Visualizing Abstract Calculus Concepts by Performing Virtual Electronic Laboratory Experiments

Jose Amieva, Apolinar Zapata

Engineering Technology Department
University of Texas at Brownsville

Abstract

Teaching calculus has been a big challenge for the Educational Community. Many schools and instructors fail to recognize that students have different learning styles. There is a noticeable resistance to change the traditional methods of teaching calculus by new methods that incorporate techniques that makes learning calculus a less painful experience. Some schools continue to teach calculus using traditional methods, which require little or no use of recent technology; these traditional methods are only adequate for a small portion of the student population. There are concepts in calculus that are difficult to understand by the kinesthetic learner. However for the auditory learner, the concepts will be easy. For the visual learners, they will need to "see" something in order to learn such concepts. The instructor needs to make sure that the students understand the fundamental concepts of calculus. Some students learn to memorize formulas and manipulate symbols only, and this creates a problem for them to succeed in further math courses. Calculus instruction must accommodate for students that are visual learners, auditory learners, and kinesthetic learners. It is a fact that the students learn better when they participate in the learning process, instead of just playing a passive roll. Using present technology, there is the possibility to visualize the abstract calculus concepts by performing virtual experiments that will permit the active participation of the student in the learning process. This paper presents two virtual electronic laboratory experiments to be used as an aid in the visualization of two of the main calculus concepts: Integration and Differentiation. These experiments were designed using electronic simulation software and contain, as main elements: operational amplifiers, resistances, and capacitors to perform the integration and differentiation operations.

Introduction

The actual generation of students, whose background is mainly of the visual domain, seems to enjoy, and learn the abstract concepts of mathematics better, when they are involved in classes that include laboratory experiments. The students learn by doing in a laboratory environment where they are involved in performing hands-on projects.

For most students in mathematics, science, and engineering, calculus is the entry–point to undergraduate mathematics [1]. Unfortunately, in some Universities, more than half of the students fail the introductory course in calculus. One of the problems encountered in teaching calculus identified at the Tulane Conference was “students performing symbolic manipulations with little understanding or ability to use calculus in subsequent courses”. Another cause identified was “mathematics lagging behind other disciplines in the use of technology.” [2,4]

Over the past decade, considerable resources have been devoted nation-wide to assessing and reformulating the teaching of calculus in the first year university curriculum [5]. This reformulating movement is called “calculus reform.” Now is the time to do for math what we have done for reading. That is why Governor Rick Perry has proposed a new math initiative [3].

Adding laboratory experiments to the calculus class is like adding a needed dimension in the learning process; it is like finding the connection of the abstract world to the real world. This paper proposes two laboratory experiments that use electronic components that will help the students to understand the basic concepts of integration and differentiation.

The objective of laboratory experiment #1, The Integrator Circuit, was to perform the integration function to the input waveform. To do this, a square wave of 1 Khz, 5 V was applied to the input of the circuit, and a triangular waveform was obtained at the output of the circuit, with the same frequency. The objective of laboratory experiment #2, The Derivative Circuit, was to perform the derivation of a waveform applied at its input. A triangular waveform of 1 Khz, 5V was applied to the circuit. A square wave was obtained at the output.

OP- AMP Virtual Laboratory Using Circuit Maker®
Laboratory # 1
Integrator Circuit

Purpose

This experiment will visually demonstrate that an Integrator circuit will provide an output, that is proportional to the area of the input waveform (the circuit will convert a square wave into a triangular wave).

Materials

(1) LM741 Op-Amp
(1) Capacitor 0. 1 uF
(2) Resistors : 1 Kohms ,100 Kohms

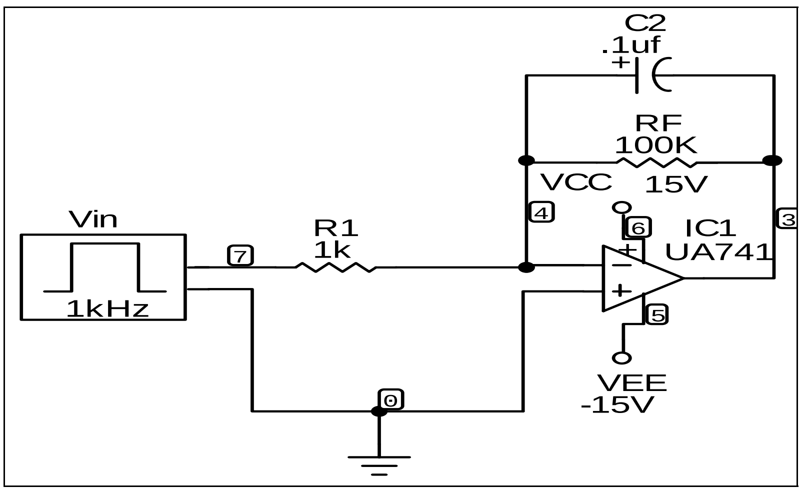

Figure 1 Integrator Circuit

Procedure

1.Create the integrator circuit using Circuit Maker® Software
2.Use the Transient/Fourier Analyses Setup
3.Run the Analyses.

Results

When the "Run Analyses" button is selected, the oscilloscope will display the input waveform to be a square wave and the output waveform to be a triangular wave

OP- AMP Virtual Laboratory Using Circuit Maker®
Laboratory # 2
Differentiator Circuit

Purpose

This experiment will visually demonstrate that the Differentiator circuit will provide an output, that is proportional to the rate of change (slope) of its input waveform (the circuit will convert a triangular wave into a square wave).

Materials

(1) LM741 Op-Amp
(1) Capacitor 0. 05uF
(1) Resistor 1 Kohms

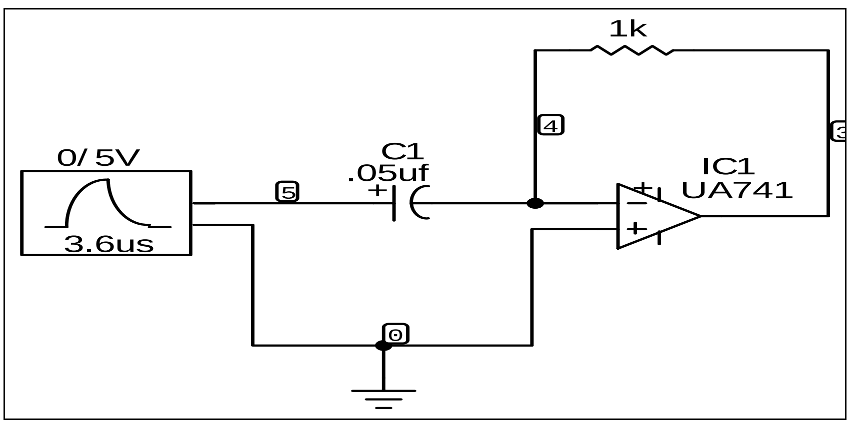

Figure 22 Differentiator Circuit

Procedure

1. Create the differentiator circuit using Circuit Maker® Software
2. Use the Transient/Fourier Analyses Setup
3. Run the Analyses.

Results

When the "Run Analyses" button is selected, the oscilloscope will display the input waveform to be a triangular wave and the output waveform to be a square wave.

Mathematical Analysis

For the analysis of these two circuits we are going to consider an ideal op-amp which has the following characteristics:

1. The inverting input (-) is held at virtual ground
2. The input impedance of the op-amp is infinite, so the same current that goes through the resistor also goes through the capacitor.

The Integrator Circuit Theory

If we perform a KCL at node 4, and considering that the current through the resistor is the same current through the capacitor, then:

$$i_R = i_C \qquad \text{(equation 1)}$$

If we know that the current through a resistor is $i_R = \frac{v_o}{R}$, and that the current through the capacitor is $i_C = -C\frac{dv_i}{dt}$, and if we substitute these in equation 1 we have:

$$\frac{v_i}{R} = -\mathrm{C}\frac{dv_o}{dt} \qquad \text{(equation 2)}$$

Solving for dv_o, results the following:

$$dv_o = -\frac{1}{RC}\, v_i \; dt \qquad \text{(equation 3)}$$

If we integrate both sides and consider the initial voltage to be equal zero, then:

$$vo = -\frac{1}{RC}\int_0^t v_i\,(\mathrm{t})\; dt \qquad \text{(equation 4)}$$

This means that the output signal voltage will be the integral of the input signal voltage as represented by equation 4.

The Differentiator Circuit Theory

The same considerations of virtual ground and infinite impedance of the inverting input will be applied in this analysis. If we apply KCL to the node #4 of the differentiator circuit, we find that the same current flowing through the resistor is the same current flowing through the capacitor. Therefore the same equation 1 applies.

$$i_R = i_C$$

The current through the resistor is:

$$-\frac{v_o}{R} = C\frac{dv_i}{dt}$$

Solving for v_o, results the following equation:

$$v_o = -\mathrm{R}C\frac{dv_i}{dt} \qquad \text{(equation 5)}$$

Equation 5 demonstrates that the output signal is the derivative of the input signal.

Conclusion

The two laboratory experiments were simulated using Circuit Maker® virtual schematic design software, by our UTB sophomore Electronic Engineering Technology students, enrolled in the Instrumentation Laboratory class, the observation from the students was that the output signal of the OP-AMP was inverted. The reason for this was that we are applying the input in the inverted input of the OP-AMP. The comments from some of the students were: "performing these experiments we can see the results of the integration and derivation functions".

The same two laboratory experiments were assembled and tested, by the same students, using the real components. The comments from most of the students again were "these laboratory experiments make the connection of the calculus concepts, with the real world. We learn by doing."

References

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JOSE AMIEVA
MSEE Jose Amieva currently serves as an Assistant Master Technical Instructor, at the University of Texas at Brownsville, Electronic Engineering Technology Dept. He is, at this time chair of the scheduling committee and supervisor of the laboratories of Engineering Department. His research interests are in control systems, simulation, robotics and automation.

APOLINAR ZAPATA
Dr. Zapata currently serves as a Lecturer of Mathematics at the University of Texas at Brownsville. He is, at the moment, participating as supervisor of the computer laboratory in the Engineering Department at the same university. His research interests include engineering optimization by using models of networks, simulation, genetic algorithms, and neural networks.