Session 3226

Data Sampling Techniques for Fourier Analysis

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Abstract

Fourier analysis methods and data sampling techniques are introduced in two laboratory courses in the Mechanical Engineering Technology curriculum. Data acquisition with personal computer hardware permits high speed sampling and analysis of large quantities of data obtained from various transducers, strain gages, and accelerometers. Data sampling methodology determines the efficacy of the results. Sampling frequency and the number of data points acquired strongly influence the resolution of frequencies and their amplitudes in the spectra calculated for a signal. The use of simple laboratory structures for which experimental and analytical frequencies are readily obtained enhances the understanding of vibrations, data sampling, and interpretation of Fourier analysis results. Since structural vibrations may produce closely spaced harmonics, an understanding of the presented method is critical for a priori determination of frequency resolution.

Introduction

Much can be learned about the characteristics of a vibrating structure by experimental determination of dynamic strains or kinematics. Often, extremely high loads can exist due to impact loading or excitation of a structure near one of its resonant frequencies. High speed data acquisition with personal computer hardware permits sampling and analysis of large quantities of data from strain gages or accelerometers for comparison to an appropriate model or verification of a finite element analysis. Fourier analysis provides a powerful tool for obtaining both a qualitative and quantitative understanding the dynamic behavior of a structure.

In the Mechanical Engineering Technology program at the University of Pittsburgh at Johnstown (UPJ), the techniques and application of data acquisition and analysis are taught in a sequence of courses intended to produce a student capable of acquiring and manipulating appropriate, useable, quantities of high-speed data from a transducer. There are pitfalls in data taking and interpretation that can be identified, and the methodology can be tailored to provide optimum results. In the course sequence, the basic techniques of Fourier analysis are introduced, and a methodology for data acquisition suited to optimizing the usefulness of the resulting frequency spectrum is presented. Classroom examples from the authors' laboratory and professional experiences illustrate the methods, problems, and outcomes.

Background

Data acquisition of experimental measurements results in a set of sampled data at regularly spaced times as illustrated in Figure 1. The continuous analog signal x(t) from a transducer is fed through an analog to digital converter to give discrete values of $x_i(t)$ at discrete times $t=i\Delta$, where Δ is the sampling interval (generally dependent on the data acquisition hardware), and $i = 1,2,...,N$ where N is the total number of samples. Analysis can now be conducted on the sampled data to determine its characteristics. In many mechanical and structural applications, the primary interest is the frequency content of the signal and the magnitude and location of the system resonances. To accomplish this, the signal is converted from the time domain to the frequency domain.

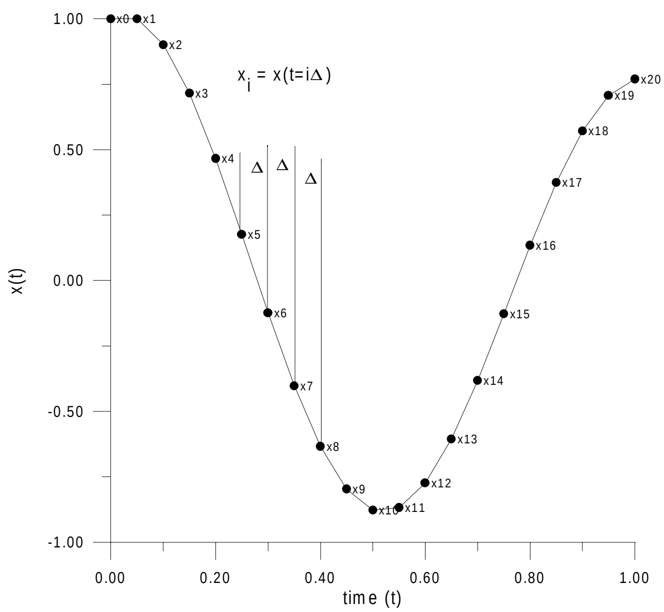

Figure 1: Sampling a Continuous Function of Time at Regular Intervals

An estimate of the frequency spectra from the measured data can be obtained using a discrete Fourier series approach. The advantage of this approach is that it is conceptually simple, yet yields usable results. The classical Fourier series of a periodic functions gives the approximation of the continuous function *x(t)* in terms of an infinite series given by

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left[a_k \cos\left(\frac{2\pi k t}{T}\right) + b_k \sin\left(\frac{2\pi k t}{T}\right)\right]$$

where $k = 1$ to ∞ are the number of terms in the series and T is the period. The coefficients a_k and b_k, are found by formal integration if an analytic function for the signal is known. For a sampled data signal, *x(t)* is known only by its discrete time series approximation, and the period of the signal is not known. To calculate the coefficients for a sampled data set, the total sample time is assumed to be the period so that $T=N\Delta$, and the integrals are replaced by finite sums. This means that integration is replaced by a summation over the sample. Using the principle of orthogonality, the coefficients can be determined by summations over the data in the form

$$a_k = \frac{2}{N}\sum_{i=1}^{N} x_i \cos\left(\frac{2\pi k i}{N}\right) \quad ,k \ge 0 \qquad\qquad b_k = \frac{2}{N}\sum_{i=1}^{N} x_i \sin\left(\frac{2\pi k i}{N}\right) \quad ,k \ge 1$$

$$\text{where} \quad c_k = \sqrt{a_k^2 + b_k^2} \qquad\qquad \text{and} \quad \phi_k = \tan^{-1}\left(\frac{b_k}{a_k}\right)$$

The coefficients a_k and b_k are the components of the signal in a frequency bin centered at the frequency $f_{cent} = k/T$. In a typical presentation, the magnitude c_k and phase ϕ_k are plotted as a function of frequency to provide the spectra of the given signal. For any sampling interval, there is a special frequency called the Nyquist critical frequency which occurs at $k=N/2$. The Nyquist frequency determines the maximum frequency component that can be extracted from a given signal, because it takes a minimum of two samples to define a cycle. If frequency information is presented above the Nyquist frequency, aliasing of the signal will occur.

Practical determination of the frequency spectra from a signal is not accomplished with the discrete Fourier series calculations defined above. It is instructive to teach the method above and require students to write a program to generate the coefficients and spectral plots for small data sets with limited frequency range. Alternately, a spreadsheet technique communicates discrete transform concepts effectively [1]. However, it is imperative to teach that there are more efficient methods called Fast Fourier Transforms or FFT's. The detail of the FFT and its implementation in software are not emphasized, but a literature search will locate various published FFT computer programs which have been assigned and effectively demonstrate the computational efficiency of the FFT over the discrete transform [2,3]. The FFT is implemented in a wide variety of analysis, plotting, and spreadsheet programs. Versions of the FFT algorithm most typically used by the Technology students at UPJ are Mathcad [4], MATLAB [5], or from a spreadsheet such as Microsoft Excel.

Data Sampling Primer

There are some data sampling techniques that will optimize the usefulness of the spectrum resulting from an FFT of a set of sampled data. The critical factors that determine the range, frequency resolution, and accuracy of an FFT are the number of data points taken and the speed at which the data is taken. There are also different applications of the Fourier analysis techniques that may determine how it is used. Some applications in vibration analysis may only require that the natural frequencies of a structure are known, yet some applications, such as machinery monitoring, require accurate values of the signal amplitude for selected frequencies for trend following. The selection of sampling parameters greatly influences the accuracy and repeatability of these measurements. The objectives of a particular sample must be clearly understood before the sampling methodology is selected. Structural vibrations applications are at extremely low frequency when compared to electronics, and encompass a large part of the mechanical engineering practice.

A convenient guide for data acquisition to be used for Fourier analysis is shown in Figure 2. This graph shows the relationship between the data sampling rate, the size of the data sample, the frequency resolution, and the Nyquist cut-off frequency. It is important to note that all FFT algorithms require a number of data points equal to 2^n, thus giving samples sizes of 512, 1024, 2048, 4096, etc. The graph dispels the notion that faster data acquisition is always better. If no change is made in sample size, a faster sampling rate increases the size of the frequency bins and reduces the discrimination of frequency content. Larger sample sizes are always better, but some data acquisition or spectral analysis hardware is sample size limited to a 1024 or 4096 point sample. Typically, for most mechanical systems with relatively low resonant frequencies, the Nyquist limit is not a severe constraint.

Conventional wisdom in the form of a rule-of-thumb used by many numerical and experimental analysts is to record a minimum of ten data points per cycle of the desired signal. Selection of sampling frequency ten times the highest expected frequency contained in the object signal results. Application of the guideline yields the same results as the "faster is always better" philosophy: either huge sample sizes become necessary or the frequency resolution is severely compromised. Unquestionably, a rule-of-thumb based sampling rate is required if an accurate curve-fit of the transient history of the signal is desired. However, better frequency resolution and manageable data files will result if the sample is acquired at a significantly reduced sampling frequency.

Frequency Resolution: The FFT finds coefficients of the harmonic signal at an incremental frequency, Δf, which is determined by the data sampling rate divided by the number of points acquired. This Δf can be interpreted as the width of a frequency bin that is centered on f_{cent}. The smaller the width of the bin, the higher the resolution of frequency. It is important to note that changing the sample size or sampling rate will change the resolution, and that changing sampling rate alone modifies both the Nyquist cut-off frequency and the center frequency.

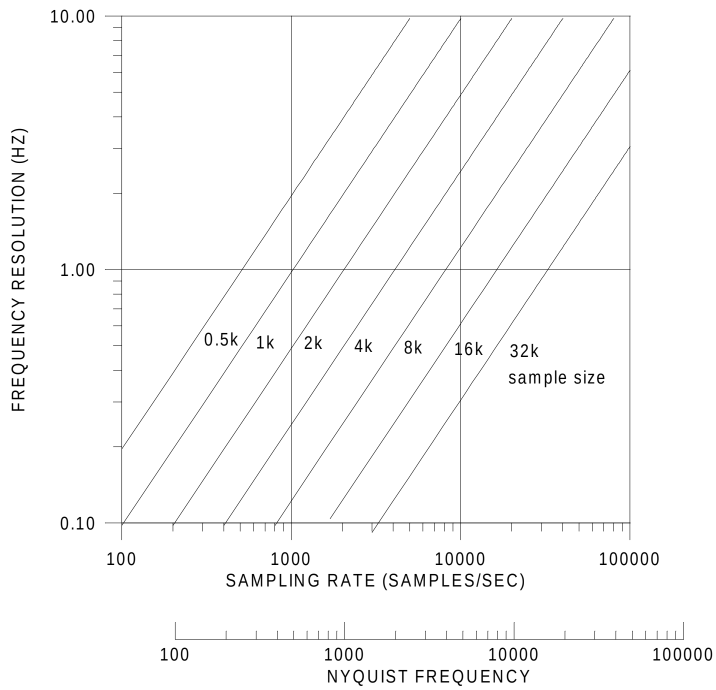

Figure 2: Frequency Resolution for a Sampled Data Set

Amplitude Resolution: The interpretation of the amplitude of a signal at a given frequency is not as clear. This is due to the smearing of a signal over adjacent frequency bins. How much smearing occurs depends on the width of the frequency bin and how well centered the bin is on the natural frequency in question. Typically, multiple samples of data are averaged to achieve the best accuracy on both frequency and amplitude. Another complicating factor concerning the amplitude of the results is that some of the Fast Fourier Transform routines contain multipliers of N, N^2, or sometimes a multiple of π, depending on the derivation of the specific transform pair. Because of this, extreme caution must be exercised when using an unfamiliar software package. Students are encouraged to perform an FFT of a known signal to determine any multiplying factors before application to an unknown signal.

An illustrative example of the effects of sampling parameters is shown in Figures 3 and 4. A time domain signal was acquired from a vibrating shear structure using an accelerometer as described below in the applications. To show the effects of sample rate and size on the resolution of the spectrum, a discrete Fourier series was obtained using an FFT algorithm from Newland. The two curves shown in Figure 3 represent the magnitude of the Fourier coefficients of the same signal sampled at two different rates of 200 samples /second and 10,000 samples/second with the same sample size of 4096 points. Figure 4 shows two samples at the same rate of 1000 samples/second with the sample sizes of 512 points and 4096 points. As discussed, both high sample rates and small sample sizes smear the spectrum over a broad range of frequency making the determination of resonant frequency difficult. Another obvious discrepancy between the plots in each figure is the magnitude of Fourier coefficient.

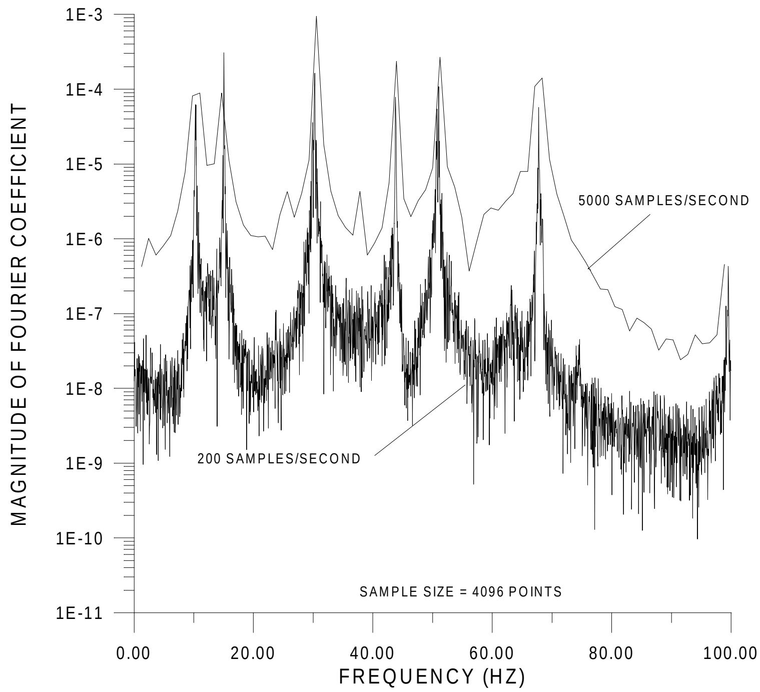

Figure 3: Spectrum Comparison for Two Sampling Rates at a Fixed Sample Size

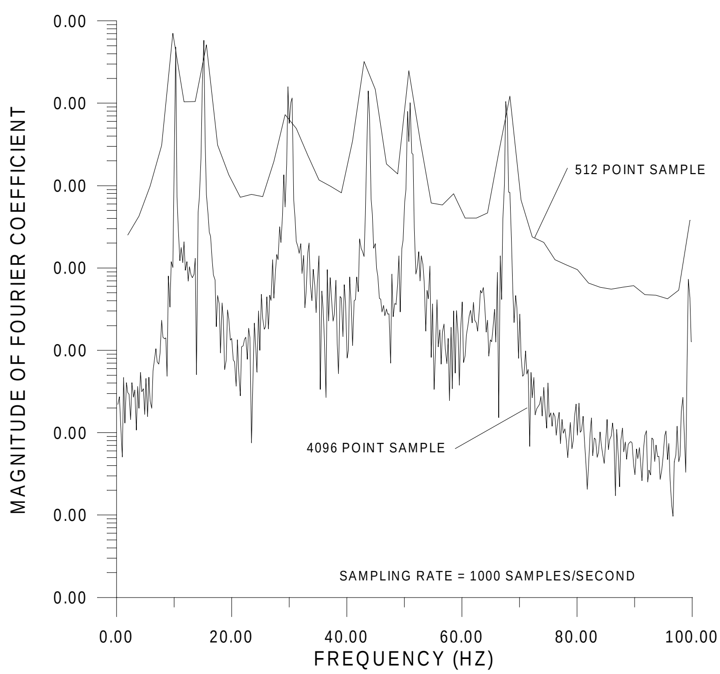

Figure 4: Spectrum Comparison for Two Sample Sizes at the Same Sampling Rate

Applications

In the Mechanical Engineering Technology Department at the University of Pittsburgh at Johnstown, the Fourier analysis method and data acquisition techniques are introduced in the Junior year in a survey course on mechanical measurements, and are followed up in an application in a Senior course in mechanical vibrations. At the introductory level, the method and its uses are introduced in conjunction with computer data acquisition. At this level, the emphasis is on acquiring the signal and manipulating the data into a useful spectral plot. The details of file manipulation, frequency resolution, and scaling are addressed and practiced. For example, data for known, pure signals are collected using data acquisition boards and the basic packaged software that accompanies them. The resulting files are imported into Mathcad or Excel and compared with student written or commercial software. Scaling problems between software packages are resolved. Plotting of amplitude and phase results is practiced with verification to known frequency. A complex signal is also collected and analyzed to illustrate the full capability of the methods and its application in determining fundamental frequencies of a structure, the frequency content of engine noise, human speech, or other available sources. Demonstration of white noise and its result is also appropriate.

At the advanced level, the student is expected to be able to analyze the behavior of a vibrating structure, predict its natural frequencies and corresponding mode shapes, and correlate the predicted to the experimental natural frequencies. To accomplish this, the Mechanical vibrations laboratory at UPJ has designed a four story shear structure consisting of flat square steel plates mounted on four threaded steel rods. The four plates are moveable allowing modification of the natural frequencies of the structure from one term to the next. Since the plates are square and uniform, there are only four horizontal, translational degrees-of-freedom, and four rotational degrees of freedom. The simple symmetrical geometry with the limited number of degrees-of- freedom make the structure very simple to analyze. The translational motion is essentially decoupled from the rotational motion, so an analysis of two separate four-degree-of-freedom models yields very good results. Some of the students who are also enrolled in the Finite Element course have also completed finite element analysis of the same structure for comparison of the methods and results.

The modeling in the vibrations class begins with the assumption of lumped mass and elasticity for the structural elements. The equations of motion are extracted, either from the free body diagram by direct application of Newton's law, or, more typically, from influence coefficients. For small displacements, linear system behavior permits expression of the structural displacements as harmonic functions $q(\chi,t) = Q(\chi)\sin(\omega t)$. This represents the matrix problem in the familiar eigenvalue form. At this point, the student is required to find and use an eigenvalue program to obtain the analytical predictions of natural frequency and corresponding mode shape. The natural frequencies are the specific values of ω, the eigenvalues, that solve the equation set, and for each frequency, there is a corresponding eigenvector or mode shape $Q(\chi)$. Typically, the students have used either canned software, student generated programs in FORTRAN or QuickBASIC, or even the eigenvalue solving capability of the Hewlett-Packard calculators. The software available in the student labs that is capable of performing eigenvalue analysis includes both MATLAB and Mathcad. These analysis predictions of resonant frequencies from simple rigid body-lumped elasticity models are compared to the experimental data acquired from accelerometers attached to the structure. This data is acquired using the techniques described above and converted to the frequency domain. The students then draw comparisons between the experimental and analytical values of resonant frequency, which typically agree within a few cycles per second. Direct comparisons of the magnitudes of the components are not made due to the inability of the existing laboratory equipment to provide accurately controlled excitation of the structure. It is being planned as a future improvement to extract mode shape data from four channel data for comparison to the analytical predictions.

Conclusions

The total experience requires the use of the computer, both as an experimental tool, to acquire and manipulate data, and as an analytical tool to analyze large quantities of data and to conduct sophisticated matrix analysis of structures. Some conclusions to be made concerning the outcome of the effort.

1. The usefulness of a frequency spectrum obtained from Fourier analysis of data is highly sensitive to the data acquisition methodology. Desired characteristics of the spectrum such as frequency resolution, amplitude resolution, or signal replication may dictate sampling rate, sample size, number of averages, frequency cutoff, etc.

2. A simple graphical representation of the important variables related to frequency resolution has been described. Unfortunately, no such concise representation yet exists for amplitude resolution.

3. The interweaving of experimental data acquisition, analysis and interpretation with a course based on simple theoretical mechanical modeling concepts significantly adds to a students understanding of the physics of dynamic systems. This combination of Measurements and Vibrations courses produces a clearer picture of how the engineering methods works.

References

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