

Excel optimization pedagogy using Van Hiele learning model of spatial abilities with Force Concept Inventory Test MRI and haptic learner data for COVID-19 online challenge

Dr. sunil Dehipawala, City University of New York, Queensborough Community College

Sunil Dehipawala received his B.S. degree from University of Peradeniya in Sri Lanka and Ph.D from City University of New York. Currently, he is working as a faculty member at Queensborough Community College of CUNY.

Dr. Dimitrios S. Kokkinos, City University of New York, Queensborough Community College

Dr. Dimitrios Kokkinos is an Associate Professor of Physics at Queensborough Community College of CUNY since 2017. He Completed his Electrical Engineering degrees (BE, ME, PhD) at CUNY and undergraduate in Physics in Europe. He worked in industry for AT&T Bell Labs and Verizon Communications for 23 years as a telecommunications engineer specialized in fiber optical system research and development. He is teaching pre-Engineering Physics courses, conducts research in fiber sensors and mentors student research projects.

Dr. Rex Taibu

Dr. Rex Taibu has taught studio and lecture classes for several years. His teaching experience has shaped his research focus. Currently, Dr. Taibu is actively engaged in research that involves development and assessment of students' lab skills and research regarding the role of language in conceptual understanding.

Mr. George Tremberger Jr

Prof. Tak Cheung

Tak Cheung, Ph.D., professor of physics, teaches in CUNY Queensborough Community College. He also conducts research and mentors student research projects.

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City University of New York Queensborough Community College Physics Department

Abstract

The van Hiele learning model of spatial abilities has been shown to effectively assess the preparedness of students learning geometry. Moreover, Force Concept Inventory (FCI) Test MRI data compiled on the neural networks of engineering students showed activation beyond the neural networks associated with regular math operations. The recently reported qualitative study of students' verbal responses to problems on the FCI in the framework of the van Hiele learning model and Redish cognitive resources model has been expanded by our group to include a quantitative study of students' math responses. The use of Excel Solver optimization pedagogy in introductory physics courses for engineering and algebra-proficient students during the COVID lockdown and reopening challenge was performed by our group. The selection of the optimization applications was designed to be consistent with the activation of the neural networks reported in MRI studies on engineering students, physics professors and haptic learners. The effectiveness of the optimization approach would confirm the assertion put forth in an ASEE previous presentation that engineering physics is a universal donor degree. It would also provide a means by which to implement the recommendation presented in another previous ASEE paper in which the engineering students' conclusion was "the learning of physics being irrelevant in their third semester after completing introductory physics". The contrast between the van Hiele learning model and Bloom's taxonomy model on educational learning objectives in the learning of physics is discussed. The use of the spatial-numeric tool provided by Excel in terms of the plagiarism prevention and equity issues is discussed.

Keywords

Spatial-numeric abilities, van Hiele learning model, Excel solver

Introduction

The learning of introductory physics requires spatial abilities with graphic visualization, vector manipulation, Newtonian force free body diagram, etc. The applications of the van Hiele learning model for geometry with spatial abilities have been documented [1, 2]. One of the reports included the implementation of the van Hiele learning model for Force Concept Inventory with assessment being done through qualitative questions [3]. The spatial-algebraic method in analytical geometry has been a standard tool in calculus physics for engineering students and physics major students. Our community college requires Calculus One as a co-requisite in Calculus Physics One. The use of the spatial-numeric tool facilitated by Excel would be acceptable as a helping tool to the use of analytical geometry, especially when Microsoft Office 365 is already included in the student fee in our CUNY system. The pedagogy of using the van Hiele learning model in physics with quantitative questioning had not been popular as far as we know. The use of the placebo comparison method to validate a new pedagogy would

generate ethical issues in education. Therefore the deployment of the van Hiele learning model in calculus physics with spatial-numeric tool would need justifications which include brain scan data. Among the MRI reports focused on the learning process, the conformation that neural networks with less segregation would support more creativity, using an explanation in terms of less brain energy utilization and more brain usage efficiency, is an important guideline consistent with the Redish theory on cognitive resources [4, 5]. The MRI data on engineering students taking the force concept inventory test showed that novices and engineering students would register distinct and overlapping patterns of neural activity [6]. The studied engineering students showed that abstract concept knowledge is driven by real world stimuli with multivariate neural response patterns. Another MRI report on the identification of the specific neural regions related to the assessing consilience of other and/or firmer knowledge, causal reasoning that are not apparent, and knowledge management in the processing of physics could be used to implement the van Hiele learning model [7]. The specific neural regions related to the reasoning about intangibles such as quantum physics beyond first year calculus physics is discussed in the Assessment section. Another MRI report stated that the studied neural score could be mapped to the test scores, $N = 33$ Dartmouth engineering students [8]. The more detailed justifications from the studied MRI reports (and haptic learner reports) are provided in Appendix-One for those interested, consistent with the suggestions of the conference proceedings reviewers. The Excel files described in the Appendices are available by contacting us (sdehipawala@qcc.cuny.edu and/or tcheung@qcc.cuny.edu) because putting Excel files on the open access Github may not be helpful. When students could download answers from Github, then plagiarism could prevail. Of course, a few Excel templates should be given to the students as guidance, and all of the Excel file images are always available to the students.

Spatial abilities pedagogy

The use of graphic solution instead of algebra-based solution to promote spatial abilities can be implemented. The abstraction process can be illustrated in terms of necessity and sufficiency in the van Hiele learning model. Examples include the use of the law of sines in solving relative velocity problems, Euclidean geometric square root method in solving projectile problems, namely, the graphic solution of the equation $v_f^2 - v_0^2 = 2ax$ (with v_f, v_0, a, x stands for the final velocity, initial velocity, acceleration, distance respectively). When geometry construction with compass and protractor is perceived by non-design-drafting-major students to be tedious, a not-to-scale sketch with Excel numeric calculation could be used as well. For instance, in the relative velocity problem, the use of either Excel solver or Excel graphic is sufficient to give the numeric plane angle. The relative velocity problem could be further formulated as an asking of the change of plane angle relative to the change of target. A short table with a few pairs of plane angle and target angle numeric values would be sufficient. Numeric examples are shown in Appendix-A (relative velocity in kinematics) and Appendix-B (geometric square root method in kinematics) for the interested instructors to use in their classrooms. Moreover, the plane angle exercise represents a class of problems that involve angles such as the solving of a force vector equation using a tracing of the (x, y) coordinates, without transforming the vector equator into two scalar-magnitude equations in the vertical and horizontal components. Such spatial abilities in the handling (x, y) or $(\text{radius}, \text{angle})$ coordinates would have similarity to the mind-seeing ability observed in blindfold chess players [9], and would also provide a training to the game

engineering major students. The spatial-numeric association would train the working memory as well [10]. For instance, the cause and effect in a collision would impose ordinality on the variables. Ordinality describes the location of a step in relation to other steps in an analytical sequence. The (x, y) information tracing would require the learning of an ordinal sequence, which was shown to be enhanced when pairing spatial complexity with relational information, N = 46 college students [11]. An angle calculation example in Newtonian force is included in Appendix-A.

The launch angle for maximum range in projectile motion can be studied using the law of cosines when the sideways diagonal length is recognized to be proportional to the range, shown in Figure 1. The sideways diagonal length would be maximum when the angle becomes 90 degrees in the law of cosines. An alternative geometry method can be used when the vertical diagonal length is recognized to be proportional to the flight time, discussed in our previous publications [12, 13]. For the optimization question discussed here, the vertical diagonal length multiplication with the v_0 horizontal component would give an area proportional to the range. The maximum area would be the fixed-length v_f vector multiplication to the fixed-length v_0 vector at 90 degrees. Another sufficient method is to use a semi-circle to find the maximum area, shown in Figure 1. The law of cosines reasoning is a sufficient condition and the maximum area reasoning is another sufficient condition. The fixed-lengths of v_f and v_0 vectors are from the energy conservation requirement, which is a necessary condition. The Euclidean geometry method of finding square root can be used, shown in Appendix B. The necessity and sufficiency conditions in the abstraction level of the van Hiele learning model are illustrated.

The elastic collision of two equal mass objects also would yield a situation in which the two final velocity vectors are at an angle of 90 degrees (angle of separation, Open Stax College Physics page 303). The geometry shown in Figure 1 clearly shows that the 90 degrees condition would guarantee the Pythagoras theorem application such that $p_{\text{total}}^2 = p_1^2 + p_2^2$, with energy proportional to p^2 , with p as the momentum. Details of using geometry representation in solving collision problems for an understanding of high energy physics at the introductory physics level has been published by us recently [14].

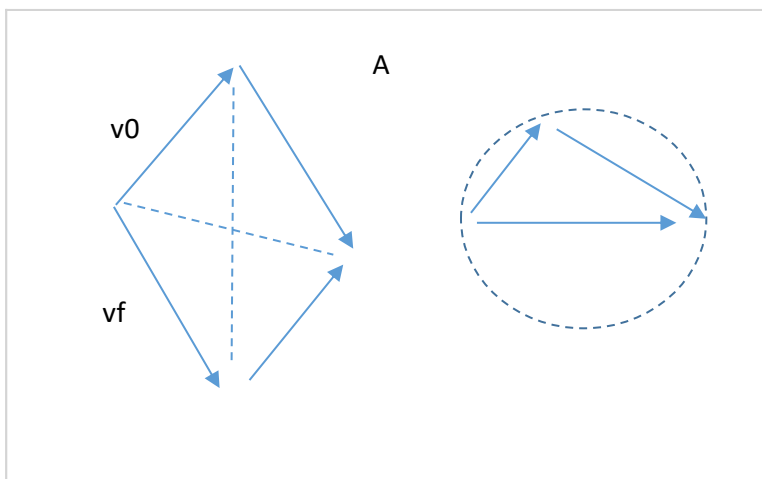


Figure 1: A graphic geometric representation of the projectile motion problem. The assessing consilience with other and/or firmer knowledge is supported by the cluster 6 left medial frontal and cluster 2 right middle front regions in the MRI data reported in Reference 7.

Using geometry spatial abilities and causality thinking to transform a problem of vertically stacked blocks to the chute-chute train model with horizontal blocks would offer another level of abstraction. A YouTube video by Lewin talked about the solving of a difficult kinematics problem. A transformation to the standard chute-chute train model would allow students to understand that each method by itself is sufficient, shown in Figure 2. The cause and effect would dictate that the effect should be cascaded as the following, lower right block A, upper right block B, upper left block C, and finally lower left block D. The transformation to the chute-chute train model would render a standard solution. The solving of another vertically stacked blocks linked by a pulley mechanism using such causality transformation is also illustrative, the numeric details are described in the Appendix-C.

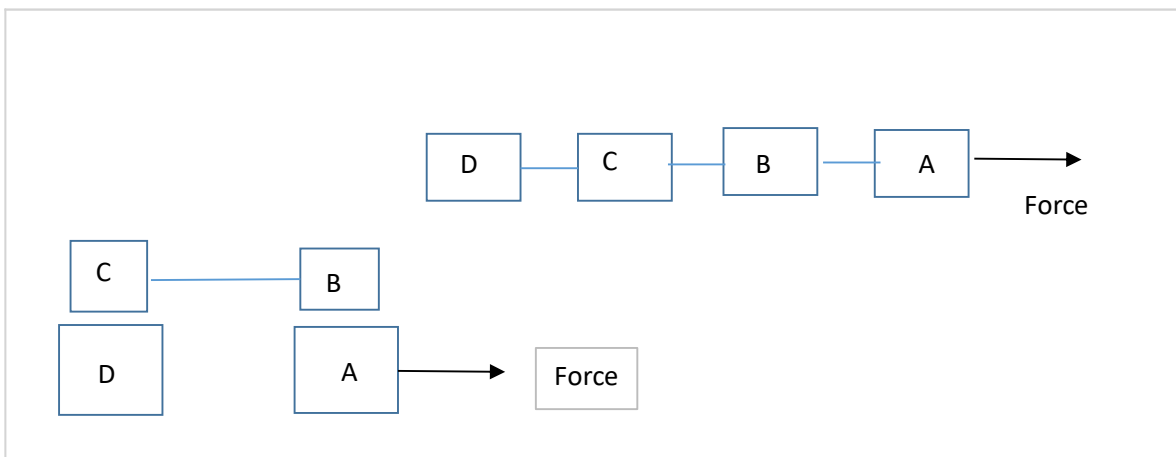


Figure 2: A graphical explanation of the transformation of a problem on vertical-stack blocks to the problem on the chute-chute train using the cause and effect perspective, A to B to C to D. The causal reasoning that are not apparent is supported by the cluster 8 left supra-marginal, cluster 1 right superior parietal, cluster 3 right middle frontal, and cluster 4 right inferior orbital frontal regions in the MRI data reported in Reference 7.

The use of the van Hiele learning of spatial abilities to foster the students' ability to understand a physics differential equation in terms of Excel difference equation approximation is an important pedagogy objective. One of the most simple spatial ability question would be the use of two ramps to replace a single ramp, shown in Figure 3. The location of (x, y) for the shortest time in a gravitational field can be solved by the Excel solver. This exercise was delivered just before the discussion on an object falling inside a fluid, Open Stax university calculus physics page 308. The spatial abilities of relating distances and angles are required. Equally important, the realization that the ending velocity in the first ramp would become the starting velocity in the second ramp is necessary. The time in the second ramp could be solved by using a single kinematic quadratic equation or two linear kinematic equations. The understanding of spatial

partition in the two-ramp shortest time problem was found to be helpful for the understanding of temporal partition in the Excel difference equation for an object falling inside fluid. This kind of metaphor or analogical reasoning using relational reasoning helps students to understand the differential equations when calculus is a co-requisite. A numeric example of the two-ramp shortest time is illustrated in Appendix-D. A numeric example of object falling in fluid in Excel difference equation is illustrated in Appendix-E. A numeric example of rocket velocity in Excel difference equation used for assessment is illustrated in Appendix-F. Three examples of momentum conservation in Excel spatial-numeric computation for assessment is shown in Appendix G.

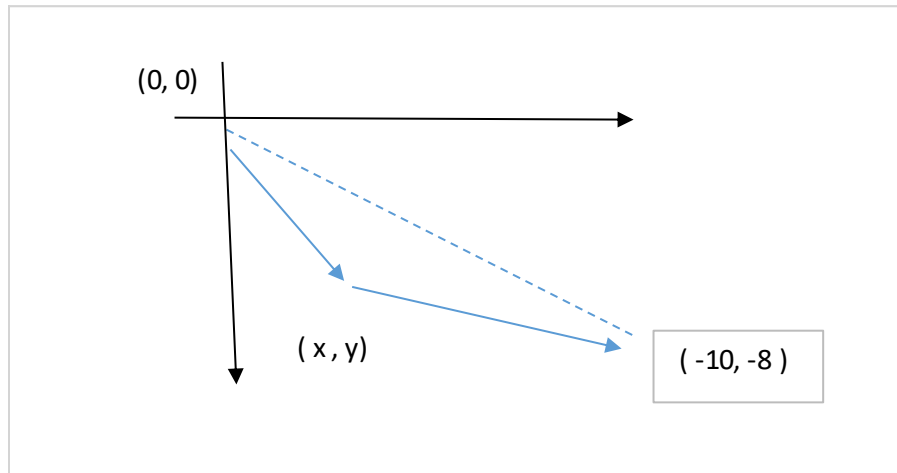


Figure 3: Graphic representation of the shortest time problem in a gravitational field. The knowledge management of the multiple columns in Excel is supported by the cluster 6 left medial frontal and cluster 7 left parietal frontal regions reported in Reference 7.

Assessment

A holistic assessment of the skills in the use of Excel difference equation approximation for differential equation was conducted. An assessment rubric example is shown in Table 1, driven by fMRI data collected from the studied engineering students that abstract concept knowledge is driven by real world stimuli with multivariate neural response patterns [6]. The plane angle in the relative velocity problem was used to assess the relational reasoning when the plane angle could be in any quadrants depending on the input numeric values. The assessment task would correspond to the activation of the cluster 6 left medial frontal and cluster 2 right middle front regions in the assessing consilience described in Reference 7. The two-ramp shortest time problem was used to assess the working memory capacity in the manipulation of multiple Excel data columns, shown in Appendix-D. The task would correspond to the activation of knowledge management neural regions described in Reference 7 as the cluster 6 left medial frontal and cluster 7 left parietal frontal region. The Reference 7 causal reasoning activation of cluster 8 left supra-marginal, cluster 1 right superior parietal, cluster 3 right middle frontal, and cluster 4 right inferior orbital frontal regions are assessed in the difference equation modeling task of the differential equation. These two Excel exercises in relative velocity and shortest time problems are pre-requisites to the difference equation representation of differential equation. For a small

in-person class of eight calculus physics students in open book situation, three students were at high competent level and three students were at need-improvement level (with two drop-outs). For Covid online algebra based physics class, the plane angle in relative velocity problem in open book situation showed 30% need-improvement students, $N = 42$. Assessment data on the use of shortcut solutions in solving problems were encouraging. For instance, the Appendix-G shortcut solution using momentum conservation, to replace the relative velocity approach on the dog-walk-on-boat problem, showed about 90% students at competent level, $N = 42$. The extension to the movable wedge problem showed about 70% students at competent level, $N = 42$ when using a different geometry orientation. The spatial-numeric details are shown in Appendix G. The conjecture that Excel optimization could offer an independent tool to facilitate spatial abilities in solving introductory physics problems received a positive affirmation in the studied samples. The small sample size limitation could be addressed with assessments in future semesters.

Table 1: Assessment rubric example, driven by the Reference 7 MRI cluster numbers in the assessing consilience, knowledge management and causal reasoning, described in the narration.

Deliverable	Highly competent	Competent	Needs Improvement
Plane angle relative velocity using Excel solver (25%)	Provided the correct angle value when in any quadrants	Contained one mistake such that the angle value was only correct when in three of the quadrants	Contained two mistakes such that the angle value was only correct when in two of the quadrants
Two-ramp shortest time (25%)	Provided the correct coordinates for the shortest time	Provided the coordinates with 10% deviation from the shortest time	Provided the coordinates with more than 10% deviation from the shortest time
Object falling in fluid difference equation delta-time interval modification (25%)	Constructed Excel difference equation answer within 5% error of the answer in the analytical solution of the differential equation	Constructed Excel difference equation answer within 20% error	Constructed Excel difference equation answer more than 20% error
Rocket velocity difference equation solution (25%)	Constructed Excel difference equation answer within 5% error of the answer in the analytical solution of the differential equation	Constructed Excel difference equation answer within 20% error	Constructed Excel difference equation answer more than 20% error

The use of a spatial-numeric tool can incorporate the use of a clicker tool (or student response system) in classroom. In fact, the active learning pedagogy in the use of the spatial-numeric tool via Excel would be deployable in the in-person and online situations, using the clicker setup or active classroom discussion [15]. The assessment of attention, participation (anonymous in in-person classroom) and game-based learning with enjoyment in the use of clickers would have value in promoting the intent of learning in all students, particularly more important for those who may be failing the class. Classroom discussion during the COVID challenge using the Blackboard threads could be implemented for asynchronous class, while the use of student polling surveys to simulate clickers did not work well for asynchronous online setting in our experience. A discussion on the comparison of the spatial-numeric method to the standard method shown in a Youtube video on the same problem seemed to generate the most discussion participation. The comparison of two Youtube video presentations on the same topic also generated some discussion participation. Whether the increase of discussion or clicker participation would better support success in the subsequent traditional courses in the use of difference equation, integrand building, etc. would be another set of assessments. The spatial abilities in relationship to the understanding of wave superposition needed in the new quantum information science is another important assessment in future studies. For instance, the reasoning of intangibles, with an activation of the cluster 8 left super-marginal and cluster 5 right inferior temporal regions, such as quantum process would be supported by the cluster 8 left super-marginal region activated in the causal reasoning that are not apparent [7].

Engaging students in active learning with X could be an intervention pedagogy. The X-tool is a function of time, but not a magic pill. Engaging students in active learning with full participation credit, for instance, submit discussion-voice threads onto Blackboard, could be described a carrot policy approach. Engaging students in active learning with responsibility credit, for instance, study the theory at home in a flipped classroom, could be described as a stick policy approach. Between the carrot and stick approaches, there are mixed approaches such as full participation credit when working on the difference equation of a damped oscillator and responsibility credit when working on the Excel solver. In any event, the assessment of a spatial-numeric pedagogy could show no overall class improvement even though the already-passing students would have learned divergent thinking (described in the first sentence in the Discussion section) while the failing students could have received minimal benefit from the spatial-numeric pedagogy.

Discussion

A 2022 review on the training of creativity stated that divergent thinking can be increased with training that expands working memory, fosters analogical reasoning, improves association fluency, promotes diverse mix-and-matching from mental sets, nurtures combinatorial play, and leverage the diverse neural networks in brainstorming into a focused output [16]. The Excel organization of multiple data columns would train working memory. The review also stated that the focused output in divergent thinking can then be described as convergent thinking, critical thinking, causal winnowing and problem solving. The Bloom's taxonomy pyramid of educational learning objectives has six layers with memorization as the first layer; understanding

as the second layer, followed by the higher cognitive processes in application, analysis, evaluation, and creation [17]. The van Hiele learning model level 2 inductive reasoning could correspond to the Bloom's taxonomy pyramid from memorization to evaluation. The van Hiele learning model level 3 abstraction, necessity versus sufficiency, could correspond to the Bloom's taxonomy pyramid from memorization to creation. The van Hiele learning model level 4 deduction cause and effect, what-if, could correspond to the Bloom's taxonomy pyramid from memorization to creation. Hence the van Hiele learning model offers better applicability for physics pedagogy utilizing spatial abilities and Excel spreadsheet fluency (not requiring the Excel VBA engine).

The literature contains the use of haptic device technology to help students to learn and physics [18, 19, 20, 21,22] and astronomy [23]. On the one hand, the application of haptic device technology in physics pedagogy could be described as helping students to achieve the memorization, understanding, and applications layers according to the Bloom's taxonomy pyramid classification, which are combined together as the levels of visualization and inductive reasoning in the van Hiele learning model classification. On the other hand, the application of the spatial-numeric tool would include the abstraction third level in the van Hiele learning model classification. The traditional pedagogy of the lecture component with formula derivation using deduction and the lab with hands-on experience were found to be of little value in the implementation of experiential learning for all students, a mandate in New York State for community colleges offering STEM subjects. The decline of student preparation in the high school system has been a challenge for instructors in the open admission community college setting. The reports on the use of haptic device technology described better assessment results in the topics of friction and Coulomb's law (via BEMA assessment instrument). During the COVID lockdown, it had been very difficult to deliver the tactile experience in the friction lab in our community college setting. In the current COVID semi-reopen, there is a budget issue in extending the haptic device technology pedagogy to the Coulomb's law. It is a conjecture that the spatial-numeric tool would help students to visualize the step-by-step skill in the formulation of a solution from our limited assessment data. The spatial-numeric tool could provide step-by-step problem solving skills leading up to the difference equation in rocket velocity, etc. Whether the coupling of the spatial-numeric tool with haptic device technology pedagogy could replace the traditional lecture component of formula derivation for the purpose of solving problems is an interesting assessment inquiry in future studies. At least the use of the spatial-numeric tool would do-no-harm as far as we know, with the bonus of encouraging divergent thinking, consistent with the findings described in the Reference-16 neuroscience review with brain scan data.

The ASEE paper position on "Engineering Physics being the universal donor program" would put the learning of physics at the foundation [24]. The use of alternative solutions in divergent thinking would encourage a solid foundation. Divergent thinking is likely to involve diverse neural networks with less segregation for better brain energy utilization, consistent with MRI data on visual divergent thinking [25]. An ASEE paper reported that "Engineering students found that the learning of physics is irrelevant in their third year" [26]. The necessity and sufficiency focuses in the van Hiele learning model could offer an explicit connection of introductory physics to engineering courses when students understand the abstraction process in the van Hiele learning model of spatial abilities. The deduction process at level 4 of the van

Hiele learning model would fit into the cause-and-effect and what-if questions. Moreover, the learning of difference equations in the engineering courses could remind the students that they had an introduction of difference equation using spatial-numeric tool in the introductory physics course.

The difference in the solving of spatial problems in female versus male has been attributed to puberty change in a mouse model [27]. Whether the effectiveness of using the van Hiele learning model of spatial abilities in the learning of physics would depend on male or female is an important project for future studies. Excel was used by more than 1 billion people according to a Microsoft survey conducted in 2016. The fact that there are more women students using Excel in MBA programs than women students in our calculus physics class for engineering majors would suggest a conjecture that using Excel as a spatial- numeric tool could address the equity issue to recruit more women students in calculus physics for engineering majors.

The advances in neuroscience could help instructors to design pedagogy with divergent thinking exercises, with the van Hiele learning model of level 3 abstraction and level 4 deduction, riding on the level 2 inductive-empirical reasoning analysis and level 1 visualization. The Open Stax calculus physics book has about 200 problems in Newtonian forces (Chapters 5 and 6). An instructor could assign 20% of them as homework for students to learn the inductive reasoning process in a traditional classroom setting or explicitly solve the problems in a flipped classroom with theory learning as homework. In both traditional and flipped classroom scenarios, the formulas are learned with physics context from the 20% problems assigned or solved by the instructor. In the van Hiele learning model, the various spatial abilities levels are clearly explained such that the formulas are learned with physics context and spatial abilities. The advances in computer technology with visualization would continue to support spatial ability reasoning, given that scalar-magnitude component equations had received full support in command prompt computer technology years ago.

An exercise for grading purpose has been known to generate anxiety, which would not support learning. Asking students to re-work the graded exercise for higher scores would push the students to cheat in about 50% of the cases in our experience. Our university purchased the Respondus LockDown Browser technology for instructors to proctor the Blackboard online examinations around March 2021. We have been concerned about the equity issue when some students were having the resources for multiple internet accesses and private physics information services. Our university is planning to purchase the McGraw Hill Publishing Proctorio technology. Student Government Newspapers articles were up in arms with objections, consistent with more anxiety. The student cheating and plagiarism issues in online courses during the COVID challenge could be alleviated using the divergent thinking exercises described above. The asking of a graphic solution is less likely to be found on pay-for-answer websites such as Chegg.com, and the asking of a graphic solution would then be more likely to prevent plagiarism. While grading is a legal contractual obligation, the assessment at second priority is a professional obligation. Exercises with full participation credit for assessment would promote learning without anxiety associated with testing.

Conclusions

The current paper described the use of the van Hiele learning model in physics pedagogy with justifications from MRI data and neuroscience advances. The MRI identification of neural regions for the assessing consilience with other and/or firmer knowledge, causal reasoning that are not apparent, and knowledge management supports the pedagogy design of using the geometry related activities, cause and effect emphasis, and optimization and difference equation exercises with multiple Excel columns. The spatial-numeric exercises are presented in the Appendices for the interested instructors to deploy in their classrooms. The impact of the spatial-numeric pedagogy in the third and fourth year engineering courses would be an important assessment in the future. An inference that the game engineering major students would find physics as relevance in their subsequent courses is expected. Future studies on the use of the spatial-numeric tool in Excel to overcome the language difficulty in the teaching of weight [28] is important, since “weight is a very non-intuitive and very trick thing” according to MIT [29].

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Appendix One Background (justification via MRI data)

The MRI instrumentation has been recruited into the question of how humans would learn. The MRI scan study in the search of cognitive creativity showed that those neural networks with less segregation would support more creativity, using an explanation in terms of less brain energy utilization and more brain usage efficiency [A-1]. The MRI scan study of the neural networks of physics professors and students showed correlation with various physics topics [A-2, A-3, A4]. In terms of saving brain energy, the cognitive shortcuts in numerosity perception could start after one year of education in the studied children [A-5]. However, the uses of “empirical rules of thumb based on observations alone may be of dubious values” have been asserted in an article published by *Physics Today* [A-6]. The use of shortcuts or cognitive heuristic would be encouraged only if the rules of thumb are the logical consequences based on physics theorems and deviations, and not based on empirical observations. For instance, the cognitive shortcuts based on physics conservation laws in the solving of collision problems have been advocated by Khan Academy on their video presentations [A-7]. Moreover, the brain usage efficiency from MRI data has been consistent with the Redish hypothesis on cognitive resources in the study of the learning of physics [A-8]. A working conjecture can be formulated as the following. The specific physics topics that trigger less neural network segregation would promote brain efficiency and could be grouped together for lesson delivery in the pedagogy of introductory physics for engineering and technology students.

Given that the relational reasoning components (analogy, anomaly, antinomy, and antithesis) that were found to be positively linked to STEM achievements, the manifestation of the relational reasoning in spatial ability has been reported to be important in the pedagogy of math, biology and geoscience [A-9]. Testing is a part of pedagogy. On the one hand, high scores in educational tests would not increase g-factor of fluid intelligence, stated in the 2014 MIT study [A-10]. On the other hand, when intelligence is accepted as the ability to reason, plan, solve problems, think abstractly and understand complex ideas, learn quickly and learn from experience, then the intelligence in the subject matter of spatial ability can be acquired in academic geometry [A-11]. Moreover, working memory capacity could drive high score in academic geometry, so educators should not overload the working memory capacity of their students [A-11]. The measurement of the g-factor for fluid intelligence showed negligible differences between females and males in the studied human population, although males were

reported to be neuronal efficient (having less brain activation in the MRI data) than females in spatial cognitive tasks [A-12]. Recently, it was reported that brain scan can be used to measure an attention index, which could influence pedagogy [A-13]. The development of working memory would be an essential pedagogy for students to acquire spatial abilities through classroom interactions [A-14].

MRI-based research has been conducted on the effect of math education on the brain neural networks [A-15]. It has been shown that elementary logical operations would not activate the linguistic networks in mathematicians, with a conclusion that abstraction is supported by a non-linguistic cortical network [A-16]. The first two levels of visualization and analysis in the van Hiele learning model of spatial abilities should be routine processing for competent students. The second level of analysis in the van Hiele learning model includes inductive reasoning in which a few examples would be used to formulate some general practices [A-17, A-18, A-19, A-20, A-21]. The third level of abstraction in the van Hiele learning model of spatial abilities includes an understanding of necessity and sufficiency with deductive reasoning. Such abstraction process should have a signature in the MRI data. The MRI data on physics professors processing equation-based physics showed that the processing of equation-based physics concepts would engage the parietal regions known as the language-based fact-retrieval region with functions related to calculation tasks [A-2, A-3]. A 2019 MRI report stated that a neural score based on the patterns of activity across the brain was able to predict individual differences in a dataset of 33 Dartmouth College engineering students [A-22]. The tasks were Newtonian force concept knowledge tasks in the physics and engineering domain with Free Body Diagrams, Statics Concept Inventory and Force Concept Inventory tasks. The MRI data on engineering students taking the force concept inventory test showed that novices and engineering students would register distinct and overlapping patterns of neural activity. The MRI data on haptic learners also showed active intraparietal sulcus and lateral prefrontal cortex regions [A-23]. The use of optimization perspective could add abstraction to the straight forward calculations in equation based physics processing in the introductory calculus physics course. Given the conjecture of saving brain energy, the van Hiele learning model should be deployed skillfully to avoid maxing out the working memory capacity of a student. Continued with the above working conjecture of less neural segregation for efficient brain energy utilization, a second working conjecture that Excel optimization could offer an independent tool to facilitate spatial abilities in the solving of introductory physics problems could be testable in assessment.

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Appendix-A Relative velocity problem (Excel solver and Excel graphic methods)

The standard spatial-numeric solution on the relative velocity problem without 90-degree condition is shown.

newton apps centripetal-friction-fluid-integral-dampig - Excel (Product Activation Failed)

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T9 =IF(AND(S7>0,T7>0),T8,"not 1st quad")

	P	Q	R	S	T	U	V	W	X	Y	Z	AA	AB	AC
2														
3		relative velocity: find plane angle for fix-wind vector and fix-plane speed												
4		magnitud	angle	x	y									
5	wind-fix	120	230	-77.1337	-91.9261									
6	plane	300	102.2708	-63.7612	293.1459									
7		Add		-140.895	201.2198									
8	target 125deg	polar		245.6436	-55									
9		ans	length		not 1st quad									
10					125									
11					not 3rd quad									
12					not 4th quad									
13		change plane angle for target 125 deg												
14		guess plane angle so target angle in 2nd quadrant												
15		For instance plane ange = 123 deg												
16		Solver : Objective T10 cell= 125 deg by changing R6 cell												
17		Ans: plane angle 102.27 deg at 245.6 mph effective speed												
18														
19														
20														
21														
22														
23														
24														
25														

Solver Parameters

Set Objective: \$R5\$8

To: Max Min Value Of: 125

By Changing Variable Cells: \$R5\$6

Subject to the Constraints:

Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Solve

Figure Appendix-A-1: The conditional statement to determine which quadrant is necessary. The Excel solver setup is shown.

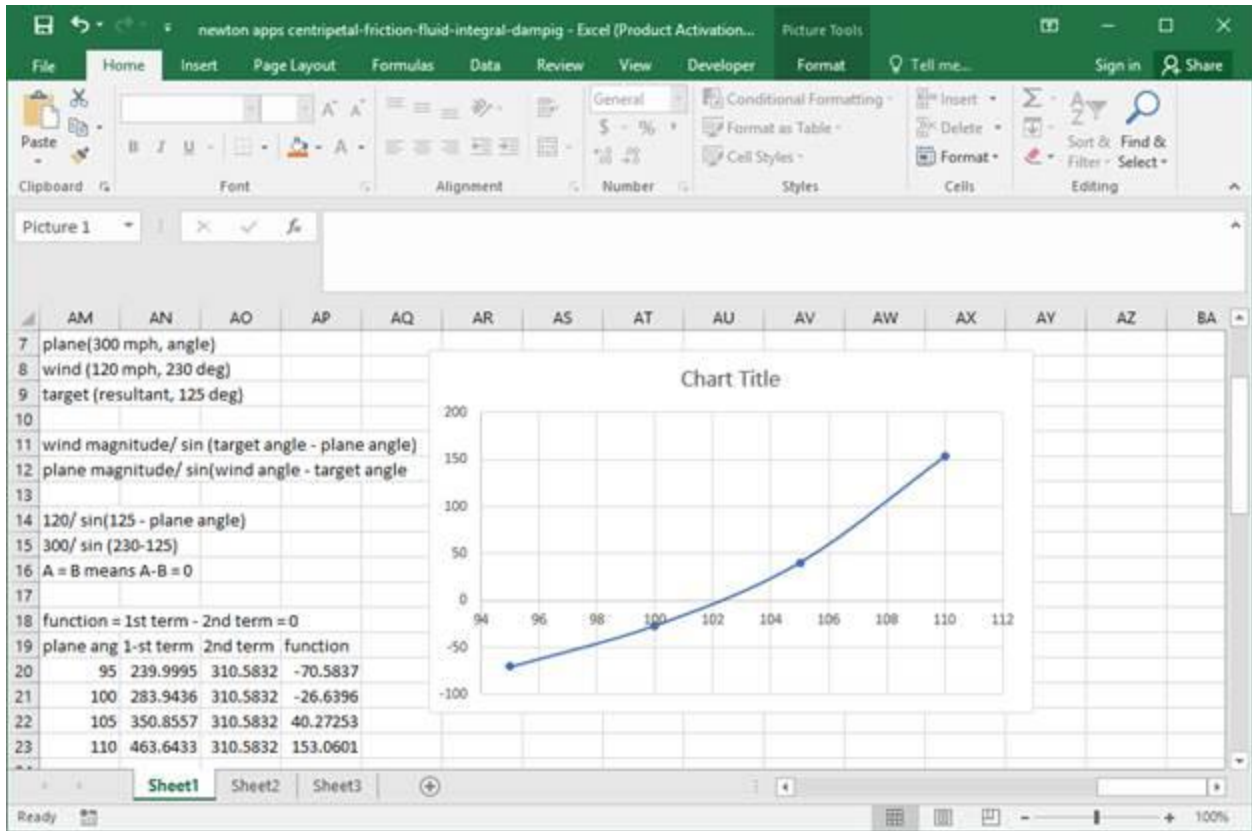


Figure Appendix-A-2: The graphical plotting method could be used without the Excel solver. In fact, the accuracy could be improved using trial and error around the 102-degree neighborhood in this example. The concept of uncertainty would be reinforced when comparing the graphical answer in visualization and the Excel solver answer in several decimal places.

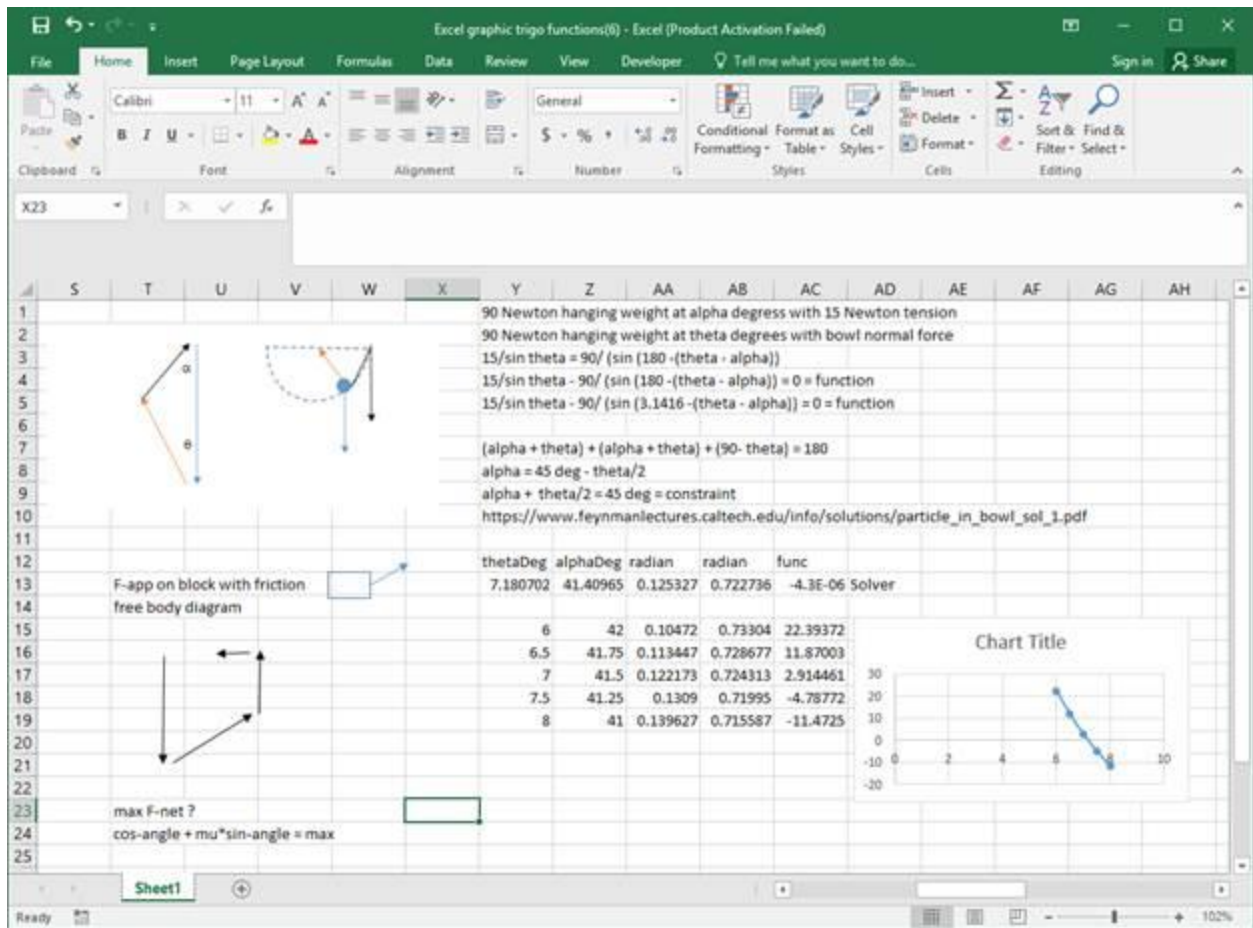


Figure Appendix-A-3: The relative velocity solution method, with the law of sines application being simplified by the isosceles triangle geometry imposed by the string, $\alpha + \theta/2 = 45$ degrees, could be applied to the bead on the bowl problem from Caltech Feynman Lectures. The force on a block problem is shown in the lower right corner. The tracing of coordinates in the free body diagram would generate an equation for the optimization of the angle for maximum acceleration.

Appendix-B Geometric square root method in projectile motion problem

The geometric square root method could be used to address the expression $v_f = \sqrt{2 \cdot 9.8 \cdot \text{height}}$ in projectile motion problem

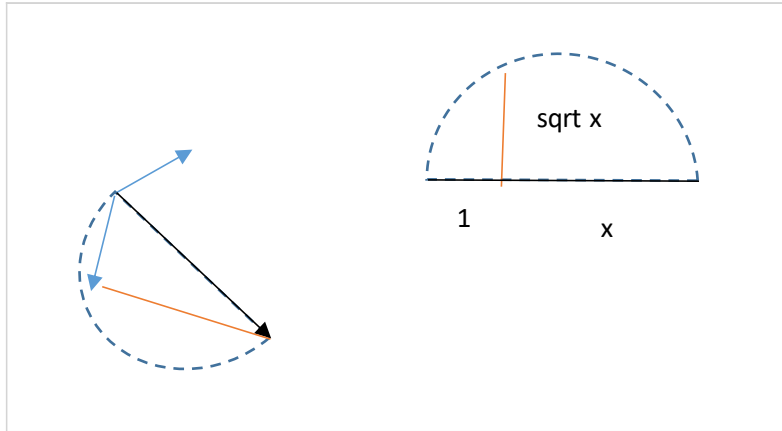


Figure Appendix-B-1: Using the geometry square root method to address the projectile motion in the lower left corner. The black is the final velocity with the blue as the initial velocity and the orange as the $\sqrt{2 \cdot 9.8 \cdot \text{height}}$ at 90° to the blue. The upper right corner diagram explains the Euclidean square root method.

Appendix-C Two vertically stacked blocks with a pulley

The transformation of a vertical-stack block problem to a chute-chute train model problem using cause and effect. Lewin viewed that as a not-easy problem. A transformation using the cause and effect perspective to the chute-chute train model would render an obvious solution.

Solution Problem #16 - Difficult High School Physics

Lectures by Walter Lewin. They will make you ♥ Physics. 2017

<https://www.youtube.com/watch?v=W0kSuEVK-sM>

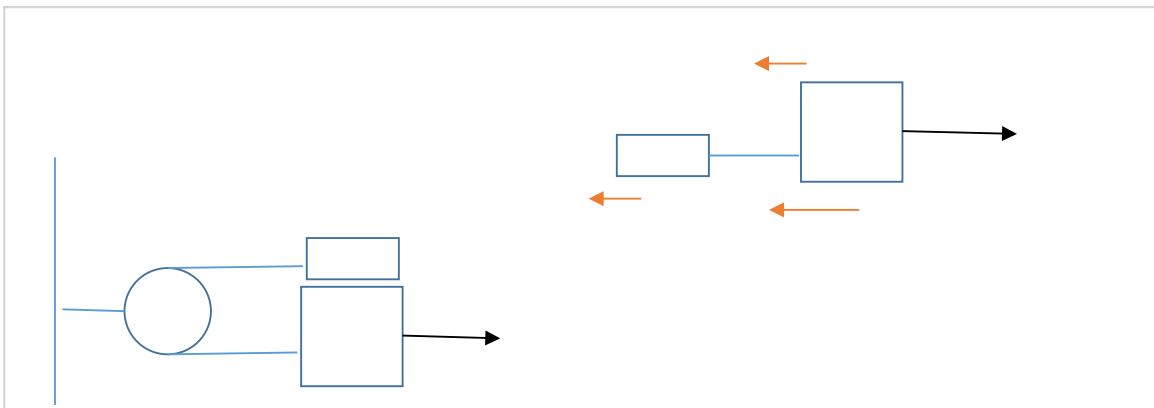


Figure Appendix-C-1: The setup of the vertical-stack problem.

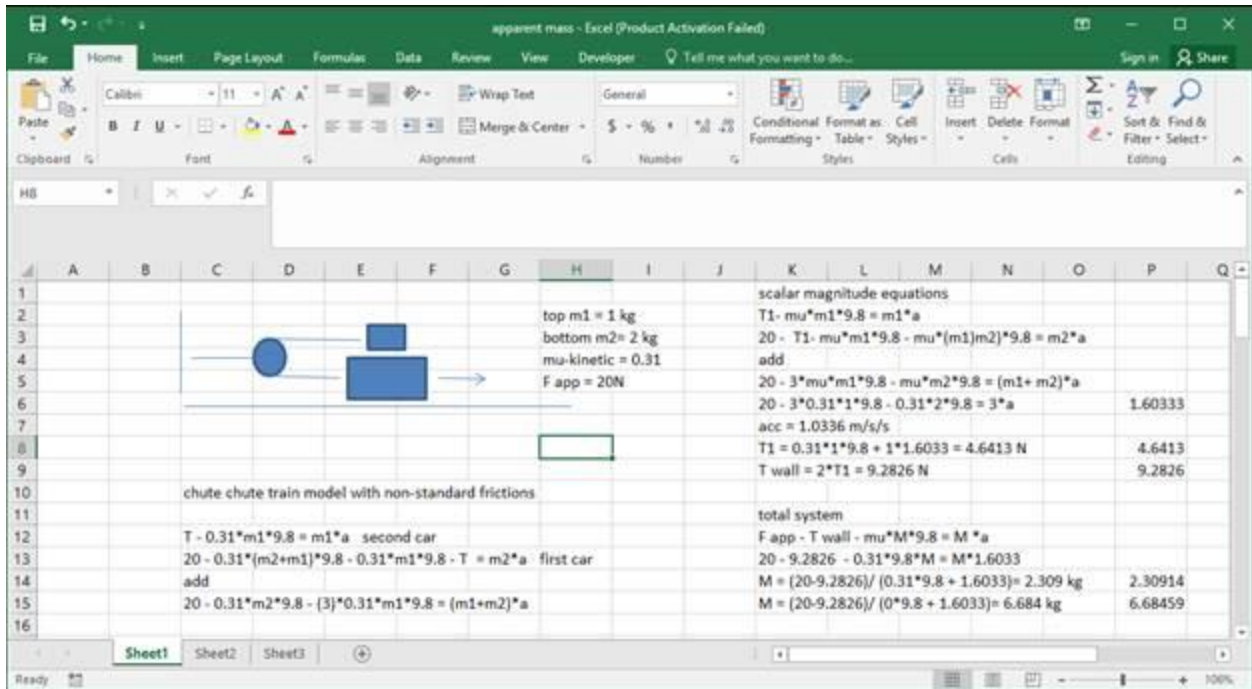


Figure Appendix-C-2: The Excel spatial-numeric tool solution of the Figure Appendix-C-1 problem. The chute-chute train model solution is shown in the lower left corner.

Appendix-D Two-ramp shortest time

The two-ramp shortest time problem is described. The multiple data columns would train working memory.

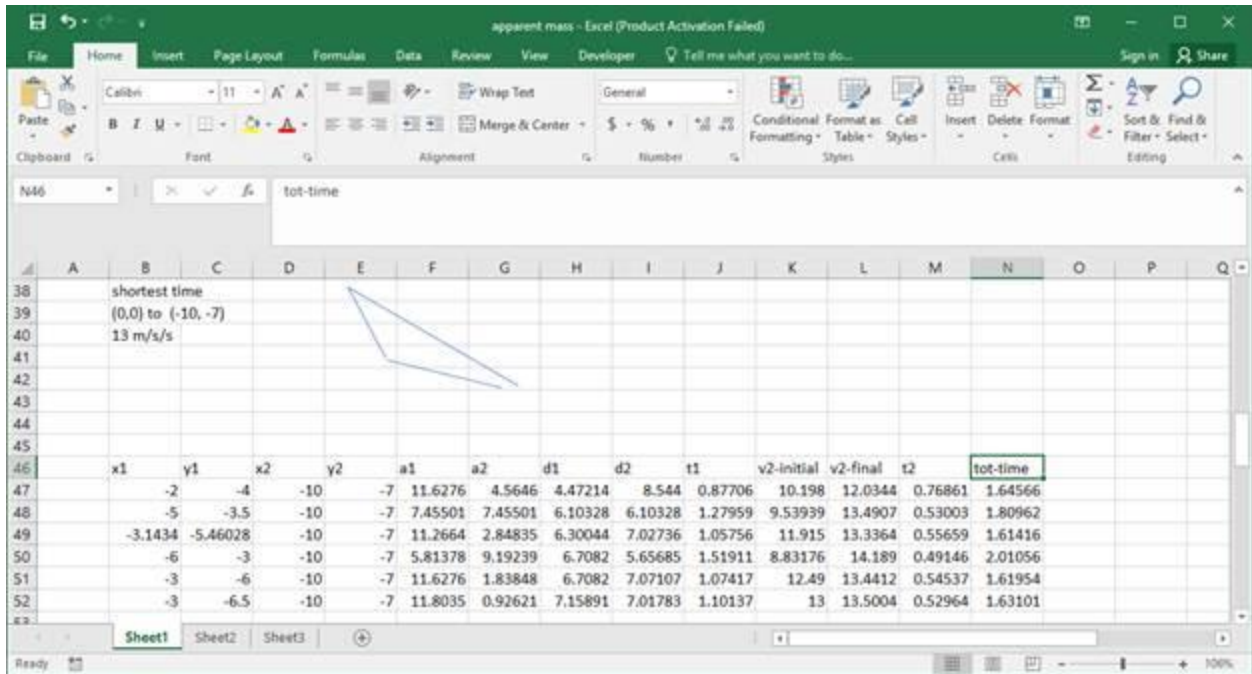


Figure Appendix-D-1: The Excel spatial-numeric tool setup with multiple data columns in the two-ramp shortest time problem for the training of working memory.

Appendix-E Object falling inside fluid in Excel approximation

The object falling in oil problem can be solved using difference equation when Calculus One is a co-requisite for calculus physics.

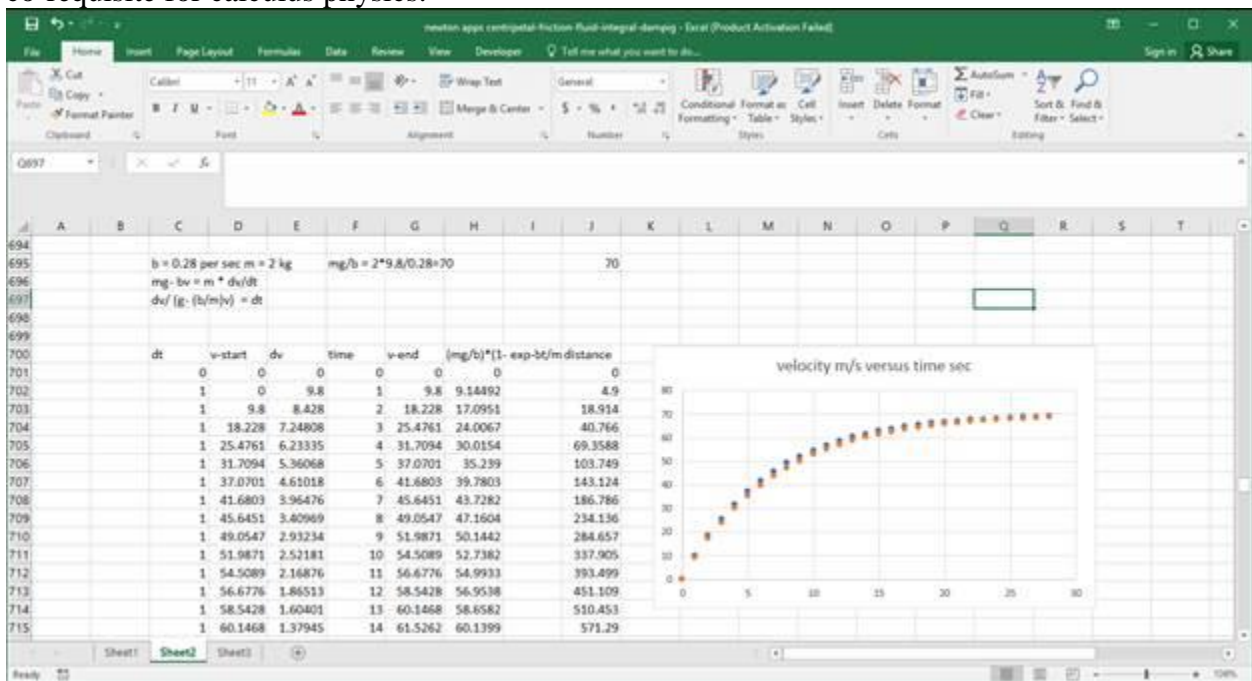


Figure Appendix-E-1: Excel difference equation solution for the differential equation expressed as $dv / (g - (b/m)v) = dt$. The blue upper curve is the difference equation solution.

Appendix-F Rocket velocity in Excel approximation

The rocket velocity problem can be solved using a difference equation when Calculus One is a co-requisite. Michel van Biezen Physics - Test Your Knowledge: Momentum (13 of 20) The V2 Rocket: Variable Mass <https://www.youtube.com/watch?v=Y3Q8WSuxk6E>

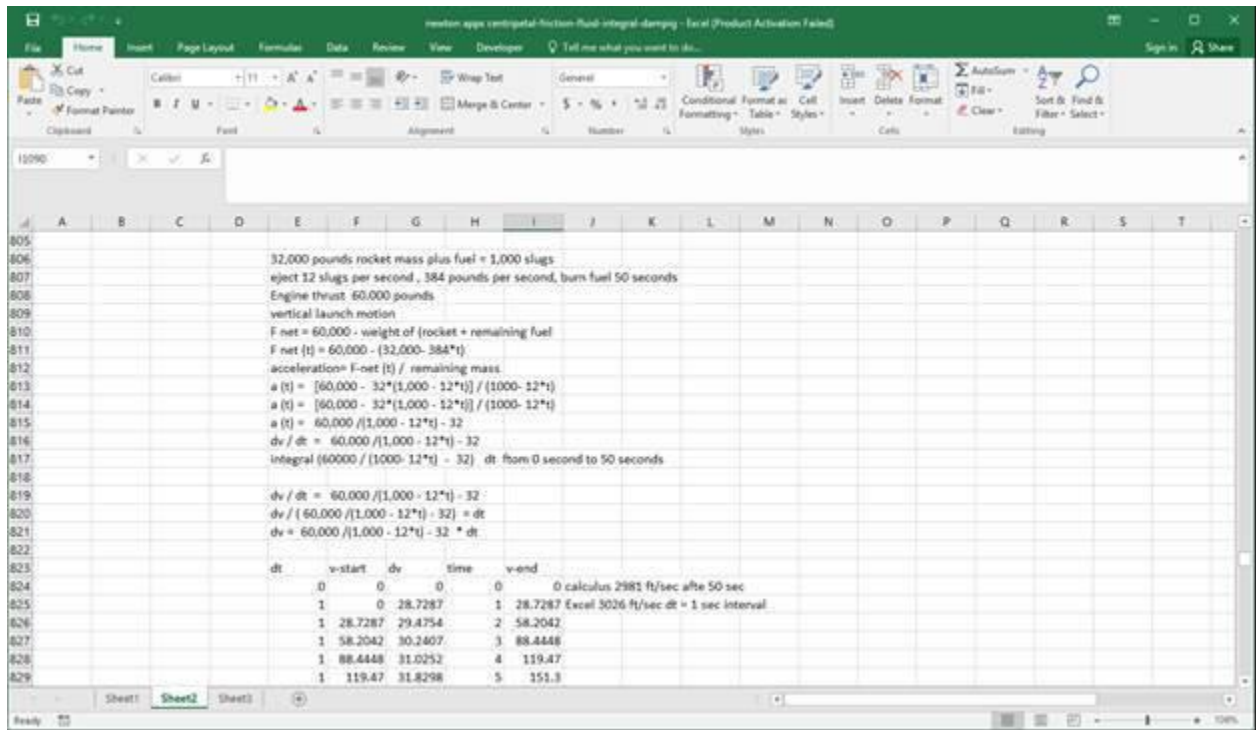


Figure Appendix-F-1: Excel difference equation solution for the rocket velocity differential equation expressed as $dv = 60,000 / (1,000 - 12t) dt$.

Appendix-G dog-walk-on-boat problem

The dog-walk-on-boat is a standard problem in the chapter on center of mass. The problem can be solved using momentum conservation. For instance, a 4.5-kg dog waked 2.4 meter on a 18-kg boat. We can let the dog distance be x and the boat distance be y in a certain time duration. A deduction using momentum conservation would be $4.5 * (\text{dog distance}) = 18 * (\text{boat distance})$. The answer of 18 meters for the dog and 4.5 meters for the boat would fit. For a smaller time duration, $(18 - x)$ for dog and $(4.5 - y)$ for boat must be still equal to $18/4.5$. Then knowing the relationship of $x + y = 2.4$ meters would solve the two unknowns with $x = 1.92$ m and $y = 0.48$ m. The spatial-numeric representation is illustrated in Figure Appendix-G-1. The concept of a smaller time duration is important. The method would help students to convert a differential equation into a difference equation in the analysis of time intervals.

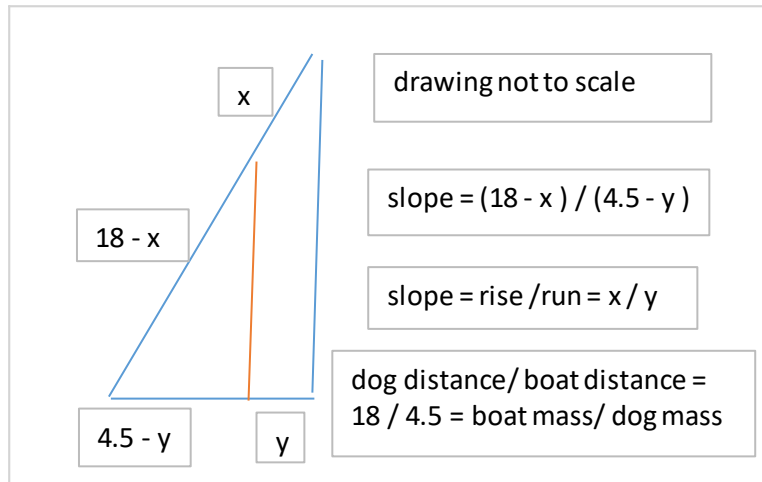


Figure Appendix-G-1: A graphic representation of the ratio of dog distance to boat distance using momentum conservation in two time intervals.

The use of relative velocity concept would also solve the dog-walk-on-boat problem. For instance when a student on a bus walked 2.4 meter towards the end of the bus while the bus moved y meters forward, the student would be $(2.4 - x)$ meters relative to a tree on the sidewalk. The one equation momentum conservation is shown in Figure Appendix-G-2. The same approach can also be used for the case of angular momentum conservation in which a merry-go-around rotates in the opposite direction to the direction of a person walking along the rim with an angular velocity magnitude relative to the merry-go-around. An extension to the movable wedge problem is included using the spatial-numeric tool in Excel shown in Figure Appendix-G-3. Using the law of scaling in the spatial-numeric method is shown in Figure Appendix-G-4. A geometry construction as an approximate solution (with graphic uncertainty) was proposed by us in a previous ASEE paper in terms of transferable knowledge. Note that a familiarity with geometry is applicable when solving orbital velocity vector problems. The orbital velocity vectors trace a circle in a hodograph (Wikipedia discussion of the Laplace–Runge–Lenz Vector) involving $V_a * V_p = V_b * V_b$ (with V_a and V_p magnitudes adding together as diameter) when V_a , V_p and V_b represent the velocity magnitudes at the positions of slow-aphelion, fast-perihelion, and semi-minor axis respectively. The spatial-numeric tool in Excel would offer an accurate solution with numeric inputs. The trend question on how the wedge speed changes as the mass moving along the ramp changes could be solved without the explicit use of calculus formulas.

Sunil Dehipawala, Vazgen Shekoyan, George Tremberger, Raul Armendariz, David Lieberman and Tak Cheung. An experiential learning strategy in introductory mechanics using transferrable knowledge from daily examples and feedback inquiry in the development of an innovative mindset. 2018 ASEE Mid-Atlantic Section Spring Conference.

<https://peer.asee.org/29453>




<https://peer.asee.org/an-experiential-learning-strategy-in-introductory-mechanics-using-transferrable-knowledge-from-daily-examples-and-feedback-inquiry-in-the-development-of-an-innovative-mindset>

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	AI	AJ	AK	AL	AM	AN
1						
2						
3						
4						
5						
6						
7		dog 4.5 kg walked left 2.4 meters on boat				
8		boat 18 kg moved right x meter from shore				
9		dog 4.5 kg walked left (2.4 - x) from shore				
10		assume per unit time, say 1 second				
11		momentum conservation $4.5 \cdot (2.4 - x) = 18 \cdot x$				
12		therefore $x = 0.48$ m/s for boat				
13		initial distant dog was 6.1 m from shore				
14		final distance = $6.1\text{m} - 2.4\text{m} + 0.48\text{m} = 4.18\text{m}$				
15						
16		Dog on a boat center of mass problem				
17		Physics Ninja 2018				
18		https://www.youtube.com/watch?v=BYqMURluf8s				
19						

Sheet1

Ready 102%

Figure Appendix-G-2: Dog-walk-on-boat problem solution as a precursor to the movable wedge problem solution.

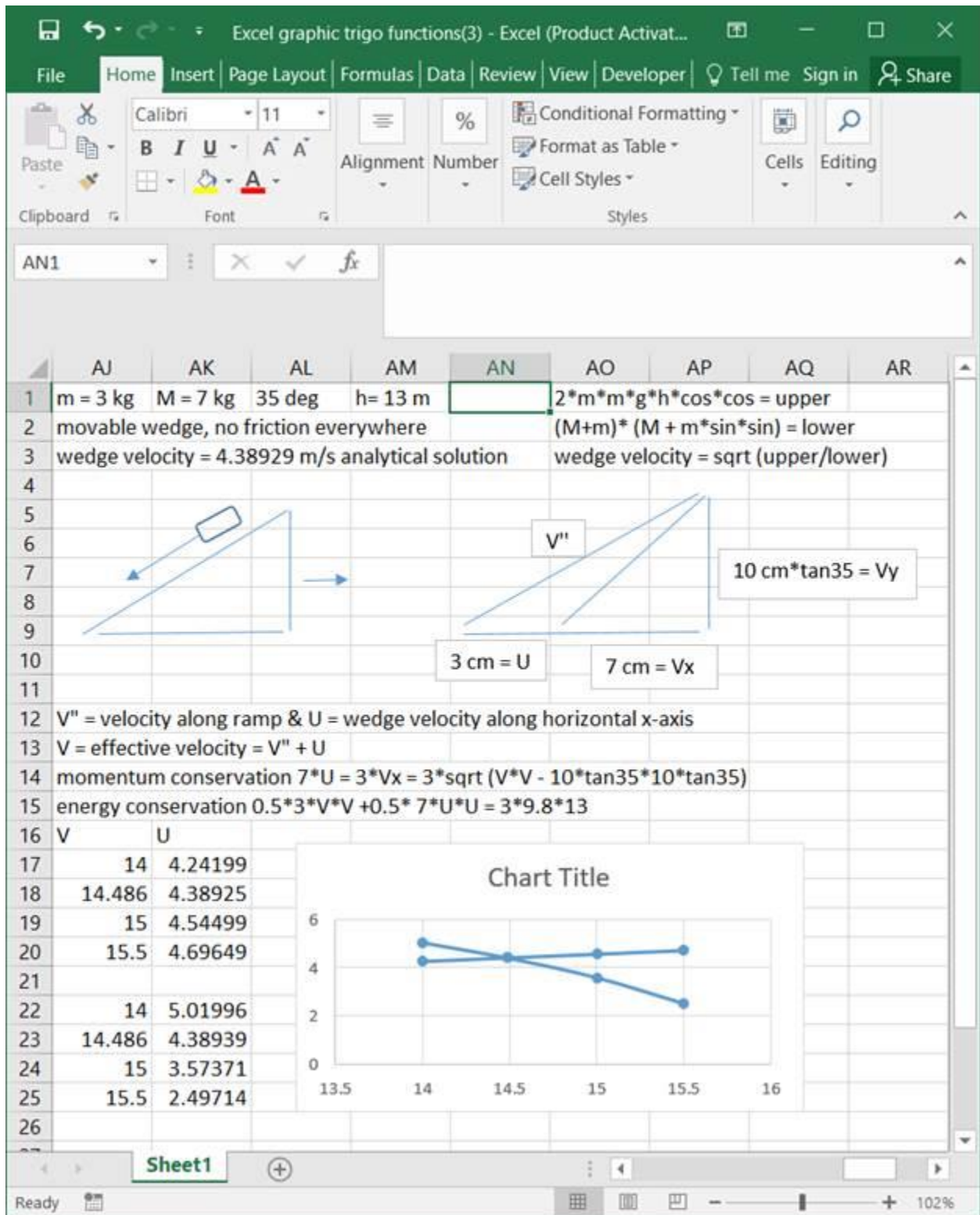


Figure Appendix-G-3: The movable wedge problem solution using the spatial-numeric tool in Excel. Youtube has a video of the movable wedge problem analytical solution offered by Professor Mooc Bogazici 2013.

<https://www.youtube.com/watch?v=xuXwWjbnVPg>

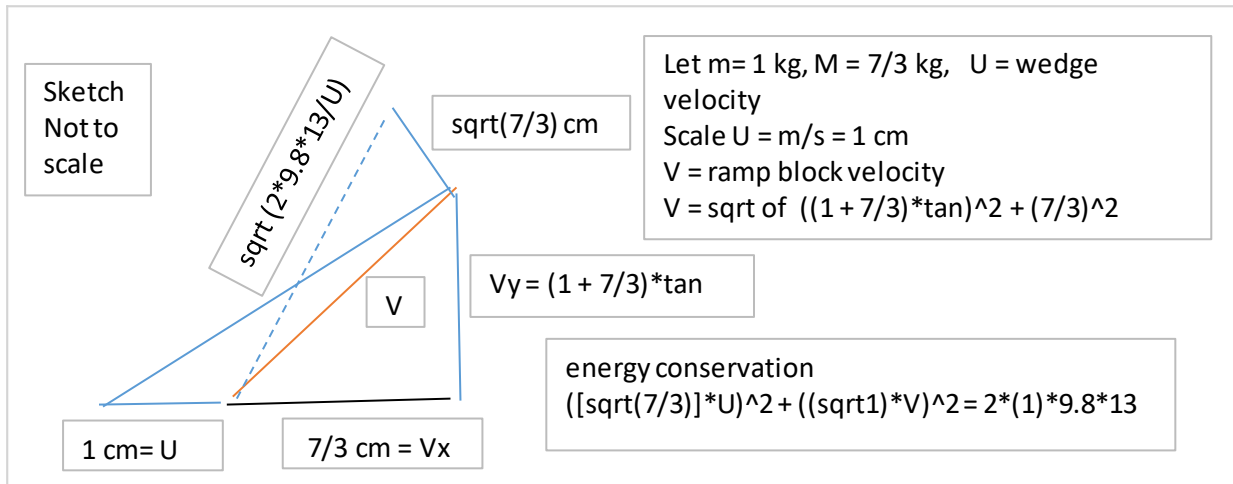


Figure Appendix-G-4: The movable wedge problem solution using the law of scaling with ramp block mass = 1 kg and wedge mass = 7/3 kg.

End of Appendices