Replacing Rigid Body Dynamics with Dynamics & Vibrations: A Perfect Introduction for Undergraduate Civil Engineers

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Abstract

The civil engineering department at Texas A&M has adopted a course in Dynamics & Vibrations as the standard introductory undergraduate dynamics course. The original concept and course was developed by Dara W. Childs, Leland T. Jordan Professor of Mechanical Engineering (details on the course were presented previously under the title: Developing A New Differential-Equation-Based Dynamics/Vibration Course, [1] at ASEE Gulf-Southwest Section 2001 ANNUAL CONFERENCE "Changing the Engineering Profession" March 28-30, 2001, Texas A&M University, College Station, TX.). The course emphasizes model development and the use of general kinematic equations and differential equations of motion for problem solving. The authors have adapted the course content to incorporate civil engineering examples and applications, and to place more emphasis on vibration. Course projects are based on realistic civil engineering examples, with an emphasis on the assumptions required to develop the analytical model. The projects are team assignments and rely on numerical analysis, a co-requisite for the course. The increased emphasis on the vibration material keeps our civil engineering students more engaged in the course. This paper presents the adaptation and implementation of this course for all civil engineering undergraduate students. Course materials (including projects); student acceptance and performance; and course assessment and evaluation will be addressed in the paper.

Introduction

Most civil engineering undergraduate curricula traditionally include either separate statics and dynamics courses, or a single combined statics and dynamics course. Texas A&M has done it both ways, but currently follows a *mostly* statics first course with a *combined* dynamics and vibrations course. The origins of this *combined* dynamics and vibrations course were reported by Dara Childs [1] during the 2001 ASEE Gulf-Southwest Annual Conference. The purpose of this paper is to describe the current adaptation of the course for *civil engineering students* (see the Appendix A for course description and learning objectives).

The *statics* course includes exposure to particle kinematics in Cartesian coordinates, Newton's laws, conservation of energy, and conservation of linear momentum for a particle. This *statics plus a little* approach allows some of the extra time required to expand the second course to include *vibrations.* Much of the remaining gain comes from the guidelines which were established for developing this *dynamics and vibrations* course:

(i) Prerequisites include both statics (and particle dynamics), and differential equations;
(ii) The course would cover dynamics of particles, rigid bodies (planar motion only), and 2DOF vibration; and
(iii) Reliance on MATLAB™ assignments and projects for the solution of **many** of the differential equations of motion (including the solution of linear simultaneous equations, solution of nonlinear algebraic equations, eigenanalysis, etc.)

The textbook [2] also contributes to the ability to include *traditionally* advanced topics (those most easily tied to civil engineering) by:

- kinematics coverage that emphasizes direct differentiation of vector components to obtain velocity and acceleration relationships in Cartesian, polar, or path coordinate systems; **and** transformation of answers to the remaining two systems;
- a parallel analysis of the same examples using *free-body diagrams, conservation of energy* approaches to derive the equations of motion; and
- an "equal-time" policy when introducing planar kinematics, working examples alternately with traditional vector approaches and then the geometric approach, **and**
- including vibrations material and examples (including eigenvalues and eigenvectors).

Content Modification for Civil Engineering Students

One of the challenges of teaching dynamics to civil engineering students is motivating them as to the relevance of the topic to their profession. Traditional undergraduate courses use examples from the mechanical engineering field have no vibrations content, which is most related to civil engineering problems. To address these issues, the authors have adapted the existing course content to incorporate civil engineering examples and applications, and to place more emphasis on vibrations. An important part of this change is a discussion of how civil engineering systems can be modeled as simple systems that superficially appear to be purely mechanical. For example, when first going through the derivation of the equation of motion for single degree of freedom systems, there is an initial resistance to learning the material when all students see are box-spring examples. Instead of starting with the simplified model, a one-story building is presented to the class and the first step in solving the problem is the development of the analytical model for the system (see Appendix B). Once students are shown how a building can be modeled as a system of boxes and springs, student interest sharply increases.

In order to be able to present civil engineering specific course content, some of the material is not covered in the same depth as that in the mechanical engineering sections of the course, such as dynamics of linkages. Minimal time spent on those topics that are only minimally relevant to civil engineers which allows for greater discussion of additional topics vibrational response, such as damping in multi-degree of freedom (MDOF) systems and model reduction. While the bulk of the course is the same for both mechanical and civil engineering sections, there is enough difference in content emphasis that enrollment into discipline specific sections is strictly enforced.

The course also uses three computational projects using civil engineering applications. These projects have several objectives: (1) to allow students to tackle a larger and more realistic civil engineering dynamics problem, (2) expose students to computational tools used in solving dynamics problems for which a closed form solution does not exist, (3) evaluate critical thinking and communication skills. The projects are designed to solved by student teams, who are told they are acting as consultants on the project posed. The projects are all centered on different real civil engineering systems and present a discussion of how to create a simple model for that system. Particular emphasis is paid to the assumptions made in the modeling process. MATLAB is then used as the framework within which the numerical solution will be achieved. The students are given template MATLAB scripts that must be customized to their particular problem. A co-requisite for this course is a numerical methods coursed where MATLAB is also used, exposing our students to the necessary skills to use this tool. The student teams are required to evaluate at least 2 possible designs and make a recommendation in their final report. This approach forces the students to think about the significance of their results, rather than blindly crunching numbers. A sample computer project is given in Appendix C.

The projects also allow for the introduction to advanced engineering concepts, such as seismic response. One project looked at seismic response of a three story structure. One of the possible design alternatives available to the students was a highly-simplified linear base-isolation mechanism. Initially, students are overwhelmed by the project. However, as they start to break it down into the required pieces they begin to realize that they can solve it. This past semester, students were allowed to create and solve their own computational project based on a civil engineering application for extra credit. Student teams tackled topics ranging from the dynamics of offshore platforms to the response of buildings with tuned-mass-dampers. While the analysis of these systems was not perfect, they demonstrated the understanding of the basic dynamics concepts involved. Additionally, the fact that the students were interested and willing to tackle such challenging concepts was indicative of success in motivating students to take the material learned to solve new problems.

Student Reception

Although we still get a few "as a civil engineer I will never need or use this!" comments; more often we hear:

- "I finally am taking an engineering course where I feel like I can or will use this in the 'real-world'"
- "One of the best things was when you showed how this directly applies to a civil engineering problem" (referring to how to model civil systems using a masses and springs MDoF model)
- “The projects really helped to bring concepts to life”

- "The projects were the heart of my interest for the class. Without civil application I could care less about dynamics."

Mid-term and final course evaluations for this class reflect that, though students find the course challenging, they indicate that this course is one where they see how the material relates to the practice of civil engineering. The results from three questions in the final course evaluations from the Fall 2003 are presented in Table 1. Students were asked to rate the following statements on a scale of 1 to 5, with 5 indicating strong agreement.

Table 1. Results from Final Course Evaluation in Fall 2003

Rank	5	4	3	2	1	Average
Course emphasizes understanding vs. memorization	26	31	12	2	0	4.14
Use of CE examples played a large role in learning the material	52	16	2	1	0	4.68
I have learned a great deal in this course	24	36	8	2	1	4.13

The average ranking for all questions lie above a 4.0. These results indicate that students do perceive that the use of civil engineering applications as a major contributor to understanding the concepts, and the distribution is the most heavily skewed one. The emphasis on model development and the use of general kinematic equations and differential equations of motion for problem solving is also well received by the students as indicated by the students perception that the course emphasizes understanding versus memorization. Both these factors contribute heavily to the strong perception by the students that they have learned a significant amount over the course of the semester.

Conclusions

The course involves the development and analysis of mathematical models for mechanical systems. The governing equations to be developed include kinematic equations and kinetic (differential) equations of motion. Goals are to determine the dynamic response of systems using mathematical analysis, and to provide knowledge and practice in understanding the behavior of dynamic systems. These goals are the same for ***both*** the civil engineering and mechanical engineering sections of the course.

The additional goal for the civil engineering section is achieved by adding of vibrations, ***and*** by including civil engineering examples. In addition, student acceptance of the course is aided because the vibrations coverage also provides a connection to the students' knowledge in previous courses in strength of materials (axial and torsional loading of bars, and Mohr's circle for principal stresses); and in structural analysis (stiffness matrices). The eigenanalysis material *even* ties back to their math course! The ABET outcomes [3] of the course also are enhanced via these connections.

References

1. Childs, Dara W., "Developing A New Differential-Equation-Based Dynamics/Vibration Course", ASEE Gulf-Southwest Section 2001 ANNUAL CONFERENCE "Changing the Engineering Profession" March 28-30, 2001, Texas A&M University, College Station, TX
2. Childs, D.W., *Dynamics in Engineering Practice*, *5th Edition, printed by John Wiley Custom Services, 2004*

Accreditation Board for Engineering and Technology, Guidelines for (*ABET*) Criterion 3, 2003

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Appendix A: Course Description & Learning Objectives

Course Description: Application of Newtonian and energy methods to model dynamic systems (particles and rigid bodies) with ordinary differential equations; solutions of models using analytical and numerical approaches; interpreting solutions; linear vibrations.

Objective #1- Planar kinematics for particle motion: Student should be able to use Cartesian, polar and path-coordinate kinematics to define the velocity and acceleration components of a material point in motion. Student will learn to use coordinate transformations to shift back and forth between the three coordinate systems (Cartesian, polar and path). Student should be able to mathematically differentiate functions of time and space coordinates to determine desired functional forms.

Objective #2 - Physical modeling of particle dynamics (1 DOF): You should be able to identify the fundamental components of mechanical systems into generalized lumped mass (inertia) M, stiffness K, damping C elements. Determine the degrees of freedom and/or the constraints present on the system. Establish the equivalence of Kinetic and Potential (Strain) Energies in Conservative systems. You should be able to derive the fundamental equations governing the motion of lumped-parameter (1 DOF and 2 DOF) systems in general plane motion. Fundamental knowledge of the kinematics and kinetics of planar rigid body motion: rectilinear motion and rotational motion about a rigid axis. Concepts of relative velocity and acceleration should be mastered.

Objective #3 - Mathematical Modeling of 1 DOF systems: Student should be able to determine analytically the dynamic response (Solutions) of 1DOF systems described by the linear ODE and given initial conditions. Be able to explain the concept of natural frequency ωn. Determine the free (transient) response to initial conditions and the dynamic response to Impulse and Step loads. Be able to discuss the concepts of transient and steady state responses, and the effect of viscous damping ratio (and logarithmic decrement) on the amplitude and decay speed of system response. Derive the dynamic response to periodic (harmonic) external forcing functions and discuss about the regimes of operation: below, close to, or above its natural frequency. Be able to obtain the Frequency Response Function (FRF) for sustained periodic excitations and explain the effects of system parameters and frequency on the Amplitude of motion and Phase lag. Use FRF for appropriate design considerations and reliable operation of vibrating systems.

Objective #4 - Mathematical Modeling of 2 DOF systems: Student should be able to derive the EOMS for 2- or M-DOF lumped parameter systems. You should be able to linearize the EOMs about an equilibrium or operating point and determine the linear system of ODEs: . For undamped MDOF systems Student should be able to determine analytically the eigenvalues and eigenvectors of . Be able to explain the concept of modal (natural) coordinates and mode shapes. Student should be able to use the transformation {to uncouple the EOMS in physical coordinates and determine (analytically) the free and forced response of both undamped and damped MDOF systems to arbitrary initial conditions, step and periodic loads.

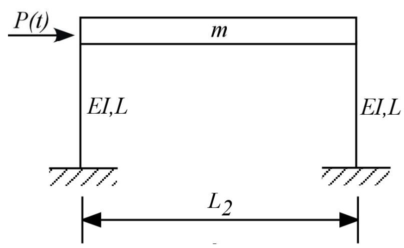
Objective #5 - Planar Kinematics for Rigid Bodies: Learn and be able to use two-coordinate systems to define the velocity and acceleration of a point in plane motion. Learn and be able to develop general kinematic equations for planar motion of rigid bodies and systems of rigid bodies including planar mechanisms. Learn the concepts of degrees of freedom, generalized coordinates, and constraint equations.

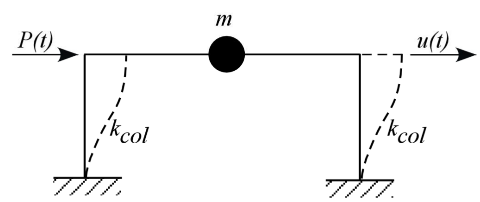
Objective #7 - Planar Kinetics for Rigid Bodies: Learn how the moment of inertia is defined and learn how to use the parallel-axis formula. Learn and be able to develop dynamic models for planar motion of rigid bodies. Equations will be developed using both Newtonian and work-energy approaches. Learn and be able to conduct analysis and simulations for planar motion of rigid bodies. Learn how to develop models for two-degree of freedom planar-kinetics examples and for planar mechanisms from free-body diagrams.

Objective #8 - Numerical Modeling of structural systems: Student should be able to use computational software to solve linear and nonlinear algebraic and differential equations describing the motion of 1- or M-DOF systems. You should be able to apply knowledge gained in numerical methods to select appropriate numerical techniques with due consideration for time steps and procedures (algorithms) ensuring accurate, numerically stable, and cost efficient system response. Student should be able to interpret numerical calculations (predictions) to explain system behavior (motion), identify possible failure mechanisms due to excessive amplitudes of motion or reaction forces, etc.

Appendix B: Example of Modeling a One Story Structure

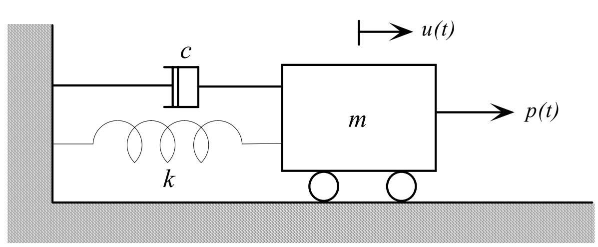
Take a one story structure where the roof is a heavy rigid concrete slab supported by flexible columns. The beams supporting the slab are so deep as to be considered rigid in bending. You want to determine the response of this structure to a lateral load and expect that the behavior will be completely planar.





As the slab is rigid and no bending is allowed in the beams, the only response is the lateral motion of the slab. Once we know where any one point of the slab is, we can determine where the other points are through geometry as all points move together and have the same acceleration. As such, the system can be modeled as a single degree-of-freedom system (SDOF). All the mass can be lumped at the center of mass of the slab. The mass of the columns is either deemed negligible of lumped with the mass of the slab. The columns act to resist the lateral force just as if the force were applied statically. Though no single physical element is a damper, one can expect some damping to exist in the real system due to friction at connections and cracking of concrete.

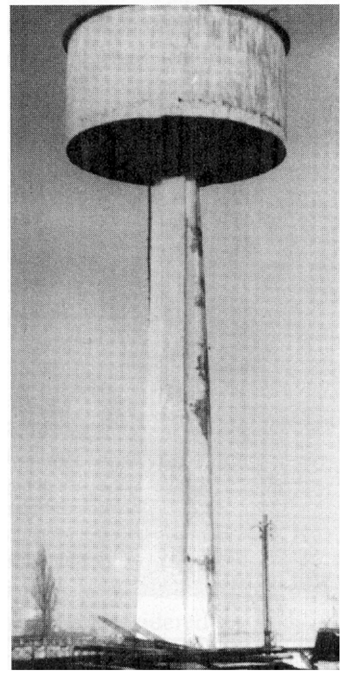
Given all those assumptions, then the portal frame can be modeled as a mass-spring-damper system as show in the figure bellow. If we consider the spring and damper to be massless, the mass to be perfectly rigid, and all the motion to be along the x-axis, then the system has a single degree of freedom. The stiffness of the spring comes from the contribution of both columns.



Appendix C: Sample Computer Project

System Description:

Water tanks, such as the reinforced-concrete tank shown, can accurately be modeled as a single-degree-of-freedom (SDOF) system, where the degree of freedom is the lateral displacement at the center of mass of the tank. This water tank rests on single concrete column that is 40 feet high. The weight of the tank when full is 175 kips. The modulus of elasticity for concrete is given by:



$$E = 5700\sqrt{f_c'}$$

where f_c' = 4000 psi for this structure. (You must enter the value in psi, and the resulting units of *E* are also psi). The cross-section of the column has a moment of inertia, *I*, of 1×10^9 in^4.

A free-vibration test is conducted on the water tank. A cable is attached to the tank, which is then pulled perfectly *horizontally 5 inche*s. The cable is then suddenly cut, and the resulting free-vibration is recorded for use in determining the damping of the structure. The data is stored in a text file titled `free_data.txt` and available in the class web site for download. The format of the data is two values per line in the file. The first gives the time the measurement was taken and the second gives the lateral displacement of the structure at that time.

This water tower is also exposed to heavy winds. As a first-order modeling strategy, the effects of the wind are modeled as a *harmonic force* acting on the center of mass of the tank with *peak value of 30 kips and a frequency of 4.19 rad/sec* (corresponds to period of 1.5 seconds).

Project Goal and Objectives:

1. Determine dynamic properties of existing system
2. Evaluate the free vibration response
3. Evaluate the response under wind load
4. Suggest design change to either mass or stiffness properties so that the response meets the following response criteria:
    - Peak response

Detailed Tasks:

<u>Part 1</u>

A free-vibration test is conducted on the water tank. A cable is attached to the tank, which is then pulled perfectly horizontally 5 inches. The cable is then suddenly cut, and the resulting free-vibration is recorded. If x is the variable describing the lateral motion of the center of mass of the tank, then complete the following:

a) Determine the initial conditions for the system – initial displacement and initial velocity (Individual and Team)

b) Write the second-order differential equation for this system

$$m\ddot{x} + c\dot{x} + kx = 0$$

as a system of 2 first-order differential equations. (Individual and Team)

c) What is the undamped natural frequency, ω_n, of this system? (Individual and Team)

d) Using the results from the free vibration test, determine the damping ratio for the system. Compute this property using the logarithmic decrement both at adjacent peaks (Equation 10) and by using peaks that are at least 4 cycles apart (Equation 12). Do you get different answers depending on the method? If so, explain why you think the differences arise. Does it matter if you look at peaks at the beginning or at the end of the test? Why or why not?(Team)

e) Solve for six cycles the response of an unforced system given by the following critical damping values: $\zeta = \{0, 0.01, 0.025, 0.05, 0.1, 0.25, 0.5, 1.0\}$. Use numerical integration such as Runge-Kutta method implemented by ODE functions in MATLAB. Develop a plot for the solutions corresponding to all values and comment on the plots obtained. Does the response with increasing damping seem reasonable? Why or why not. Does the test result make sense in comparison with these simulations? What are differences and similarities? (Team)

Part 2

This water tower is exposed to heavy winds. As a first-order modeling strategy, the effects of the wind are modeled as a harmonic force acting on the center of mass of the tank with peak value of 30 kips and a frequency of 4.19 rad/sec (corresponds to period of 1.5 seconds).

a) Write the second-order differential equation for this system

$$m\ddot{x} + c\dot{x} + kx = F$$

as a system of 2 first-order differential equations. (Individual and Team)

b) Utilizing the same initial conditions as for Part 1, solve for six cycles the response of an unforced system given by the following critical damping values: $\zeta = \{0, 0.01, 0.025, 0.05,$

0.1, 0.25,0.5,1.0}. Use numerical integration such as Runge-Kutta method implemented by ODE functions in MATLAB. Develop a plot for the solutions corresponding to all ζ values and comment on the plots obtained. Does the response with increasing damping seem reasonable? Why or why not. How does the result for same level of damping compare with the previous result under free vibration? Does amplitude of oscillation change? Does "shape" of response change? What is the frequency of the response? Does it match or is closer to the undamped natural frequency, the damped natural frequency, or the frequency of the excitation? (Team)

c) Discuss how performing the test under windy conditions might impact your results when trying to estimate damping. (Team)

Part 3

Your consulting group is being asked to improve the performance of this water tower. Two possibilities under consideration are: (a) limiting the amount of water stored in the tank and (b) adding bracing members for lateral support. Option (a) reduces the mass of the system and option (b) increases the stiffness. The desired response criteria under the wind load are:

- Peak response equal to or less than 7 inches

a) What is the effect on the natural frequency and period of these changes? Do these properties increase or decrease? (Individual and Team)

b) What would be your recommendation for meeting the response criteria? Give specific values for your new mass and/or stiffness. Why did you choose that property to change rather than the other? You may want to consider system capacity and cost in your deliberations. (Team)