Integrating Engineering Design and Applied Probability

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ABSTRACT

Integration of engineering design into an applied probability course is discussed. The course, which is required for the industrial engineering major at Northern Illinois University, presents basic probability, simple reliability models, Markov chains, the Poisson process and simple queueing systems — standard fare for industrial engineering majors. Engineering design activity has been incorporated into the course through

- 1. homework and test problems that emphasize parametric analysis, variations of standard models, and comparison of alternative systems;
- 2. loosely stated open-ended problems intended to allow creative response, with the hope that the students will model the physics of the problem situation, identify economic and ethical constraints, and find a way to base decisions on quantitative analysis.

This paper focuses on the second type of problems, labeled "design problems" in the course syllabus. Examples are given, with descriptions and analyses of the students' responses and their suggestions for improving the content and presentation of future design problems. Special attention is given to an exceptionally alert and well-motivated class that was willing to discuss the nature of design and evaluate the course. The paper also contains examples of homework and test problems intended to encourage creative, independent thinking.

INTRODUCTION

Combining theory and design in a single course is a difficult challenge. There seems to be an innate conflict between obtaining a sound theoretical basis and actively practicing design. If too much time is devoted to theory, students may acquire only a vague idea of how the concepts can be related to engineering practice. As a result, they will not initiate applications of the theory. If too much time is devoted to design, students may not acquire the deep and solid understanding needed for effective design.

To resolve this conflict, I believe that the entire course should be taught with a "design attitude." That is, the entire course should be supportive of design activity. The material should be

presented from a design point of view, and the students should be given problems that foster a design point of view. More specifically, the following three strategies should be used:

- 1. In so far as possible, the theory should not be presented as a fixed body of knowledge, but as something that is being constructed for engineering purposes and therefore subject to manipulation and change the product and object of design activity. Historically, this is the case for a large number of the mathematical concepts taught to engineers. This strategy places more emphasis on "open mathematics" than "closed mathematics" as recommended in Kapadia and Borovcnik⁵.
- 2. Homework and test problems should emphasize parametric analysis, variations of standard models, comparison of alternative systems, cost analysis, and decision criteria.
- 3. The students should be given loosely stated open-ended problems intended to allow creative response, and they should be allowed to respond experimentally without the fear of lowering their grades.

The rest of this paper enlarges on these strategies as applied to teaching applied probability to industrial engineering students.

THE COURSE

I am integrating engineering design into an applied probability course that is required for the industrial engineering major at Northern Illinois University. This course is called Operations Research II. Its prerequisites are an introduction to statistics and a course in deterministic operations research models. The contents of the course are basic probability, simple reliability models, Markov chains, the Poisson process, and simple queueing systems. The basic probability portion of the course is intended to reinforce and deepen knowledge of the probability concepts taught in the preceding statistics course. The reliability portion is meant to further strengthen knowledge of basic probability concepts, while at the same time introducing an important application. The remainder of the course is completely new material for most of the students. It is a standard part of the industrial engineering curriculum, although I suspect that very few students who learn it in a traditional way will ever apply it.

As theory, probability is rich and beautiful, and students should be taught to appreciate it. They should also, however, be taught to connect the theory to engineering problems. Engineering students have an active interest in problem solving. This makes it tempting to assume that they will make automatic connections between theory and applications. In my experience, however, they have the same inability to do this that students have in general. (See Gardner⁴ for a discussion of this inability.)

To help engineering students make a connection between theory and the engineering workplace, theory should be connected to engineering activity not only through illustrative examples, but also in the way that fundamental concepts themselves are presented. For example, the concept of a *sample space* is fundamental to probability theory. (A sample space is the set of all possible outcomes in a random experiment.) Since decision analysis is important for industrial engineers, I present a sample space as something a decision maker might create when mapping out all the possible scenarios confronting him or her when a decision must be made. I also emphasize the

fact that probabilities are choices that can be manipulated by actions, and that they change as more or less information becomes available. Similar connections to industrial engineering are made throughout the course.

SHORT PROBLEMS

The homework and test problems are intended to build intuition and confidence, and to encourage a manipulative and investigative attitude. This is done by emphasizing parametric analysis, variations of standard models, comparison of alternative systems, cost analysis, and decision criteria. For example, the venerable six-sided dice with one to six dots on the sides can be changed to have an arbitrary number of sides, nonconsecutive numbers on the sides, colors on the sides, or any combination of these characteristics. Further, decision problems can be formulated: if the numbers are variables and we can pay to set them, what numbers should we choose?

In the appendix, Examples 1 through 3 further illustrate this approach.

DESIGN PROBLEMS

The third strategy for integrating design activity in the course is to assign loosely stated openended problems. These are labeled "design problems" in the course syllabus. The students are given minimal help with these problems. To make them feel safe to make genuine personal responses, any written response to a design problem is guaranteed a grade of at least C. To make them realize the importance of the design problems, the design problems count for twenty percent of the course grade. (Another twenty percent of the grade is based on the rest of the homework.) There are usually four design problems per semester. My hope is that the students will model the physics of the problem situations, identify economic and ethical constraints, and find a way to base decisions on quantitative analysis. An example of a design problem is given in Example 4. This problem parameterizes a problem in Clarke and Disney's² textbook. This problem has a straightforward solution that leads to puzzling decisions — there is something wrong with the decision criterion. Over several years, only one student has recognized this and suggested a way to fix it.

THE CLASS OF SPRING 1995

Initial Expectations

At the beginning of the semester, the students made written responses to the question "What do you hope to get out of this course?" Their responses show a mix of attitudes and levels of sophistication, but a decided overall interest in problem solving. Here is the complete set of responses, with minimal editing:

- I just hope that I don't leave this course saying that this course was a waste and that I'll never use this stuff ever again.
- Problem solving techniques which can be applied to my future work interests.
- A better understanding for operations research and a grade.
- I hope to learn more about modeling and solving all kinds of real world problems.

- A broader understanding of Industrial Engineering. To learn new techniques for problem solving.
- Accumulation of information on how to research into a problem and then how to relate this into design of a simple system to solve.
- I hope to gain insight on stochastic models and eventually use this information when I'm actually working in the field.
- An A.
- I hope to build on what I learned in the previous course and to end up with the complete understanding of operations research and its various applications. And of course a good grade wouldn't hurt any.
- I hope to broaden my knowledge of operations research techniques, increase my problemsolving skills, and give myself a strong base in operations research for future courses and my career.
- The ability to interpret and solve any dynamic programming problem presented to me.
- Useful information in solving operations research problems.
- I hope to get a better understanding of what operations research is and where operations research can be applied in real life settings.
- As much as possible.
- Some practical techniques that I can apply to my job. I also would like to receive a grade of an A or B.

Discussion of the Design Process

Before assigning the first design problem, I led the class in a short discussion on the nature of engineering design. The class lightheartedly decided that design is a process that takes "Nothing," runs it through a black box called "Creative Powers," thus producing "Something." I decided to let them do more or less exactly that.

The Sequence of Design Problems

The first design problem (Example 5) was accompanied by some statements on the nature and purpose of design, including the ABET statement on engineering design and its importance in the curriculum. The design problem itself is faulty: it is difficult to relate it to the control of a realistic production process. Some of the students made good suggestions for controlling production processes, but most did not try to extract a real problem from the problem statement. Consequently, for the second design problem the students were asked to produce an improved description of the problems hinted at in the first design problem, including all the things that need to be quantified and some methods for quantifying them. Their responses were much clearer. Discussion of what should be included in responses to design problems continued throughout the remainder of the semester.

The third and fourth design problems were much more clear cut than the first two. The third problem was to decide how many safety switches should be installed on a device subject to catastrophic failure. The fourth was to redesign an inventory ordering policy after a change in the demand pattern.

The following was observed in the students' responses to all four design problems:

- Identification of options beyond those clearly implied in the problem statement.
- Identification of flaws in the problem statements.
- Creation of imaginary situations in which to house their solutions.
- Identification of costs, benefits, and tradeoffs.
- Creation of symbols for costs and other variables.
- Creation of formulas (correct or incorrect) for probabilities, expected costs, and so forth.
- Recognition of the need to collect data.
- Creation of the kinds of solutions that might be found in textbooks.
- Extensions of the problems to make them more meaningful.
- Identification of situations where the problems are real.
- Introduction of decision criteria.
- Recognition of ethical and safety issues.
- Intuitive understanding.
- Vague answers and just plain wrong answers.

Students' Evaluations of the Design Problems

In the spring of 1996, when the students were nearing the end of their senior design project, I asked them their opinion of the design activity. (One or two of the respondents had taken the course before 1995, but had still been assigned design problems.)

The students were given a memo from me whose subject was "Improvements to Operations Research II." The memo contained the following preamble:

The design problems you did in Operations Research II were supposed to:

- shift your focus away from books and teachers onto your own ideas;
- give you an opportunity to think experimentally in a relatively risk-free environment:
- encourage you to develop problem solving skills;
- encourage you to adopt a manipulative, investigative attitude towards the concepts you learn in school;
- show you that the above four goals are important.

In the spring of 1995, there were four design problems:

- the blisters and the mold:
- the blisters and the mold refined;
- a reliability design problem (how many switches?);
- an inventory policy design problem.

I am interested in your reaction to this kind of course activity.

This preamble was followed by these questions:

- 1. Do you agree with the purpose of the design problems? Why or why not?
- 2. Do you think the design problems served their purpose? Did they help you to adopt an independent creative attitude? Were they helpful preparation for your senior design project? Why or why not?
- 3. Do you think the design problems were helpful in any way at all, possibly not related to their intended purpose? If so, how?
- 4. Do you think the design problems were harmful in any way? If so, how?
- 5. Do you think the design problems should be continued? Why or why not?
- 6. Please list any suggestions you may have for improving the design problems or the design activity.
- 7. Please list any ideas you may have for future design problems.
- 8. Please list *any* ideas you may have for improving the course.

The overall response was positive, although there may be some bias due to the fact that I maintained a strong rapport with most of these students after the course was over. The entire set of responses to questions 1 and 2 is given below:

Responses to Question 1

- I do believe it helped take our focus away from the book and allowed us to look at problem creatively. They help explore our problem solving skills.
- Yes because it sounded like real life situation.
- Yes, the design problems did accomplish these goals to an extent. I think the focus of the problems should *somehow* be made clearer to try and avoid the *confusion* of the students.
- Yes, they were analytically involved problems. Each was capable of pushing one to think out a logical end to an open ended question.
- Yes, I totally agree with the purpose of the design problem! It is good to steer students away from the concepts of pouring information into them and then having them chug it back up! Creativity and problem solving skills is essential.
- Yes, it brings real-world type problem solving skills into the classroom.
- Yes, I agree with the purpose(s) that the design problems were geared towards. The problems made us think analytically, and outside the constraints of the textbook format.
- Yes, I agree with the purpose. But the structure in order to ensure the purpose should have included presentations maybe.
- Yes, they helped you apply course work to a real life situation.
- Yes, it gave us a chance to look at practical problems and solve them to the best of our ability. There were not any right or wrong answers.
- I was always confused as to the purpose of the projects.
- Yes. These problems did give us as engineering students, the ability to use our own thinking processes instead of following set formulas.
- Yes, I agree with these purposes. The design problems were an important element of the course and stimulated creative thinking.
- Yes, they made you think about an open ended problem. Something more "real-world" than text book problems.

- I agree with these problems. It does give the students an opportunity to think independently on their own. In other homework, there are references to books, etc. and examples of those types of problems.
- Yes, the purpose of the design problems was effective. It was the first real exposure we had of the types of problems we would face in the future.

Responses to Question 2

- Yes. It allowed me to look at the problem from several different way, and find what I thought to be the best solution.
- I wish we had more variety of design problems to cover more of the IE diverse areas.
- The creativity is definitely there, but the preparation for senior design is difficult because no one understands what senior design really involves.
- For myself, I think they were a reinforcement for a way for thinking and learning.
- I think they tried to serve their purpose. It did make me somewhat think about how to solve problems. I don't think they were helpful in doing senior design because there are more guidelines to follow.
- The problems seemed a little abstract. I couldn't relate them to my experiences in school or in work.
- I think if the design problems were worth more of the grade, the student would put more effort and creativity!! I know that you're trying to implement a risk free thinking environment. But honestly, most students only care to get it done!!! As I look back I realize that it was more than just "getting it done."
- Yes, yes, the real world work force is not out of books, it's out of people's ideas.
- Yes they did. The problems weren't related to my senior design project, but someone with similar goals/problem descriptions could benefit and should have benefited from the design problem essays.
- It helped in getting me away from books. As far as being creative, I don't see exactly where that came into play on a short term project. Yes, they were helpful for senior design. It helps to take what you've learned and apply it to real life.
- I think they did in a way. But the answers you wanted were too open-ended. I think a more structured approach would make students put more time into them. They were not helpful in preparing for senior design because my project is completely different.
- Even though I am not using the course in my senior design, the class did help me to think of a project as coming up with a solution and not a right or wrong answer.
- They did adopt creativity, but I wish the problems were more interesting.
- Yes, again, they assisted us to think for ourselves to determine alternative solutions.
- The design problems were of benefit to me in my senior design project because they gave me practice in structuring an "open-ended" problem, and gave me the confidence that I could do so.
- Yes, they were a good stepping stone between school work and senior design because of the fact there was no right answer and you had to be creative.

SUGGESTIONS FOR IMPROVEMENT

I see my experience with design problems so far as partially successful. I think the quality of learning will improve if students participate more in constructing the learning environment. To accomplish this, I am considering the following:

- more open discussion of creativity,
- having the students keep journals (as suggested by Countryman³),
- class presentations of solutions,
- classroom discussion of design problems,
- design problems done as teams,
- classroom sessions using techniques to stimulate creativity, such as mindmapping (Wycoff⁶).

CONCLUSION

It is not enough to present theoretical material with the assumption that the students will be able to apply it later when they encounter engineering problems. Engineering educators must seek to develop skills and attitudes that will empower engineers to respond to ill-structured problems intelligently and responsibly. This need is recognized in the ABET requirement for strong design content throughout the curriculum, and in the ASEE¹ call for engineering education to become relevant, attractive, and connected.

I believe that students who engage in active, creative problem solving that links theory to engineering design will be better able to apply the theory to the problems they will encounter in practice.

APPENDIX

Example 1

Problem with parametric analysis, comparison of alternative systems, and decision criteria.

Suppose a system may be constructed in only one of two ways (not both). It may either be constructed as a single component with reliability p_1 , or it may be constructed as two components in parallel, each with reliability p_2 . Each of these alternatives has roughly the same construction cost.

- a) If $p_1 = 0.95$ and $p_2 = 0.90$, which of the two alternatives should be chosen? (Systems with higher probability of being operational are preferred.)
- b) If $p_1 = 0.95$, for what range of values of p_2 is the single component preferred to the system with two components?
- c) Below is a picture that shows all the possible pairs of values of p_1 and p_2 . These values are divided into two regions, region I and region II. For p_1 and p_2 that lie in one of these regions, the single component is preferred. For p_1 and p_2 in the other region, the system with two components is preferred. Which is which? Justify your answer. (The curve is $1 (1 p_2)^2$.)

(The picture mentioned in part (c) was hand-drawn, and not reproduced here.)

Example 2

Variation of a standard model for questions about elementary probability

The sides of a seven sided die are inscribed with the numerals from 1 to 7. The sides of a four sided die are inscribed with the numerals 4, 5, 6, 8. Each die is weighted so that each of its faces has equal probability of being on the bottom when the die is thrown. The dice are thrown in rapid succession. Let *X* be the numeral that is face down on the seven sided die, and let *Y* be the numeral that is face down on the four sided die. *X* and *Y* are random variables. Assume that the throws have no effect on each other.

Example 3

Problem involving variation of a standard model, cost analysis, and decision criteria.

Consider the following variation on the Poisson process: The rate at which events occur between 2 and 5 is a variable, A, but the rate at which events occur is 5 for all other time periods. Except for this single change, the process has all the properties of the ordinary Poisson process.

Suppose that every event that occurs between 2 and 5 costs \$10000. It is thus desirable to have A as small as possible. The designers of the process can choose a value of A. However, the cost of choosing any particular value is \$50,000/A. If no choice is made, A will be equal to 5. Explain how you would approach the problem of choosing a value of A.

Example 4

Sample Design Problem

Automobile dashboards have lights that are intended to warn the driver of impending engine problems. Ideally, each light should turn on when problems are imminent, and stay dark otherwise. However, it is usually only possible to approximate this behavior with high probability. Suppose A stands for the event "Engine Problem Exists" and B stands for the event "Light Turns On". Suppose $P\{B \mid A\} = p$, and $P\{B^c \mid A^c\} = q$. Suppose that $P\{A\} = 0.01$. Assume that 0 < p, q < 1.

- a) How large should p and q be if the designer wishes to have $P\{A \mid B\} >= 0.99$ and $P\{A \mid B\} <= 0.02$?
- b) Suppose that $P\{A\}$ is also a design parameter, equal to r. For each possible r, determine the values of p and q that give $P\{A \mid B\} \le t_1$ and $P\{A \mid B^c\} \le t_2$, where t_1 and t_2 are chosen by the designer.

Example 5.

The first design problem for Spring semester, 1995.

A manufacturer is producing plastic parts using an injection molding machine. Occasionally, the molding machine will produce a part that has a surface blister in the central area of the part. From past experience, the manufacturer knows that this is caused by a slight loosening of the knockout pins on either the left or the right hand side of the mold. The probability that both the left and right knockout pins are loose is so small that the manufacturer wishes to ignore it. Also,

the probability of a blister when neither the left or right knockout pins are loose is so small that the manufacturer wishes to ignore it.

The manufacturer has assigned a probability of 2/3 to the occurrence of a blister given that the **left** hand knockout pins are slightly loose, and a probability of 4/5 to the occurrence of a blister given that the **right** hand knockout pins are slightly loose. At any particular time, the probability that the left hand knockout pins are loose is 0.05, and the probability that the right hand knockout pins are loose is 0.08.

In the above problem, suppose that the probability that the left knockout pins are loose is a parameter p_1 , (rather than 0.05), and that the probability that the right knockout pins are loose is a parameter p_2 (rather than 0.08). Also suppose that the probability that a blister will occur given that the left hand pins are loose is a parameter r_1 (rather than 2/3), and that the probability that a blister will occur given that the right hand pins are loose is a parameter r_2 (rather than 4/5).

The reason for doing this is that the manufacturer has several molding machines, with differing probabilities, and wishes to have a policy for fixing a mold when a blister occurs. The time required to check either the left or right hand knockout pins for looseness is T_C . The time required to repair the pins if they are loose is T_R .

Your job is to produce the requested policy.

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BIOGRAPHICAL SKETCH

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