Teaching Critical Thinking Through Simple Experiments

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Students are challenged to think critically regarding the simplifying assumptions of a powerful yet simple analytical model. Results of analytical and experimental investigations are compared for three different devices specifically chosen so that the results would have a wide range of agreement. Students are then asked to explain why the analytical results, which all had the same simplifying assumptions, had such varying agreement.

Although this technique was developed and implemented within the context of a three lecture hour per week introductory fluid dynamics course, the idea of marrying analysis with simple experiments to encourage critical thinking is applicable to most engineering disciplines. The following sections cover the motivation behind the development of these exercises, the specifics of the exercises, student perceptions, and concluding remarks.

1 Motivation and Background

Enabling future engineers to think critically is one of the most challenging aspects of engineering education. Most engineering students are strategic learners (Woods, Hrymak, and Wright, 2000), meaning that they will think deeply about a problem only if they are forced to. This is especially true of the simplifying assumptions that are made. Students learn that when performing certain types of problems a certain set of simplifying assumptions are always made. Most will not independently think about the validity of the simplifying assumptions.

If they are strategic learners, they will also verify their answers in the most expedient way which is typically one or more of several artificial means, including comparing results with provided numerical answers, with classmates' answers, and with solution manuals. Of course these techniques are not available to engineers outside the classroom setting. Additionally, these methods mask the effects of the simplifying assumptions because as long as the numerical answers agree the students assume they have proceeded correctly and never think about the simplifying assumptions.

This is especially true of the simplest models we use. We often can explain, at least qualitatively, many physical phenomena by making assumptions that will yield a simple equation. Of course students retain these models most readily because of their simplicity and intuitive nature. However, often these models are limited in their numerical predictive ability if numerous simplifying assumptions are required to obtain the model. If we do not explicitly do something to show students the effects of these assumptions, we can not expect them to understand their effect.

The technique presented here forces students to face a situation where one problem has excellent agreement between experimental and analytical results, another has some good and some poor agreement, while a third has poor agreement. A dilemma is set up because the same simplifying assumptions are made in all the analyses and the experiments are simple enough that the disagreement is not attributable to experimental error. The exercises have been engineered so that the differences are due to the different degrees of applicability of the simplifying assumptions. The dilemma forces the student to think critically about both their simplifying assumptions and verification techniques.

This technique has three essential parts: 1) analytical problems the students perform using a simple model; 2) experimental exercises where the quantity that was predicted from the analytical model is measured experimentally; and 3) follow–up questions were the students are led through critical thought processes to help them discover the effects of their simplifying assumptions. Each of the problems use the same analytical model with as close to the same set of simplifying assumptions as possible.

2 Exercise Specifics

In the study of fluid dynamics, Bernoulli's equation (1) is a prime example of powerful, yet simple equation because it is algebraic and it explains the functioning of everything from toilets to airplane wings.

$$p + \frac{\rho V^2}{2} + \rho g z = C \tag{1}$$

In this equation p is the static pressure, ρ is the fluid density, V is the fluid velocity, g is the acceleration due to gravity, z is the height from some common reference, and C is a constant. It relates the static pressure p (internal energy) in a fluid, the dynamic pressure (kinetic energy) $\rho V^2/2$, and the hydrostatic pressure $\rho g z$ (potential energy). This form of Bernoulli's equation can be developed from either momentum or energy principles if the flow is assumed to be incompressible, steady state, frictionless, and either irrotational or the equation is applied along a streamline.

It is so popular that some introductory physics books completely neglect to mention the momentum and energy principles and only introduce conservation of mass and Bernoulli's equation (Bueche, 1980). In Munson, Young, and Okiishi (2006) the third chapter following introductory and fluid statics chapters is entitled "Elementary Fluid Dynamics – The Bernoulli Equation". This chapter precedes material where the more complete momentum and energy principles are introduced. When asked what are governing principles of fluid dynamics, it is no surprise that many students answer "Bernoulli's equation".

However, since it has the four assumptions of incompressible flow, steady state flow, frictionless flow, and either irrotational flow or flow along a streamline its quantitative abilities are quite limited. Students easily learn Bernoulli's equation from hearing a lecture, seeing a few examples, and doing a few homework problems. However, this is not sufficient for the them to learn the effects of the simplifying assumptions and the accuracy of their numerical answers. The following set of three experiments each having the three components of analytical, experimental, and follow–up questions is one way of assisting students to learn these additional consequences.

A set of homework problems involving analysis of the time for water to drain from a can, a Venturi tube and a Pitot tube are solved using Bernoulli's equation. Experiments designed to measure the calculated quantities are then performed and the results compared. The problems and experiments have been carefully chosen so that the level of difference between the analytical and experimental results vary from within the uncertainty of the measurements in the case of the Pitot tube to as much as 35% in the case of the water draining from a can. The students then answer a set of questions designed to force them to think about the simplifing assumptions and why in some cases they have little effect and in others they have a dramatic effect.

2.1 Time to drain water from a can

The problem of determining the time required to drain water from a cylindrical can through a circular hole in the bottom so that the the water surface starts and ends at two predetermined heights is easily analyzed using conservation of mass and Bernoulli's equation. Applying Bernoulli's equation (1) at the surface of the water and then again at the exit; noting that the pressures at both locations is atmospheric and that the velocity of the water at the surface is negligible, the analysis yields Torricelli's equation for the velocity V at the exit as a function of the acceleration due to gravity g and the height h of the free surface from the bottom of the can:

$$V = \sqrt{2gh}$$

substituting this into the conversation of the mass for a incompressible fluid

$$\frac{d\forall}{dt} = V A_d$$

where $\forall$ is the volume and A_d is the exit hole area, and finally integrating from an initial height h_1 to a final height h_2 yields

$$t = \sqrt{2/g}\,(D/d)^2\left(\sqrt{h_1} - \sqrt{h_2}\right) \tag{2}$$

where t is the drain time, D and d are the diameters of the can and hole respectively.

Paint cans were chosen for the experimental apparatus since they are cylindrical, inexpensive, and easily obtained. Holes of 3/8, 7/16, and 1/2 inch were drilled in the center of the bottom of three cans. The cans were measured to be 6.53 inches in diameter and each have two formed ridges at 4.69 and 2.31 inches from the bottom of the can with were used for the h_1 and h_2 quantities. All of these values were provided to the students when they were assigned the problem of developing equation 2 so they could determine the draining times prior to performing the experiments.

It is clear from examining typical data as shown in Table 1 that the comparison between the analytical experimental results are poor. All of the analytical times under–predicted the draining times by 32 to 35%.

d (in)	Analytical Time (sec)	Average two (sec) Experimental trials (sec)	error (%)
0.5000	7.91	12.22	-35.3
0.4375	10.33	15.88	-35.0
0.3750	14.06	20.66	-32.0

Table 1: Typical results of water draining from a can exercise

Many students initially assume experimental error due to uncertainty in the measurements or mistakes in the procedures caused the disparity between the analytical and experimental results. The follow–up questions ask the students to estimate how accurately the diameters of the can and hole, initial and final water heights, the acceleration due to gravity, and finally the draining time were measured. They are then asked if it is reasonable to assume the disparity between the analytical and experimental drain times is attributable to measurement uncertainty. They typically conclude correctly that it is not.

The follow–up questions then address the simplifying assumptions: 1) flow along a streamline; 2) constant velocity across the exit hole (one–dimensional flow); 3) incompressible fluid ; 4) steady state; 5) negligible velocity of the free surface; and 6) frictionless flow. This is a long list of assumptions for students to assess one at a time. Although a detailed order of magnitude analysis is possible to determine the effect of most of these, it is beyond the scope of most introductory students who can easily get lost in the details. The follow–up questions ask them to consider: 1) the effect on the drain time would have been if corn oil had been used rather than water. Universally, they conclude the drain time would increase; 2) the critical property that differentiates the corn oil from water. They typically conclude correctly that it is the viscosity; and 3) where in the analytical model is viscosity accounted for, which of course it is not. Through these follow–up questions, they rapidly come to the correct conclusion that the assumptions of frictionless flow is very poor and accounts for large disparity between the analytical and experimental results.

2.2 Venturi tube

A Venturi tube, shown in Figure 1, is converging followed by a long smooth diverging pipe section. By measuring the pressure drop between the inflow and the smallest area (throat) the flow rate can be determined by applying Bernoulli's equation (1). Since water was used in this case it is possible to measure these pressures using piezometers which are simply vertical tubes that the water from the Venturi rises up until the hydrostatic pressure exerted by the vertical water column equals the pressure at the tap location in the Venturi. Although typically only pressure taps upstream and at the throat are used, a Venturi designed for educational purposes was used that has eleven piezometers distributed along the length of the Venturi as shown in

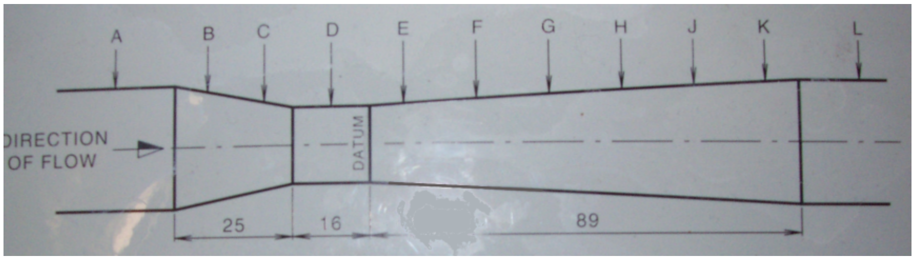

Figure 1: Schematic of the Venturi. Letters indicated locations of the pressure taps.

Figure 1.

The water height h_2 in any piezometer can be related to the water height h_1 in any other by: 1) recognizing that the volume flow rate Q at any point must be the same and therefore the velocity V at any location can be expressed as $V = Q/A$ where A is the cross sectional area at that location; 2) recognizing that the pressure at any location can be expressed by the hydrostatic relationship $P = \rho g h$ where P is the pressure, ρ is water density, g is the acceleration due to gravity, and h is the height of the water in the piezometer; and 3) applying Bernoulli's equation (1) between two points. This analysis yields the desired analytical model

$$h_2 = h_1 + \frac{Q^2}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right) \tag{3}$$

The volume flow rate was measured experimentally by recording the time required to collect a predetermined volume of water. Using the first experimental piezometer height as the reference h_1 all of the other heights can be calculated using equation 3 and were then compared to the experimental heights. Typical results shown in Figure 2 indicate reasonably good agreement between the first four most upstream locations in in the converging section, with ever decreasing agreement downstream of the throat.

The follow–up questions helped students realize: 1) the upstream piezometer heights (pressures) agree exactly because they are forced to be the same; 2) the analysis based on Bernoulli's equation predicts that since the entrance and exit areas are the same that the piezometer heights (pressures) at those locations will be the same; and 3) the disparity between the results in general becomes greater the further the pressure tap is from the reference pressure tap.

Once again it was necessary to work with the students to convince students that the differences are not due to experimental uncertainty nor error in recording the measurements. This is done by asking how carefully the diameters of the Venturi tube can be measured, how accurately they know the volume of the collection container, and how accurately they can measure the collection time. These uncertainties did not account for the disparity apparent in the results at the downstream locations. Additionally, they can not explain the agreement of measurement close to one another and the disagreement of more remote locations.

The disparities must be due to the simplifying assumptions, which are: 1) along a streamline; 2) steady state flow; 3) incompressible flow; and 4) frictionless flow. Since the first two of these

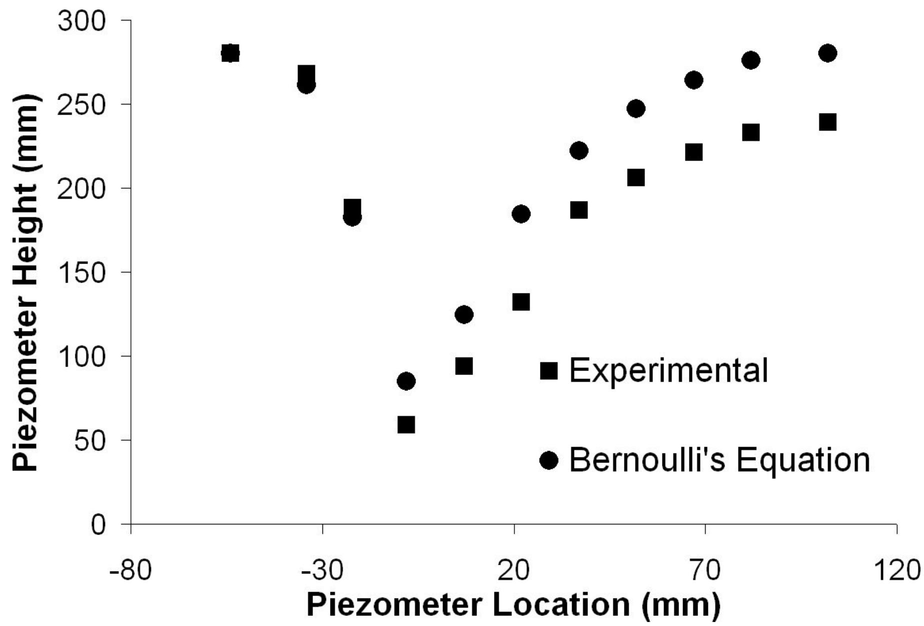

Figure 2: Typical results of the Venturi tube analysis. The flow direction is from the left to the right.

are absolutely true and the third accounts for a very small error the disparity must results from the assumption of frictionless flow. By discussing the nature of friction with the students they typically understand that the pressure loss due to friction accumulates in the flow direction due to increasing surface area and therefore the agreement of the pressures with nearby piezometers is quite good but much poorer for piezometers further away.

2.3 Pitot tube

A Pitot tube shown schematically in Figure 3 consists of two concentric tubes arranged such that the stagnation pressure (sum of the static and dynamic pressures) is sensed though the center tube at location 1, while the static pressure alone is measured through the annular region between the two tubes through holes at location 2.

A Pitot tube along with Bernoulli's equation is used to measure point velocities in a flow. By recognizing that there are no significant height differences between the the stagnation and static pressure ports, the flow has stagnated (come to a stop) at location 1 in Figure 3, and applying Bernoulli's equation it is simply shown that the flow velocity V is:

$$V = \sqrt{\frac{2\left(P_{\text{stag}} - P_{\text{stat}}\right)}{\rho}} \tag{4}$$

where P_{stag} and P_{stat} are the stagnation and static pressures respectively and ρ is the fluid density.

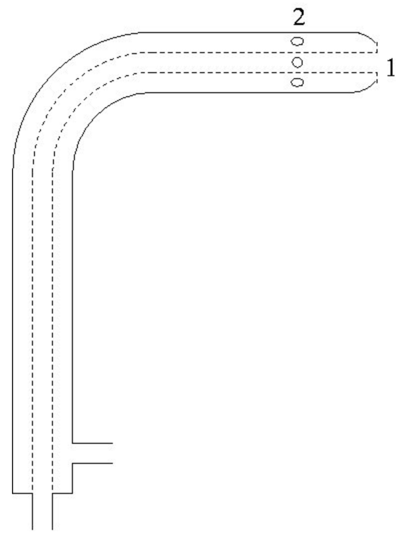

Figure 3: Schematic drawing of a typical Pitot tube.

It was difficult finding a simple experiment that could be used to independently verify a Pitot tube. Most wind tunnels use some form of Pitot tube to determine the flow velocity, therefore they can not provide an independent source. Many other devices, such as Venturis and vane anemometers measure average flow rates rather than point velocities. Hot film and hot wire anemometers are even more complicated and unfamiliar to students than are Pitot tubes and therefore were not simple enough.

After several iterations using a car's speedometer as a source of verification was found to be suitable. A Pitot tube was mounted on the front of a car as shown in Figure 4A. Students drove the car at constant speed (according to the speedometer) along a stretch of straight road and recorded the dynamic pressure $P_{\text{stag}} - P_{\text{stat}}$ as measured by a differential pressure transducer as shown in Figure 4B. Students took measurements while driving both directions on the road so that any head wind could be averaged out of the velocity data.

The typical data in Table 2 show excellent agreement between the analytical model and experimental data. Although the agreement was excellent students needed to be convinced of this, in the follow–up portion, as many of them were unfamiliar with verifications through experimental means. After they considered their experience with automobile speedometers, that it was believed that this particular car's speedometer read slightly low, and to consider flow fluctuations due to turbulence and wind, they typically agree that the results were within experimental uncertainty of the measurements.

After the students conducted the experiments and reduced the data they were in a position to consider the dilemma. Why were the same four assumptions of steady state flow, incompressible flow, flow along a streamline or irrotational flow, and frictionless flow made for all of the experiments, but the experimental results for the draining of water from a paint can poor, prediction of pressure in a Venturi tube good at some points and poor at other points, and the results for the Pitot tube excellent? Of course friction caused the poor results with the can draining and

Figure 4: Pitot tube experimental apparatus A) Pitot tube mounted on front of car B) Pressure transducer for reading the dynamic pressure

Speedometer speed (MPH)	Pitot tube speed (MPH)	error (%)
40	42.0	5.0
50	51.6	3.2
60	60.3	0.6
65	65.9	1.3

Table 2: Typical analytical and experimental data for the Pitot tube experiment

the Venturi tube, but why did it not effect the Pitot tube? Apparently friction does not play an important role for the Pitot tube.

It was sufficient to ask students to predict the speed of the air inside the Pitot tube. Initially, a significant number of students thought it would be at the flow speed. However, when asked what the air speed was at the transducer they quickly realized that the air inside the Pitot tube was static and therefore the only frictional losses occurred between the stagnation pressure tap (location 1 in Figure 3) and the static pressure taps (location 2 in Figure 3). Since the area between these is small the frictional effect is negligible, just as the frictional effect was small in the Venturi tube when the pressure taps were close together.

3 Student perceptions

At the conclusion of the follow up questions, the students were asked to complete a perception survey. When asked if they learned more than if they had only completed the analytical problems 87% agreed that the learned more. 82% agreed that they learned more than they would have if they had only conducted the experiments. The follow–up questions were not as popular as only

60% agreed that they learned more because they were included.

It is not completely clear why the students did not perceive the follow up questions as being as important, although several possibilities exist. Perhaps the follow–up questions are non–essential and students are able to synthesis these results without them. Although possible, it is unlikely, because of the questions asked during class and the clarification that was required on certain points. Perhaps the follow–up questions are necessary, but since some students are rarely forced to think critically about their work and they are uncomfortable answering them. This is a more likely cause and is supported by some of the written comments students gave on the survey. A third, and most likely cause, is that the questions need to be refined further so that the students are led to the appropriate conclusion in smaller steps. Since the follow–up questions are the third component of the exercise they have not evolved as much as the analytical and experimental parts. In retrospect some of them were too general and did not guide students well through the thought process.

4 Conclusions

The purposes of these simple experiments were to: 1) force students to think critically about their results and the simplifying assumptions that are made in a problem; 2) show students that the simplifying assumptions can have greater of lesser impact on the numerical accuracy of a model; and 3) let students experience a more realistic scenario of answer verification than they typically employ.

Student responses on the survey and subsequent exam questions indicate that after these exercises, students have a greater appreciation of the simplifying assumptions inherent to Bernoulli's equation and are better able to think critically about the effect of these assumptions than they were prior to them.

Although this technique of using simple experiments to assess the applicability of simplifying assumptions on analytical models was conducted in the context of Bernoulli's equation in an introductory fluid dynamics course, the method is applicable to any discipline were analytical models with simplifying assumptions are used.

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Biographical Information

John Iselin received his doctorate from Iowa State University in 1999. He taught at Bucknell University in Lewisburg, Pennsylvania for five year before accepting his current position at the University of Wisconsin, Platteville. His professional interests are in atmospheric transport and dispersions modeling in urban areas, and educational pedagogical research.