Freshman Calculus in an Integrated Engineering Curriculum

David Barrow, Jack Bryant, Dante DeBlassie, Howard Seidel, Arlen Strader
Texas A&M University

INTRODUCTION

We are helping to develop, implement, and evaluate an integrated engineering curriculum that emphasizes technology, active learning in the classroom, and teaming. We will describe our experiences teaching calculus, during the past two academic years, to first year students in the integrated curriculum, which also includes courses in engineering, English, physics, and chemistry. This paper will focus on the areas of integration and computers, with primary emphasis on the differences between our course and the traditional freshman calculus course.

First year engineering students typically take introductory engineering, physics, calculus, chemistry and English courses. The students often see the courses as distinct entities. This view is reinforced by the fact that usually none of the faculty members know what the others are specifically doing day to day. Also, the order of presentation of material in a particular course is not influenced by the other courses. For instance, mechanics requires vectors, elementary differentation and integration, and line integrals. In a traditional calculus sequence, vectors and line integrals are introduced in third semester calculus. Although derivatives and integrals are presented in first semester calculus, the students use the ideas in physics well in advance of when they are covered in the calculus.

To remedy this situation, the faculty team met about three times a week during the summer preceeding each school year.[1] The purpose was to lay out our respective courses for the whole year. We also met at least once a week during the academic year to keep one another apprised of what and how we were doing. As a result, we radically changed the order of presentation of material in our calculus course, even to the point of moving a large amount of traditional second year engineering mathematics into the freshman year, and postponing some traditional first year material. The guiding philosophy was the belief that a much stronger impression is made in students' minds when they encounter the same or similar ideas, applications, terminology, or notation in two or more courses at approximately the same time, especially if they perceive a "need-to-know" for the material.

In addition to changing the order of presentation of material, we were able to achieve additional integration through two weekly sessions, which we called **Calculus Workshops** and **Maple Labs**. These classes, which will be described more fully below, frequently provided superb opportunities for students to see how mathematics relates to the other disciplines.

A novel feature of our program was our mode of testing. At the end of every three weeks, we gave exams during parts of Thursday and Friday. These consisted of subject exams for each course, which the students took as individuals, and a cross-disciplinary "integrated" exam. In the integrated exam, they were given a problem and instructed to work together in teams to find a solution. The problem was constructed in such a way that no individual working alone could finish in the alloted time.

Assessment of our program is understandably incomplete, since we are in its early stages. The paper[2] describes the some first efforts at evaluating the program, mainly through pre- and post-testing of students in the program and in comparison groups. In all tests used, achievement by students in the integrated program was equal to or above that of the comparison groups. In addition, we have put test items from traditional courses on our mathematics exams. The performance of our students has been a bit better than that of students in the traditional sections. We cannot say at this time if this is due to chance or our program. We plan to track our students through their college careers and see how they perform in upper division courses relative to students not in our program.

The remainder of the paper will describe some of these features in more detail, and will conclude with some specific examples of workshop and lab activities.

KEY FEATURES OF THE COURSE

- **Rearrangement of Topics**

The driving force was physics, and to a lesser degree, engineering. In the first year of the program, physics was given in a two hour course each semester, covering mechanics and introductory thermodynamics. In the second year of the program, physics was expanded to a three hour course each semester, and included electricity and magnetism along with mechanics and thermodynamics. In both years, we introduced vectors from the start and we taught the students about derivatives and indefinite integrals of polynomial functions very early. We did this in a short period of time, postponing differentation and integration formulas for other elementary functions to later in the semester. Limits were introduced very naturally in the explanation of derivatives.

About two thirds of the way through the fall semester of our first year, shortly after covering the Fundamental Theorem of Calculus, we introduced the students to line integrals (two lectures). In our second year of the program, largely because of the expanded physics content, we included line integrals, 2D and 3D intergals, and surface integrals (six lectures). These topics are traditionally sore points for students in physics because they do not see them at all in traditional first year calculus. Our coverage was very introductory in nature. We prepared a supplement to the text to bridge the gap between the students' background at the time and the more advanced treatment of these topics in the text. To prepare for these topics, we used computer labs and workshops in weeks 3-5 to familiarize the students with parameterizations of curves and surfaces (see Example 1 below.) In the second semester we postponed the treatment of infinite series and most techniques of integration until third semester calculus, replacing them with the vector calculus needed for physics, and elementary ordinary differential equations.

There was much more emphasis in our course on numerical methods than in the traditional course, such as numerical approximation of derivatives and integals, and least square curve fitting of data. This was largely due to the needs of the enginering courses, and was reinforced by some of the integrated exams (see below). Another nice tie with engineering was with programming. Students were learning Fortran and logical processing there, and we pointed out some similarities between the computer algebra system Maple and Fortran programming, such as **do** loops, **if** statements, and subprograms.We then illustrated programming with numerical integration and Euler's method.

- **Dynamic Design of Calculus Workshops and Maple Labs**

In the traditional course, students meet once a week with a teaching assistant in a recitation class for review, homework discussion, and quizzes. In addition, there is a computer laboratory once a week, the primary purpose of which is to learn and use Maple. Our course uses the same structure, but we have a different emphasis. The recitation class is called the Calculus Workshop, and typically involves team activities. An activity is presented at the beginning of the period, to be completed during the period, using computers when appropriate. This offers numerous opportunities to make connections with their other courses. We also use this period to review for exams by giving a practice exam every exam week.

We also have a Maple Lab each week. This lab also involves a team assignment,but, in contrast with the workshop activities, is usually due a week or two later. Although the primary purpose of the lab is to learn Maple and to use it to help learn calculus, we also see it as another opportunity to increase the integration of calculus and the other courses. In the **EXAMPLES** section below we illustrate some typical Calculus Workshop and Maple Lab activities.

We wish to emphasize the dynamic, flexible nature of the workshops and labs. By this we mean that they are definitely not "off the shelf", but must be planned based on conditions which are difficult to predict. The specific activities in the other courses are always changing, and as we attempt to make connections with them and with what they are covering, we must remain very flexible. To accomplish this requires close coordination, both among the mathematics instructors and teaching assistants (the math team), and the interdisciplinary faculty team. We have weekly meetings of each group. The math team meets to plan future Calculus Workshop and Maple Lab activities, after someone on the faculty team reports what is happening (or will be happening, or has just happened) in the other courses.

In the syllabus the lectures are laid out completely. We use the labs and workshops to reinforce difficult concepts, to introduce new ideas, or to discuss tangential concepts. Some of the lab and workshop assignments were designed based upon student reactions to other assignments, such as previous labs, homework, and tests. For example, about a third of the way into the the second fall semester we learned that many of the teams had several members who did not take the lab work, and especially Maple, seriously. This resulted in those students allowing other team members to do the work while not making any attempt to learn the material themselves. This is a common problem in teaming situations so we decided to address this problem. We started giving individual quizzes and put Maple questions on the exams. Although this did not completely solve the problem, we at least convinced most of the students they needed to learn Maple themselves rather than rely on others. By the end of the semester most of the students had a working knowledge of Maple.

Another example came about as a result of student comments. Many said they wanted a concise readable description of common Maple commands. As a result, we gave an assignment which addressed this problem in a way the students had not expected. Each person had to choose an aspect of Maple and write a short report. These reports were then made available on the World Wide Web, at the address

http://calclab.math.tamu.edu/~strader/math151/proj1/

- **Integrated Exams**

Every three weeks a round of exams is given, and one of these exams is an integrated exam. The usual characteristics of the integrated exams are as follows:

1. They last an hour, and are worked by four-person teams, with computers available.
2. They are completely new problems, not variations of problems they have seen before, and involve material from two or more of their courses.
3. The integrated exam grade constitutes 25% of each team member's course exam grade, for all courses.
4. The problems were constructed in such a way that no individual working alone could finish in the alloted time.

The exams are constructed as follows. A few days before the exam, in a regular faculty team meeting, instructors are reminded of what has been covered recently in all the courses, and asked to submit possible problems. At a subsequent meeting, the submissions are discussed, and one is selected, although it usually is not exactly the same as originally submitted.

These exams provide yet another opportunity to integrate calculus with the other subjects, even after the exam has been given. For example, one exam concerned analyzing numerical data from a golf ball's trajectory, and this was the subject of the next Maple Lab (see Example 2 below).

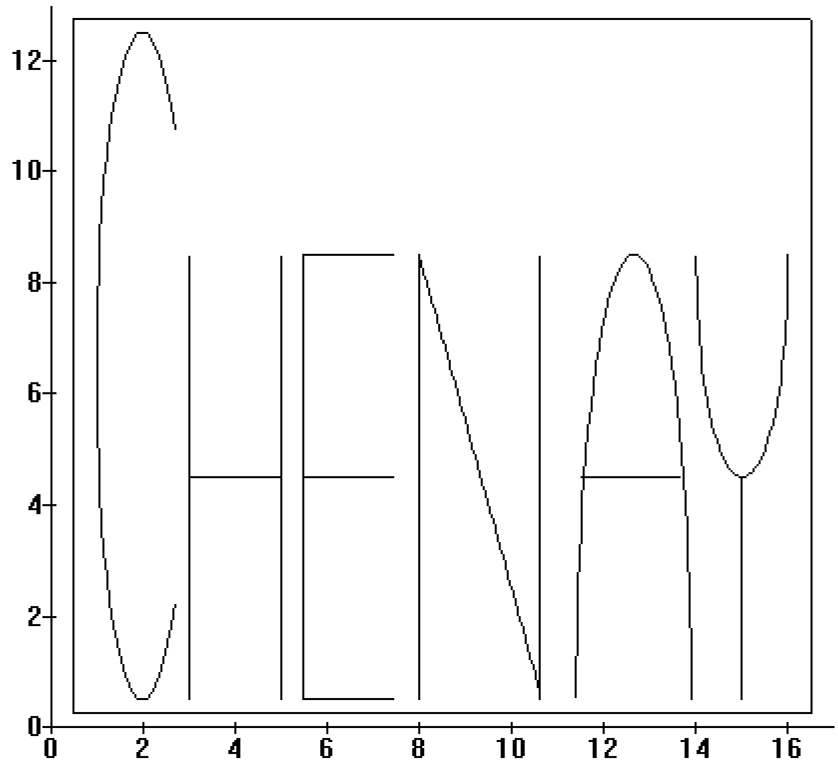
EXAMPLES

1. Using Parametric Equations to Write Names

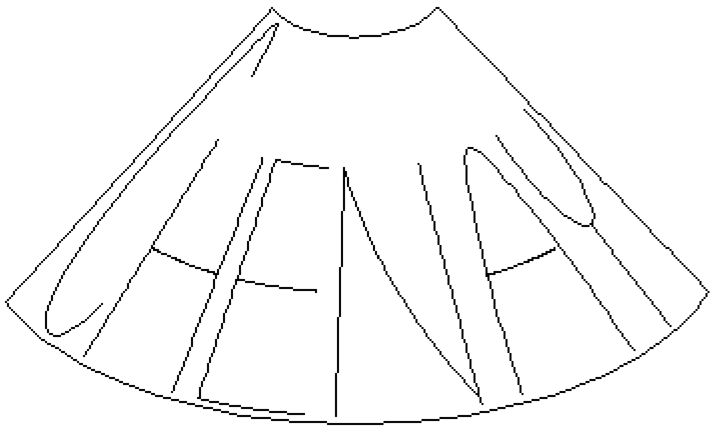
In the second year of the program, we knew we were taking a somewhat radical step in trying to introduce both line and surface integrals in the first semester, even though we felt we had been successful in covering line integrals in the first year of the program. Of course, the main reason we believed it could and should be done was that the students were seeing these concepts in their physics course. As mentioned above, we prepared supplements to the text to help provide as gentle an introduction as we could, in the tenth week of the freshman year, to these topics which are traditionally covered in third semester calculus. To help prepare the students for this material, we decided to use Maple to give them some familiarity with parametric equations of lines and surfaces (the approach we were planning to take to line and surface integrals requires representing curves and surfaces parametrically). The first exercise was for each student to find the parametric equations for curves and lines that would spell out their first name, and to plot them. The next exercise was to plot their names on a surface. Everyone was able to complete these exercises, with some turning in quite creative examples. We list below a typical student's solution to the problems:

```
>with(plots):
>a[1]:=cos(6*Pi*t/4+Pi/4)+2:b[1]:=6*sin(6*Pi*t/4+Pi/4)+6.5:
>a[2]:=3:b[2]:=.5+8*t:
>a[3]:=5:b[3]:=.5+8*t:
>a[4] :=3+2*t:b[4]:=4.5:
>a[5]:=5.5:b[5]:=.5+8*t:
```

```
>a[6]:= 5.5+2*t:b[6]:=4.5:
>a[7]:=5.5+2*t:b[7]:= 8.5:
>a[8]:=5.5+2*t:b[8]:= 0.5:
>a[9]:= 8:b[9]:=.5+8*t:
>a[10]:=8+2.625*t:b[10]:=8.5-7.875*t:
>a[11]:=10.6:b[11]:=.5+8*t:}
>a[12]:=1.25*cos(Pi*t)+12.65:b[12]:=8*sin(Pi*t)+.5:
>a[13]:= 11.52+2.145*t:b[13]:=4.5:
>a[14]:=cos(Pi*t)+15:b[14]:=-4*sin(Pi*t)+8.5:
>a[15]:=15:b[15]:=.5+4*t:
>a[16]:=.5+16*t:b[16]:=.25:
>a[17]:=16.5:b[17]:=.25+12.5*t:
>a[18]:=.5+16*t:b[18]:= 12.75:
>a[19]:=.5:b[19]:=.25+12.5*t:
>chenay1:=NULL:
>for i to 19 do
>chenay1:=chenay1,[a[i],b[i],t=0..1]:od:
>plot({chenay1},0..17,0..13);
```




```
>chenay2:=NULL:
>for i from 1 to 19 do
>z:= -1.25*Pi*b[i]/13 + 5:
>x:=z*cos(.8*Pi*a[i]/16):y:=z*sin(.8*Pi*a[i]/16):
>chenay2:=chenay2,[x,y,z,t=0..1]:
>od:
>spacecurve({chenay2},scaling=constrained,orientation=[75,-60]);
```



2. Analyzing the Trajectory of a Golf Ball

About halfway through the first semester of the second year of the program, the topic of trajectory motion was covered in the physics, math, and engineering courses. The second integrated exam was then constructed based on trajectory motion. It involved calculating the drag coefficient of a golf ball based on supplied trajectory data.

The students were presented with a data file containing 127 data records for the flight of a golf ball. Each record contained the time and x and y coordinates of the ball. From this information they were asked to produce plots of the golf ball's position and velocity, both theoretical (no air resistance) and actual plots . They were then asked to determine its acceleration and from this calculate the drag coefficient of the ball.

This problem required them to apply their knowledge of first and second derivatives to a real world type problem. Instead of being given analytic expressions for functions and taking derivatives, they had to utilize (simulated) data and use finite differences to approximate the derivatives. They also derived a theoretical function for the flight of the ball (without air resistance) based on the ball's initial velocity as estimated from the data. After plotting the data they could then compare the theoretical and actual trajectories on the same graph.

The results of the exam were not considered satisfactory by the faculty team. Most of the students either used a spreadsheet to do their finite difference calculations, or did them incorrectly. This, then, presented an opportunity for the math team to design a Maple Lab that guided them through a solution to the exam. All of the data analysis and graphics were done by using simple Maple commands and programs. They produced plots showing that the actual flight differs from the theoretical result without air resistance. The students were able to see that this departure from theory agreed with their common sense knowledge of air resistance. Most of the teams were also able to correctly calculate the drag coefficient.

Labs such as these allow the students to put their knowledge to use on problems which bear more resemblance to the real world than the typical first year fare. They also learn advanced skills, such as finite difference formulas in this case, not usually taught to students at this level. All of these factors teach them the ability to apply their knowledge in several areas to solving problems.

3. Writing and Using Student-Produced Maple Labs

This activity was used in both years of the program, and proved to be one of the most-liked lab activities. The assignment was in two parts. The first part, which comprised two weeks of their lab and workshop classes, was for each team to write a Maple Lab to teach some topic, of their choosing, to another team. We gave them some suggested topics, but required that the subject be approved and be something from either their math, physics, or engineering courses. They were to provide an explanation or background of the topic, some examples of using Maple to illustrate it, and some exercises. This was a format they were familiar with, since we had used it for all Maple Lab activities we had previously given to them.

Some of the topics selected were: applied maximum and minimum problems, the method of least squares, line integrals and work, finite difference approximation of derivatives, Newton's second law of motion, applications of derivatives and integrals, and the work-energy theorem. The second part of the assignment was to work through a Lab that had been written by another team, and to make constructive criticisms.

REFERENCES

[1]Barrow, D., Bassichis, B., DeBlassie, D., Everett, L., Imbrie, P., and Whiteacre, M., "An Integrated Freshman Engineering Curriculum, Why You Need It And How To Design It", FIE Conference, November 1-5, Atlanta, 1995 (http://fre.www.ecn.purdue/edu/fre/fie95/3c1/3c12/3c12.htm).

[2]Willson, V., Monogue, T., Malave, C., "First Year Comparative Evaluation of the Texas A&M Freshman Integrated Engineering Program", FIE Conference, November 1-5, Atlanta, 1995 (http://fre.www.ecn.purdue.edu/fre/asee/fie95/3a6/3a64/3a64.htm).

DAVID L. BARROW

David Barrow is an associate professor in the Mathematics Department at Texas A&M. He has a B.S. in aeronautical engineering from Oklahoma State and four years' engineering experience in jet propulsion with the U.S. Air Force. He has a Ph.D. in mathematics from the University of Michigan. He has co-authored two Maple laboratory manuals, and has research interests in differential equations and numerical analysis

JACK BRYANT

Jack Bryant received the Ph.D. in mathematics from Rice in 1965. Since then his interest has been centered on mathematical modeling and pattern recognition. Dr. Bryant is an associate editor of *Pattern Recognition*, and has published numerous articles on image understanding.

DANTE DEBLASSIE

DeBlassie has a B.S. in mathematics and physics from New Mexico State University. His Ph.D. is in mathematics from the Massachussetts Institute of Technology. Currently he is an associate professor in the Department of Mathematics at Texas A&M University. His research interests include diffusion processes and stochastic differential equations.

HOWARD SEIDEL

Howard Seidel worked for ten years as a systems analyst in the insurance industry attending college part-time. He received his B.S. in Mathematics in 1994 and entered Texas A&M University the same year. . He expects to obtain a M.S. in Applied Mathematics this year and will then pursue a Ph.D. in Physical Oceanography.

ARLEN STRADER

Arlen Strader graduated from Bob Jones University with a bachelors degree in Mathematics. He is currently working on a Ph.D. in Mathematics at Texas A&M University in the area of Scientific Computation. He has worked with the Integrated Engineering Coalition for three semesters.