Introducing Kinematic Fundamentals of Strain Wave Gear for Robotic Arm Joint

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Abstract

Robotic arms play an important role in factory automation. A key mechanical component in robotic arm joint is a gear speed reducer called strain wave gear. This paper presents the kinematic fundamentals of strain wave gear. Three topics will be covered. First, systematic method to calculate gear ratio for different types of the strain wave gear. Second, the unique tooth geometry used for strain wave gears to maintain the conjugacy of meshing teeth. Third, typical materials and manufacturing process to make such gears. The topics are from the authors' research in strain wave gear. This paper's goal is to break the barrier between applications in robotic arm industry and engineering education in academia. From teaching effectiveness evaluation questionnaire, 87% students established the basic concepts after one lecture. 33% students can correctly calculate the speed ratio after one lecture. Homework practices are needed for the students to use the gear ratio equation correctly.

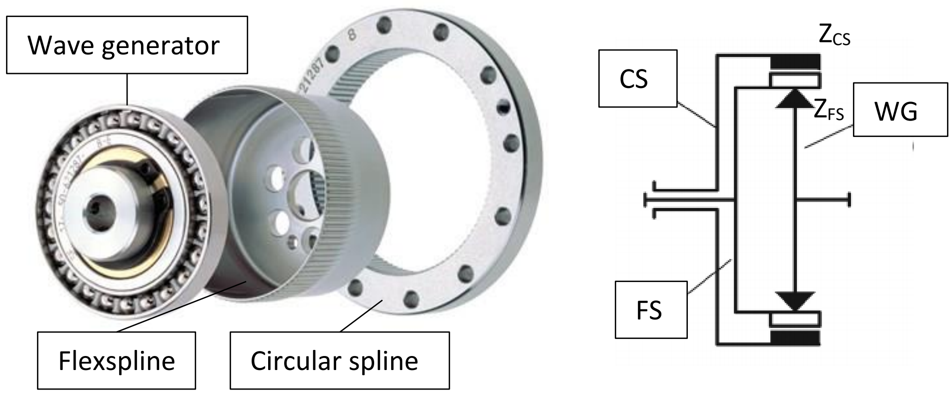
1. Introduction

Strain wave gear was invented by W. Musser in 1955 [1]. Its application is in aerospace since then. The strain wave gear is a type of mechanical gearing system that has unique characteristics comparing with cycloidal drives or planetary gears. It has the advantages of zero backlash, high precision, and high gear ratio from 30:1 to 320:1 in a compact packaging. Recent years, its main application is for robotic arm joint. Since the availability of robotic arm increases substantially due to its important role in automation, so the high-performance strain wave gear is also in need. The goal of this paper is to introduce the basic concepts of strain wave gear to students majored in Mechanical Engineering, Mechanical Engineering Technology, and Mechatronics.

2. Gear ratio

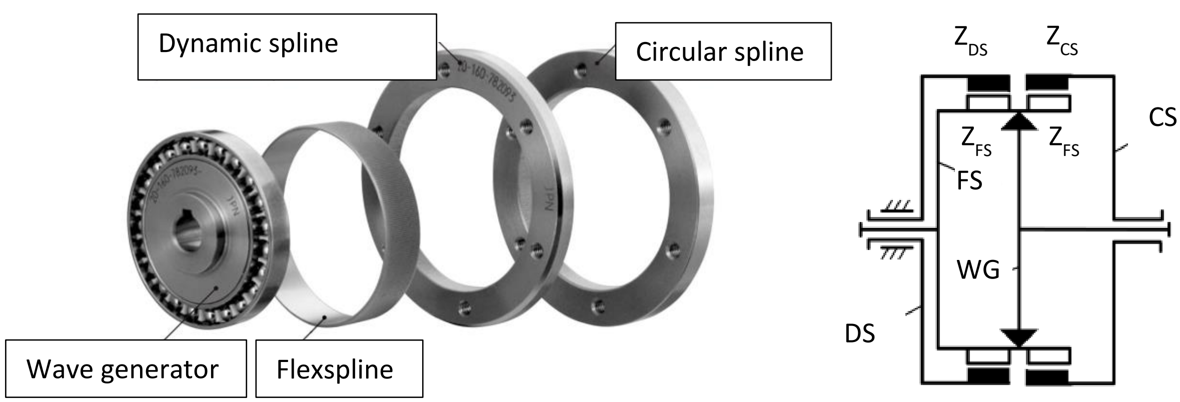
There are two types of strain wave gear commercially available [2]. One has three components as shown in Figure 1 (a). The wave generator (WG) is a ball bearing with an elliptical cam insert. Its circumference is an ellipse, which is an offset curve from the cam. The flexspline (FS) is a cup shape thin shell. The flexspline is deformable along the radial direction, but has high torsional stiffness. The flexspline has spur gear teeth at the open end and rigid flange at the close end. The circular spline (CS) is a rigid ring gear.

The flexspline will fit tightly round the wave generator, the open end of flexspline will deform with the wave generator when wave generator rotating. The teeth mesh at the major axis of the ellipse between flexspline and circular spline. The three components system has two degree of freedom as a planetary gear train. The kinematic diagram is shown in Figure 1 (b). The rotational speed between the components is in Equation (1).



(a) (b)

Figure 1. Cup style strain wave gear



(a) (b)

Figure 2. Pancake style strain wave gear

$$\frac{\omega_{CS}-\omega_{WG}}{\omega_{FS}-\omega_{WG}} = \frac{z_{FS}}{z_{CS}} \qquad (1)$$

In the equation, z is tooth number, ω is angular velocity. For example, if the flexspline has tooth number$z_{FS} = 160$, circular spline has tooth number $z_{CS} = 162$. The circular spline is connected to the fixed link of a robotic arm, angular velocity $\omega_{CS} = 0$. The wave generator is driven by servo motor as input. The flexspline rigid end flange is connected to the robotic arm moving link. The gear ratio can be calculated from Equation (1).

$$i = \frac{\omega_{input}}{\omega_{output}} = \frac{\omega_{WG}}{\omega_{FS}} = \frac{z_{FS}}{-z_{CS}+z_{FS}} = \frac{160}{-162+160} = -80 \tag{2}$$

The speed of the servo motor will be reduced 80 times to the moving link. Meanwhile, the output torque will increase 80 times, due to conservation of energy. The negative sign means the wave generator and flexspline will rotate in opposite direction.

To reduce the lengthwise dimension of the strain wave gear, the second type of strain wave adds a fourth component called dynamic spline (DS) as shown in Figure 2 (a). The flexspline is an flexible spur gear without the rigid end flange. The kinematic diagram is shown in Figure 2 (b). The rotational speed between components are shown in Equation (3) and (4).

$$\frac{\omega_{DS}-\omega_{WG}}{\omega_{FS}-\omega_{WG}} = \frac{z_{FS}}{z_{DS}} \tag{3}$$

$$\frac{\omega_{CS}-\omega_{WG}}{\omega_{FS}-\omega_{WG}} = \frac{z_{FS}}{z_{CS}} \tag{4}$$

For example, if the flexspline has $z_{FS} = 160$, circular spline has $z_{CS} = 162$. The circular spline is connected to the fixed link of a robotic arm $\omega_{CS} = 0$. The dynamic spline has $z_{CS} = 160$. The wave generator is driven by servo motor as input. The dynamic spline is connected to the robotic arm moving link. The gear ratio can be calculated from Equation (3) and (4).

$$\omega_{FS} = \omega_{DS} \tag{5}$$

$$i = \frac{\omega_{input}}{\omega_{output}} = \frac{\omega_{WG}}{\omega_{DS}} = \frac{z_{DS}}{-z_{CS}+z_{DS}} = \frac{160}{-162+160} = -80 \tag{6}$$

A strain wave gear is a two degree of freedom mechanism. If different component is used as input, such as back drive load condition, Equation (1), (3), (4) are still applicable.

3. Double circular tooth profile

Unlike planetary gear train, the strain wave gear's flexspline is not a rigid body but a complaint cup with elastic deformation. Teeth on flexspline have variant instantaneous velocities, which are governed by the shape of wave generator. The theoretical conjugate tooth profiles for the flexspline and circular spline is derived from the following procedure as in reference [3].

1) Select wave generator shape, solve the motion of a teeth on the flexspline with respect to circular spline.
2) Select a double circular tooth profile for the flexspline and calculate coordinates of points on the profile in the flexspline coordinate.
3) Calculate trajectory of the points under the relative motion between flexspline and circular spline by coordinate transformation.
4) Find the envelope of the trajectory point cloud by using analytical method or graphical method [4,5].
5) The conjugate profiles are assigned to a cross section along the longitudinal direction of the teeth.

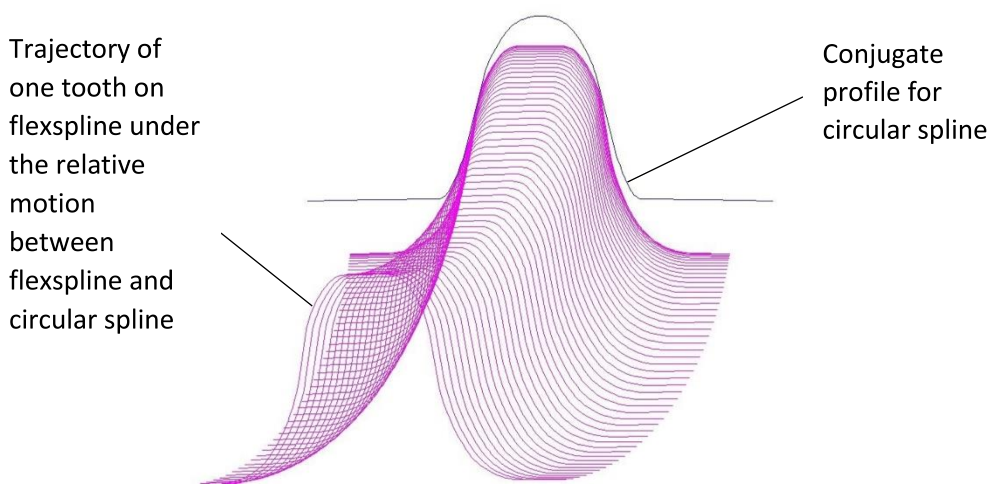


Figure 3. Double circular tooth profile

As shown in Figure 3, the conjugate designed cross section will have theoretical conjugation between circular spline and flexspline. In other words, the two profiles will be perfectly tangent to each other during the meshing process. The tooth surface is further crowned along the lengthwise direction to compensate the manufacturing and installation misalignment [3].

4. Material, manufacturing process, and testing

The strain wave gear operating theory is based on the elasticity of material. The average design life is above 10,000 hours. On the other hand, the machining requires high precision to achieve the desired tooth profile. Research [6] has focus on using aluminum with hard material coating for the circular spline. Reference [7] uses bulk metallic glass for the flexspline. Reference [8] uses carbon-fiber bonded with steel composite for flexspline. Commercially available strain wave gear uses heat treated 40CrNiMoA (AISI 4340) to achieve required design life.

Since the production of strain wave gear is high volume. High precision, high productive, fine pitch hobbing is used to manufacture the flexspline external gear teeth. For the circular spline, a new gear manufacturing process, power skiving, as shown in Figure 4 is used to manufacture the internal ring gear. Due to the non-involute profile used, the hob and power skiving tooling are custom designed.

Figure 4. Power skiving

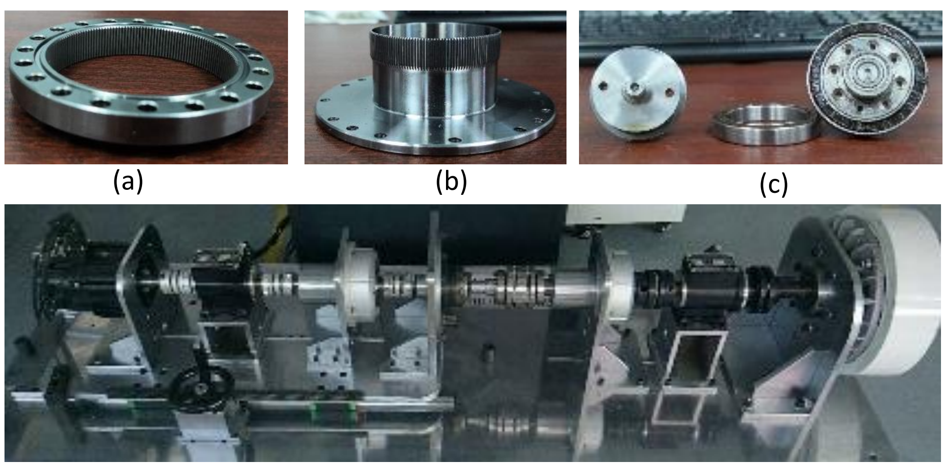


Figure 5. Strain wave gear prototype and test rig: (a) circular spline, (b) flexspline, (c) wave generator, (d) test rig

Figure 5 shows a strain wave gear prototype and test rig. The test results includes, no load friction torque, efficiency, contact pattern, torque limits, and torsional stiffness. If any of the testing results can not satisfy the design goal, modification of the tooth surface needs to be fine-tuned.

5. Teaching effectiveness evaluation

A questionnaire is designed to evaluation the teaching effectiveness, which includes the following eight questions.

1) What are the three components of a strain wave gear?
2) Name three applications of the strain wave gear.
3) What is the manufacturing methods for internal gears? Spur gears?
4) What is the gear ratio range for a strain wave gear?
5) Calculate gear ratio for strain wave gear with 162 teeth on the circular spline and 160 on the flexspline.
6) What tooth profile is used for standard cylindrical gears?
7) What tooth profile is used for the strain wave gear?
8) What standard is used for gear design?
9) There are total 15 students taking the questionnaire with the results shown in Figure 6.

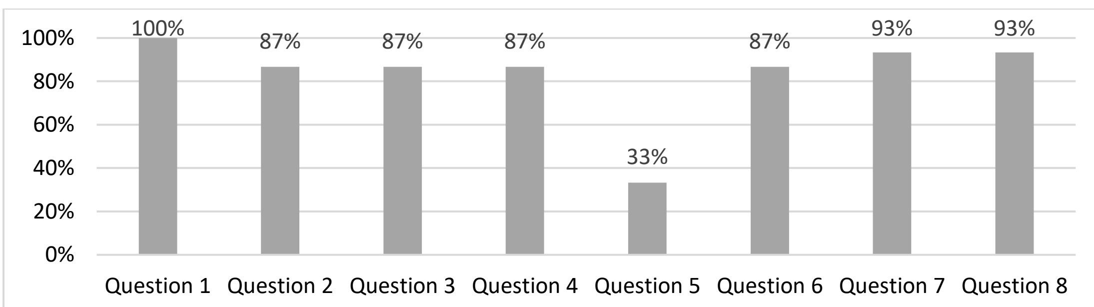


Figure 6. Percentage of correct answers

From Figure 6, above 87% students established the basic concept of the strain wave gear after one lecture. For the calculation question, Question 5, only 33% students have the correct answers. More homework practices are needed after the lecture.

6. Conclusion

This paper gives a brief introduction of strain wave gear kinematic fundamentals, tooth profile design, material, manufacturing, and testing. From the discussion above, the following conclusions can be drawn.

1) Strain wave gear is a two degree of freedom mechanism. The gear ratio can be intuitively calculated from the method proposed by the paper.
2) Strain wave gear operates differently than traditional gear, double circular profile with modification is used instead of involute or cycloid.
3) The tooling to manufacture strain wave gear are specially designed.
4) By using teaching effectiveness evaluation questionnaire, 87% students established the basic concepts after one lecture. 33% students can calculate the speed ratio.

5) The presented topic of strain wave gear will be covered in Dynamics of Machinery and Machine Design courses, more teaching effectiveness data for larger class size will be collected in the future.

Reference

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