Economic Criteria

John H. Ristroph
University of Southwestern Louisiana

Abstract

The standard method for developing economic criteria such as present worth entails extrapolating banking formulae into the more complex industrial environment, but discerning students recognize that the two environments are different and question the validity of this approach. This paper first develops a very fundamental, easily understood economic decision criterion for the banking environment in a manner that recognizes an assumption central to economic analyses: reinvestment. This basic criterion is extended to show that present worth, equivalent annual worth, and future worth produce the same decisions, so they may be used as surrogate criteria. The next step is to explicitly consider the more complex nature of industrial investments and capital accumulation. The development of industrial criteria parallels that of the banking environment, but it addresses additional issues, such as how to determine the appropriate discount rate. A final section provides a summary and conclusions.

Introduction

This paper addresses a fundamental problem in teaching engineering economics. It is common practice to derive formulas within the context of a single savings account or loan with a stated rate of interest. Then these formulas are ported to a totally different environment, that of industry wherein reinvestment occurs in a multitude of projects with different rates of return. This can lead good students to ask potentially embarrassing questions such as:

1. Why use formulas derived under one set of conditions in a totally different environment?

2. Why is the minimum attractive rate of return (MARR) used as the discount rate?

Answering these questions merely requires a few pages of reading.

Oakford and Theusen [1] provided the first empirical validation of the effectiveness of present worth (PW) analyses in the 1960's when they:

1. observed that economic analyses should seek to maximize the wealth of a entire firm at some future point in time;

2. modeled the investment process via a computer simulation and numerically determined an optimal discount rate policy; and

3. explained their logic using the concept of a unit investment of one dollar for one year.

This paper extends their work by developing a closed-form mathematical model for the future total worth (FTW) of an entire firm, its wealth at the end of the planning horizon, as a function of project selection.

The model is suitable for presentation in an undergraduate class. Its level of mathematical complexity is well within students' grasp, and it:

1. avoids developing equations in one environment and then using them in a totally different one;

2. explains why PW works (e.g., why it produces the same decisions as FTW);

3. shows how to determine the appropriate discount rate.

The objective is not to question current methods of analysis, but to present why they work.

The paper assumes that investors select projects to maximize their FTW. The logic of this approach is developed initially in a banking environment wherein reinvestment occurs in a single account. Next, modeling is extended to more complex industrial investments. The development provides a framework for understanding topics such as comparing alternatives with different lives or determining the appropriate discount rate.

Reinvestment in a Single Account

The following subsections assume that all funds for investments come from and return to a single account, such as a savings account. An example illustrates the basic logic of such investments, and then generalizing the example provides criteria for selecting projects.

Example for a Single Account

Suppose that an investor has an account paying 10% interest that initially contains $100. Only one of the three alternatives shown in Table 1 will be chosen over the next 3 years, and neither A nor B will be replaced. Project A consists of withdrawing $10 at time 0, and then returning to the account $7 at time 1 and $5 at time 2. Project B withdraws $9 to provide returns for 3 years. The null alternative rejects A and B and leaves all of the initial assets in the account, so its cash flows are all $0.

Table 1. Cash Flows

Time	Alternative		
	A	B	Null
0	-10	-9	0
1	7	3	0
2	5	4	0
3	0	5	0

Table 2 shows that A's withdrawal reduces its initial balance to $90, so its balance at time 1 is 90(1.1) + 7 or $106.00. Its balance at time 2 is 106(1.1) + 5 or $121.60, and its final balance at time 3 is 121.60(1.1) or $133.76. Similarly, the final balances for project B and the null alternative are $134.15 and $133.10. Project B is the best choice for an investor who wishes to maximize the final balance.

Table 2. Account Balances

Time	A	B	Null
0	90.00	91.00	100.00
1	106.00	103.10	110.00
2	121.60	117.41	121.00
3	133.76	134.15	133.10

All costs and benefits are known, so each alternative's final balance can be computed. The balance of both projects A and B is reinvested at time 2 and grows at 10% until time 3. The fact that project B's balance is augmented by $5 at time 3 has no bearing on the computation of project A's final balance. Similarly, the null alternative has reinvestment without returns for 3 years, and its final balance still can be calculated.

Generalization for a Single Account

If all costs and benefits of alternatives are known and reinvestment occurs in a single account, then it seems reasonable to use the *final balance procedure*:

1. Set the length of the planning horizon to the life of the longest alternative.
2. Assume future benefits and costs increase or decrease capital that is reinvested.
3. Choose the alternative producing the largest final balance at the end of the planning horizon.

Performing the balance calculations using symbols instead of numbers results in the following formula for final balance:

$$B_n = A(1+i)^n + c_0(1+i)^n + c_1(1+i)^{n-1} + c_2(1+i)^{n-2} + \cdots + c_{n-1}(1+i) + c_n \ , \qquad (1)$$

where B_n is the balance at the time n (the end of the planning horizon), A is the initial assets in the account, c_j is the cash flow at time j, and i is the account's interest rate.

Economic Criteria for a Single Account

Final balance is an easily understood decision criterion, but other criteria are more popular. They are presented below, still within the context of a single account.

Future Worth. A project's future worth (FW) is defined as the difference between the final balance of the project and that of the null alternative, so

$$FW = c_0(1+i)^n + c_1(1+i)^{n-1} + c_2(1+i)^{n-2} + \cdots + c_{n-1}(1+i) + c_n \quad . \tag{2}$$

Future worth is a surrogate criterion for final balance, since it differs only by the term $A(1+i)^n$ and hence results in the same ranking of alternatives. Note that the definition implies that the FW of the null alternative is always \$0.

Present Worth. A project's present worth (PW) is defined as the single, current cash flow that produces the same final balance as the project. The final balance for a single, current cash flow of *PW* dollars is $A(1+i)^n + PW(1+i)^n$. Set this equal to the general expression for final balance in equation (1) and solve for *PW* to obtain:

$$PW = c_0 + c_1(1+i)^{-1} + c_2(1+i)^{-2} + \cdots + c_{n-1}(1+i)^{-(n-1)} + c_n(1+i)^{-n} \quad . \tag{3}$$

or

$$PW = FW(1+i)^{-n} \; . \tag{4}$$

Present worth is proportional to future worth, so it results in the same ranking as future worth and hence the same as final balance.

Equivalent Annual Worth. A project's equivalent annual worth (EAW) is defined as the equal amounts flowing into or out of an account at times 1, 2, ..., *n* that produce the same final balance as the project. Setting the final balance for cash flows of amount *EAW* at times 1, 2, ..., *n* equal to the final balance of a project results in

$$EAW = PW\left(\frac{i}{1-(1+i)^{-n}}\right) . \tag{5}$$

Equation (5) shows that equivalent annual worth is just a re-scaled version of present worth, and hence is proportional to future worth and final balance. Thus it produces the same ranking as final balance, and is another surrogate criterion.

Note the exponent in equation (5). It must be the same for all alternatives if EAW is to be proportional to final balance. This means that the EAW for project B in the example must be computed over 3 years, not 2. This does not pose a problem for alternatives that are not replaced and for which all costs and benefits are known. Further analysis is required for other situations.

Capital Budgeting

The foregoing analyses deal with mutually exclusive projects in an unlimited capital environment. Now consider the capital budgeting problem wherein projects are independent and capital is limited. Maximizing present worth and hence final balance in this environment is a complex binary programming problem.

Benefit / Cost Ratio. Project's benefit / cost (B/C) ratios provide an easily implemented capital budgeting heuristic. Let B/C_j equal the ratio of the present worth of project *j*'s cash flows at times 1 through *n* to its first cost. Thus B/C_j equals the rate at which project *j* provides benefits per unit of cost. Selecting projects in decreasing order of their B/C ratio until funds are exhausted usually results in a selection of projects of a quality very similar to that of the binary programming formulation [1]. The heuristic does not include incremental analyses designed to insure selections identical to PW. Its success nonetheless indicates that a project's B/C ratio is an indicator of its quality in a limited resource environment.

Rate of Return. The internal rate of return (IRR) of a project is defined to be the interest rate that would be paid by a savings account receiving c_0 dollars and returning yearly flows of c_j dollars. This implies that it is the root of the following equation:

$$0 = c_0(1+i)^n + c_1(1+i)^{n-1} + c_2(1+i)^{n-2} + \cdots + c_{n-1}(1+i) + c_n \ . \tag{6}$$

The rate of return (RoR) heuristic selects projects in decreasing order of IRR until funds are exhausted. It does not include incremental analyses designed to insure selections identical to PW, but it still produces very good results [1].

Relationship between B/C and RoR. The B/C and RoR heuristics both provide good solutions to the capital budgeting problem. They do not provide identical rankings of independent alternatives, but they do produce highly similar rankings because they *tend* to be proportional. This tendency can be measured statistically using their correlation coefficient. The following experiment performed using an Excel spreadsheet found this correlation to average 97%, where 100% means that the two criteria are perfectly proportional.

The basic experimental procedure involves creating a set of 100 independent projects. Each project lasts 5 years and consists of randomly selected returns, each between 0 and 1, with an initial cost computed so that the project's IRR is a random number between 5% and 25%. Then the correlation coefficient is computed for the resulting 100 IRR's and their corresponding B/C ratios using a discount rate of 5%. Correlation coefficients also are computed for B/C ratios evaluated at 10%, 15%, 20%, and 25%. For example, Figure 1 shows a scatter plot of 100 points with a coefficient of 97.1%. The resulting 5 coefficients are stored, and the entire procedure is repeated 50 times. The average value of the resulting 50 sets of 5 correlation coefficients is 97%, with a standard deviation of 2% and individual values ranging from 92% to 99%.

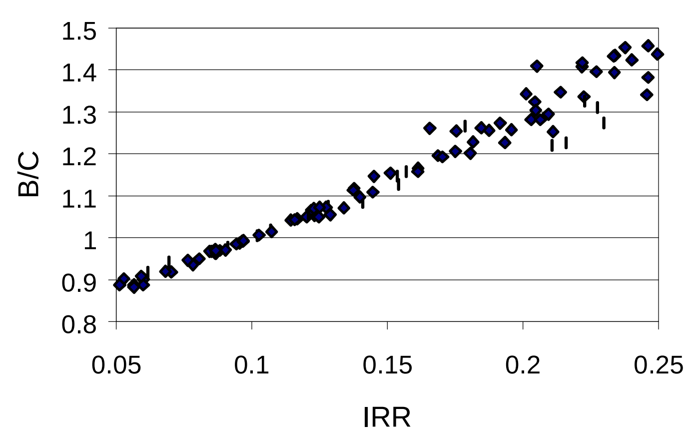

Figure 1. Sample Plot of B/C Versus IRR

Theusen demonstrated the high quality of the B/C heuristic for approximately solving the binary programming problem that describes capital budgeting. The correlation between B/C and IRR indicates that progressively less desirable projects based on the B/C criteria tend to have progressively smaller RoR's. For example, suppose that the last project accepted on the basis of its B/C ratio has a ratio of 1.4 and an IRR of 10%. Then as ratios decrease from 1.4, the corresponding IRR's also tend to decrease from 10%. They do not, for example, fluctuate widely between 5% and 25%. Thus contractions and expansions in capital would affect projects with IRR's of around 10%.

In general, the last projects that are selected or rejected due to expansions or contractions in capital are referred to as marginal projects. Their average IRR is known as the average marginal rate of return (AMRR). This important concept will be used shortly in the development of the industrial model.

Reinvestment in Industry

In an industrial environment, funds for competing alternatives do not flow from and to a single account. Instead, funds are taken from or reinvested in other projects, each with a potentially different IRR. This section examines how this impacts the worth of a company at the end of the planning horizon. The following subsections first use an example to illustrate the basic logic of industrial investments, and then generalize this to develop criteria for selecting investments.

Example of Industrial Reinvestment

Suppose that the mutually exclusive investments shown in Table 1 are made in an industrial environment having \$100 of investment capital at time 0. The objective is to chose the alternative that maximizes the capital at the end of the planning horizon, time 3. This capital at the end of the planning horizon corresponding to each alternative is its *future total worth*, or FTW.

Table 3. Investment Capital

Time	A	B	Null	Common
0	90.00	91.00	100.00	90.00
1	110.50	107.60	114.50	107.60
2	131.93	127.74	131.33	127.74
3	151.51	151.90	150.85	150.85

Alternative A initially requires \$10, leaving \$90 available for other investments, as shown in Table 3. Similarly, alternative B allows \$91 for other investments, and the null alternative results in all \$100 remaining available. All alternatives have a common funding base of at least \$90 for other currently unknown investments, and it is assumed this first \$90 will be consumed during capital budgeting by the better, non-marginal projects that provide an average non-marginal rate of return (ANMRR), say 15%.

It also is assumed that there are a limited number of good investments, and that any funds above \$90 will be used by projects of marginal quality relative to those selected with the first \$90. Recall from the preceding section that an indicator of a project's quality for capital budgeting purposes is either its B/C or IRR ranking. These two criteria produce similar rankings, so projects tend to be marginal regardless of which measure is used. In this example, it is assumed that marginal projects have an average rate of return equal to 10%, the AMRR.

The foregoing assumptions allow computation of the investment capital corresponding to each alternative at time 1. Choosing alternative A implies that the remaining \$90.00 of investment funds available at time 0 will be invested in non-marginal projects grow at an ANMRR of 15% and then be augmented by the first return of \$7, so the funds for A at time 1 are

$$F_{A,1} = 90.00(1.15) + 7 \tag{7}$$

or \$110.50. Choosing alternative B results in the first \$90.00 of investment funds growing at 15%, the marginal amount of \$1.00 (91.00 - 90.00) increasing at 10%, and then augmentation by the return of \$3, so

$$F_{B,1} = 90.00(1.15) + (91.00 - 90.00)(1.10) + 3 \tag{8}$$

or \$107.60. The null alternative results in the first \$90.00 growing at 15% and the marginal amount of \$10.00 (100.00 - 90.00) increasing at 10%, so

$$F_{O,1} = 90.00(1.15) + (100.00 - 90.00)(1.10) \tag{9}$$

or \$114.50.

Assumptions similar to those made at time 0 must be made at time 1 to compute the investment capital for time 2. At time 1, there will be investment capital of at least \$107.60. It is assumed that the first \$107.60 of investments will grow at the ANMRR of 15%. Any funds above \$107.60 are assumed to be used by less attractive investments that provide an AMRR of 10%. Then the computation of the investment capital for time 2 proceeds much as before.

At time 3, the *reinvestment assumption* is made again: Investments up to the common level grow at the ANMRR, and investments beyond that amount increase at the AMRR. Repeating this process until the end of the planning horizon allows the FTW corresponding to each alternative to be computed.

Generalization for the Industrial Environment

If all costs and benefits of alternatives are known and reinvestment occurs as described above, then it seems reasonable to use the *FTW procedure*:

1. Set the length of the planning horizon to the life of the longest alternative.
2. Assume that future benefits and costs increase or decrease capital that is reinvested, and that common amounts earn the ANMRR and amounts beyond that earn the AMRR.
3. Choose the alternative producing the largest FTW.

Developing a formula for FTW requires assigning symbols to the parameters of the investment problem explained above, writing the expressions using those symbols instead of numbers until a pattern emerges, and then algebraically rearranging the result. Let the following symbols be used, illustrated using values from the preceding example:

K_j	Common capital at time *j*, e.g., K_0 equals 90.00 and K_1 equals 107.60.
c_j	Value of a cash flow at time *j*, e.g., c_0 for alternative B equals -9.
A	Initial investment capital at time 0, e.g., \$100.
a	Average non-marginal rate of return or ANMRR, e.g., 15%.
m	Average marginal rate of return or AMRR, e.g., 10%.
n	Length of planning horizon, e.g., 3.

Then the equation for future total worth can be shown to be:

$$FTW = \begin{array}{c} A(1+m)^n + (a-m)[K_0(1+m)^{n-1} + K_1(1+m)^{n-2} + \cdots + K_{n-2}(1+m) + K_{n-1}] \\ + c_0(1+m)^n + c_1(1+m)^{n-1} + c_2(1+m)^{n-2} + \cdots + c_{n-1}(1+m) + c_n \end{array} \qquad (10)$$

In the example,

$$FTW_A = \begin{array}{c} 100(1.1)^3 + (0.15-0.10)[90.00(1.10)^2 + 107.60(1.1) + 127.74] \\ -10(1.1)^3 + 7(1.1)^2 + 5(1.1) + 0 \end{array} \qquad (11)$$

or \$151.51, as before. The FTW equation is primarily of theoretical importance, since the values of K_j cannot be determined without performing year-by-year computations.

The top line in equations (10) and (11) is the FTW of the null alternative for which the cash flows equal \$0:

$$FTW_O = A(1+m)^n + (a-m)[K_0(1+m)^{n-1} + K_1(1+m)^{n-2} + \cdots + K_{n-1}(1+m)] \qquad (12)$$

Only the bottom line of equation (10) varies from alternative to alternative, and this leads to the concept of future worth and other economic criteria for the industrial environment.

Economic Criteria for the Industrial Environment

The development of decision criteria for the industrial environment proceeds almost exactly as in the single reinvestment rate case, as shown below.

Future Worth. The future worth of an industrial project is defined as the difference between the project's FTW and that of the null alternative:

$$FW = c_0(1+m)^n + c_1(1+m)^{n-1} + c_2(1+m)^{n-2} + \cdots + c_{n-1}(1+m) + c_n \qquad (13)$$

The formula for FW in the industrial environment is quite similar to the one for the single account environment, except that now the AMRR is used in place of the account's interest rate. The interpretation of FW is the same for both environments.

Present Worth. The more popular present worth measure is defined as the single cash flow at time 0 that will result in the same FTW as the original project. Deriving the formula for a project's present worth is done by setting the FTW of a project equal to the FTW of a single cash flow of amount *PW* at time 0, resulting in either:

$$PW = c_0 + c_1(1+m)^{-1} + c_2(1+m)^{-2} + \cdots + c_{n-1}(1+m)^{-(n-1)} + c_n(1+m)^{-n} \ . \qquad (14)$$

or

$$PW = FW(1+m)^{-n} \tag{15}$$

As before, PW is proportional to FW, so it results in the same ranking as FW and hence the same as FTW. Its interpretation is the same as before, but notice that it is evaluated at the AMRR.

Equivalent Annual Worth. A project's equivalent annual worth is defined as equal cash flows of amount *EAW* occurring at times 1, 2, ..., *n* that produce the same FTW as the project. Setting the FTW for equal cash flows of amount *EAW* at times 1, 2, ..., *n* equal to the FTW of a project results in

$$EAW = PW\left(\frac{i}{1-(1+m)^{-n}}\right). \tag{16}$$

EAW is proportional to PW, and hence to FW and FTW, so it is another surrogate criterion. The exponent in equation (16) must be the same for all projects in order for proportionality to be maintained. Thus project A's EAW must be computed over 3 years, even though it lasts only 2 years.

Summary and Conclusions

This paper focuses on the case where all costs and benefits are known. There are many situations where the values of benefits are unknown, such as when pumps are compared. In this case, steps must be taken to insure that the contributions of any unknown benefits to FTW are equal, thereby allowing the benefits to be ignored. This usually is effected by choosing a planning horizon equal to the least common multiple of service lives or to some preset length that truncates one or more alternatives.

There also are cost analyses in which a planning horizon cannot be set via a least common multiple or truncation, but it can only be described as long and indefinite. In this case, a heuristic can be used if there are identical replacements. The best equivalent annual cost (EAC) of each alternative is computed by systematically varying its service life. Then the alternative with the smallest of these EAC's is chosen.

There are still other situations in engineering economics in addition to those noted above that require more detailed development. Nonetheless, the concept of future total worth provides a framework within which to consider these topics and to identify which procedures are designed to maximize FTW and which are heuristics.

There are several advantages of using FTW as a basic economic criterion. It is easily understood, and it allows precise definitions of the more popular present worth and equivalent annual worth criteria. FTW clearly reveals the reinvestment assumption, and this, in turn, explains why equal lives are not necessary if all costs and benefits are known and why the AMRR is used to compute PW and EAW.

The AMRR is slightly different from the MARR. The AMRR is a representative IRR for however many marginal projects are affected by typical fluctuations in capital, whereas the MARR technically is the absolutely lowest IRR at which investment occurs. If there are not large changes in the values of the last IRR's accepted, then the difference between AMRR and MARR is small and of little consequence in decision making. For practical purposes, it suffices to say that discounting should be done at the IRR's of marginal projects, a common interpretation of MARR.

Bibliography

1. Thuesen, G. J., "Decision Techniques for Capital Budgeting Problems," Ph.D. Dissertation, Stanford University, University Microfilms, 1968.

Biography

Dr. John H. Ristroph is a Professor of Engineering and Technology Management and a registered Professional Engineer in Louisiana. His B.S. and M.S. are from LSU, and his Ph.D. is from VPI&SU, all in industrial engineering. He has taught engineering economics for over twenty-five years, and first published in this area over twenty years ago. The material in this paper is used as a handout in both undergraduate and graduate classes.