropic materials) $\overline{D} = \varepsilon\overline{E}$, $\overline{B} = \mu\overline{H}$, and $\overline{J} = \sigma\overline{E}$ where ε, μ and σ are the permittivity, permeability and conductivity of the region, respectively. Another equation commonly referred to is the continuity equation,

$$\nabla \bullet \overline{J} = -\frac{\partial \rho_V}{\partial t}. \tag{5}$$

When reviewing these equations stress is placed on the physical as well as mathematical meaning of the equations. For instance, we note that each of the equations contains either a divergence operation (e.g., $\nabla \bullet \overline{D}$) or a curl operation (e.g., $\nabla \times \overline{E}$). As mentioned in an earlier paper (Nelson, 2006) the divergence can be thought of as a "flow source/sink finder". As such, if

- $\nabla \bullet \overline{D} > 0$ there is a net flux leaving the volume. This implies that the volume contained a source of the flux.
- $\nabla \bullet \overline{D} < 0$ there is a net flux entering the volume. This implies that the volume contained a sink of the flux.
- $\nabla \bullet \overline{D} = 0$ there is no net flux entering or leaving the volume. This implies that the volume did not contain a source or sink of the flux - rather, all of the flux lines entering the volume continued to pass through the volume.

Similarly, the curl tells one if there are any "swirling" or "vortex" fields, and also provides the source of those fields. As such, it is a "vortex source" finder. Keeping these two physical interpretations in mind, we see that equations (1)-(5) tell us that

- Electric fields have two sources: charge and time-varying magnetic fields.
- Charge is a "flow source" of electric fields – i.e., charge creates electric fields that start and stop at specific points.
- Time-varying magnetic fields are a "vortex source" of electric fields – i.e., they create electric fields that "swirl", or form closed paths.
- Magnetic fields have no flow sources. As such, they always form closed paths.
- The vortex sources of magnetic fields are current and time-varying electric fields.

We complete this review by also discussing the boundary conditions on tangential and normal components of the electric and magnetic field, pointing out when various terms are commonly included or neglected, as well as briefly discussing the general concept of Dirichlet, Neumann, and mixed boundary conditions. When we put these concepts together we can gain understanding into how electromagnetic waves propagate. Remembering these simple facts is often easier and more meaningful to students than memorizing the exact form of Maxwell's equations. The beauty of "seeing" what the mathematics means is that students seem to have greater capacity to visualize what the electric and magnetic fields might look like in a given situation.

B.) Analytical methods

In the present version of this course the focus of the course is computational methods used in electromagnetics, but several analytical methods are examined. Since the students are familiar with some concepts from their introductory course on electromagnetics, these concepts are reviewed near the beginning of the course. These include determining the static electric field from a known charge distribution using Coulomb's Law, determining the static magnetic field using the Biot-Savart Law, and solving Poisson's (or Laplace's)

Equation for the potential distribution. Following the review of static field concepts, solution of the wave equation is developed, from which the concept of modes, cutoff wave number, etc. are developed, which leads nicely to solving the wave equation using the finite difference method. Additional analytical methods are introduced throughout the class in a manner that coincides with the particular topic.

C.) Finite Difference method

Basic concepts of the finite difference (FDM) method are developed within the context of solving the wave equation. Various differencing methods (forward, backward, central) are examined, as is the categorization of various types of partial differential equations (i.e., elliptic, parabolic and hyperbolic). The particular example of solving the wave equation using an analytical solution and the finite difference method is then carried out in class, and provides a natural way to introduce the concept of stability, importance of step size, etc. This is followed by a discussion of various iterative methods (Jacobi, Gauss-Seidel, etc.)

D.) Finite Difference Time Domain method

The basic finite difference methods are then applied to the finite-difference time-domain (FDTD) method, which is often used to solve the three-dimensional, time-varying set of Maxwell's equations. The basic Yee cell (Yee, 1966) is examined and used to solve examples with various types of sources. The stability issue is addressed again, which leads to the concept of numerical dispersion. To apply the FDTD method to open regions one needs to apply absorbing boundary conditions to avoid numerical errors. The Mur (first- and second-order) boundary conditions are examined, and the PML (perfectly matched layer) absorbing boundary is discussed. This discussion naturally leads to discussion of another analytical method – namely, the solution of the problem of wave reflection at a two-layer interface. This is carried out for the general case, and provides an opportunity to discuss reflection and transmission coefficients and surface waves.

E.) Review of vector space concepts

A transition is made at this point in the course to shift from finite-difference methods to other techniques used in computational electromagnetics. Since several of the other methods rely heavily on vector space concepts, a review is provided of the concepts of inner products, norms, orthogonality, etc.

F.) Variational methods

Vector space concepts are expanded upon by introducing the concept of functionals, and how to develop a functional from a given partial differential equation. Application of the variational method is examined in a general sense for direct methods (i.e., Rayleigh-Ritz) and indirect methods (i.e., weighted-residual). The general concepts discussed at this point are then woven in with the moment method and the finite element method.

G.) Green's functions and the Method of moments

The application of the method of weighted residuals to the solution of electromagnetics problems is often referred to in electromagnetics literature as the moment method (Harrington, 1968). Detailed application of this method is applied to three example problems: (1) determining the static charge distribution for a conducting wire immersed in air and held at a constant potential, (2) determination of the unknown charge distribution for a multi-layer problem consisting of a variety of dielectric regions and conducting sections, and (3) determining the current distribution on a wire antenna. Various basis and weighting functions are discussed. Use of these three problems provides an excellent way of pointing out many of the key concepts needed to solve both static and time-varying problems in electromagnetics. For instance, Green's functions have wide application in electromagnetics, and are discussed in the context of these problems. In addition, the Equivalence and Uniqueness Theorems are discussed, as is the use of auxiliary potentials to solve for the electric and magnetic fields.

H.) Finite Element method

The concept of the functional developed earlier is then applied to the finite element method (FEM). The development of the functional, the discretization of the solution region, equation formation for each element, assembly of the equations from all elements, and the system solution are discussed. Although time limitations restrict the majority of coverage to the solution of static problems, time-varying problems and the use of absorbing boundary conditions is briefly discussed.

I.) Additional methods

As time allows, brief discussion is included of the basic ideas central to some of the methods commonly used in computational electromagnetics. For example, the Partial Element Equivalent Circuit (PEEC) method (Ruehli, 1974) is a relatively new method that formulates a given electromagnetic problem in terms of equivalent circuits. A brief examination of this method is sometimes included, which focuses on the how the wave equation can be viewed in terms of equivalent circuits.

Plans for New Two-Semester Course Sequence

As mentioned, the present offering is a 3-credit class. Although several analytical and computational methods are examined, the 3-credit time limitation poses a severe hindrance to the depth and breadth of the coverage of various methods. To address that, plans are in place to develop a two-semester course which includes greater breadth and depth of both analytical and computational methods. Although it is common to have one or more graduate courses devoted theoretical electromagnetics and one or more courses addressing computational aspects, our plan is to weave these two aspects into both courses each semester. We do this so that students have the opportunity to examine both theoretical and computational issues. We think this is more likely using the "weaving" approach for two reasons: (1) If separate courses are offered, it is anticipated that the enrollment for the "computational" course will likely exceed that of the "theoretical" course since many students are hesitant to dive deeply into theoretical aspects. (2) We hope to fashion the content order so that students can take the second course even if they have not taken the first course. Although we hope that many students will take both courses, time conflicts may prevent that.

In terms of the "big picture", we would like to describe as general an electromagnetics problem as possible, and then draw our "boundaries" on what we are doing, and show where the "special cases" come from, etc. For instance, some of the aspects that specify the general problem may include

- The Source

  In describing the source of the field, one of the first aspects is whether the source of the field is in the vicinity of the problem being addressed, or is it far away from the source. If it is far away, then the problem can typically be solved by examining a source-free region, and assuming that any incident field can be represented by a plane wave. If the source is close to the region under investigation, then one needs to ask if the source can be represented in a steady-state fashion (i.e., sinusoidal excitation) or if a transient response is needed. In addition to the type of signal (and frequency) being used, we may need to examine the shape, size and location of the source.

- The Material

  In addition to the source attributes, one needs to know the material characteristics of the region under investigation. For instance, we need to know the size, shape, and material parameters (ε, μ and σ). We need to know if the region can be approximated as linear or not, whether the material parameters are scalars (i.e., if the material is isotropic) or not, whether the materials are functions of position (i.e., if the material is homogeneous), and if not, whether we can assume the region can be divided into piecewise homogeneous regions. We also need to know if the region is bounded (i.e., if it is a closed region) or not (i.e., if it is an open region).

- The Desired Analysis

  Closely related to the source attributes, we also need to know what type of analysis is desired. For instance, is the time-domain or frequency-domain solution desired? Is a microscopic analysis needed or is it sufficient to model the problem on the macroscopic level? Are any objects stationary or can they be moving? What is it that is needed – i.e., the electric and magnetic fields, the charges and currents and voltages, forces?

Painting such a "general picture" may help the student understand why we examine various methods (both computational and analytical) and how the use of methods fits into the "big picture". For example, typical discussion of plane waves assumes that the source of the field is far away, and that the field is propagating in a source-free region. Similar assumptions are often employed when examining the modal fields in waveguides. In a similar fashion, most of the computational methods have particular strengths and weaknesses which often relate to how well the method can be used for particular types of problems. For example, finite difference and finite element methods are extremely well suited for problems involving closed boundaries, while the moment method is suited extremely well for open problems. Although both methods can be used for both types of problems, pointing these aspects out to the students right away may help put the rest of the material into context, and help them understand where various assumptions affect the analysis. We also hope to help students see where the material discussed in other courses fits into the bigger picture For instance, where does the material in NDSU ECE 455/655 ("Designing for Electromagnetic Compatibility"), 751 ("Microwave Engineering") or 755 ("Antenna Theory and Design") fit into this material? In addition, we may be able to illustrate ties to other disciplines – such as optics, thermodynamics, mechanics and fluid dynamics.

In terms of course organization, the goal will be to use specific problems (i.e., "canonical problems") as springboards to illustrate both the analytical and computational methods. Although this approach may not always work, we think it has the potential of providing a natural link between the analytical and computational methods. For example, one of the canonical problems will be determining the charge and current distribution on a conducting wire. After a review of Maxwell's equations and boundary conditions, examining this problem provides a natural introduction to the equivalence theorem, Green's functions, vector space concepts and the moment method, time- and frequency-domain concepts, and the finite difference method. We envision beginning with determining the static charge distribution on a conducting wire in free space that is at a constant potential. In solving this problem with the moment method (e.g. method of weighted residuals), we will point out that the integral contains a Green's function and use this as an introduction to discuss Green's functions. To expand the unknown charge distribution in terms of basis functions, we will introduce the concept of the inner product and norm. Developing and solving a system of equations requires examination of weighting functions, orthogonality and similar vector space concepts as well as a review of the concepts from linear algebra that are needed to solve the system of equations. Once this problem is solved, a natural extension is to that of the time-varying case and determining the current and charge distribution on a simple antenna. This provides insight into how to derive an integral equation using vector potentials and appropriate boundary conditions, as well as additional insight into Green's functions and the equivalence theorem. The solution will first be carried out in the frequency domain. Then, a time-domain solution will be pursued, using the time-domain moment method. Development of this solution requires the use of finite difference concepts – thus providing a natural lead to the finite-difference method, as well as iterative solutions and stability. We also plan to use this opportunity to review Fourier Transform concepts, so that students are equipped to convert between the time and frequency domains.

Although the syllabi for the new two-course sequence is not complete as this paper is being written, initial plans are to include the moment method (e.g., method of weighted residuals), the finite difference method and the finite difference time-domain method in one of the semesters, and the finite element method (and other variational techniques), the partial element, equivalent circuit (PEEC) method and the transmission line method (TLM) in the other semester. As time allows, additional computational methods will be included. In terms of analytical methods, we plan to introduce key theorems/concepts including image theory, duality, reciprocity, the equivalence theorem, and the induction theorem. We hope to examine the appropriate equations in both integral and differential form in both time and frequency domains, and examine the solution of Laplace's/Poisson's equations the wave equation (scalar and vector) in the three major coordinate systems. This will require a review (or introduction) to appropriate mathematical concepts such as Bessel and Hankel functions, Legendre polynomials, etc. and will also employ the use of various potential solutions (e.g., scalar potentials, vector magnetic and electric potentials, Hertz potentials, etc.)

Conclusion

A course discussing computational electromagnetics is taught at North Dakota State University and is examined in this paper. Course enrollment has included seniors and graduate students, and the content includes various topics related to electromagnetic theory. In the past, particular emphasis has been given to computational electromagnetics, and students with a wide range of interests have found the course helpful. A major revision of the course is presently underway which will expand the content into a two-semester sequence. The two-semester sequence will provide greater breadth and depth of both analytical and computational aspects of electromagnetic fields, and will be presented in a blended fashion by weaving analytical and computational aspects throughout the course. The concepts and methodology used in the course are discussed, as well as course content for both the "old" and "new" versions. Although specifically addressed to students interested in electromagnetic fields, many of the concepts used in the course are used in a variety of engineering disciplines.

References

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