

Visualizing fundamental engineering mathematics concepts through use of interactive viewports

Mr. Adomas Povilianskas, Penn State University, ESM

Assistant Teaching Professor from Penn State University; The Engineering Science and Mechanics department.

Ms. Henrietta Tsosie, Pennsylvania State University

Visualizing fundamental engineering mathematics concepts through use of interactive viewports

A. Povilianskas* and H. Tsosie*

**Penn State University, ESM*

Abstract

This is a work in progress study of implementing novel educational content delivery methods in 2nd year engineering mechanics courses. Today's students have access to a plethora of digital information and are quickly adapting to new technologies. Addressing the continuing challenges in maintaining students' motivation and success requires new approaches to teaching introductory topics in engineering courses. In typical engineering class, traditional teaching approaches, such as chalkboard talks and PowerPoint presentations are often utilized, this often presents the issue of unidirectional communication and passive learning. While students have access to many online education resources such as YouTube and Khan Academy, they vary in quality and are typically lecture based; lacking content for engaging hands-on activities.

Hands-on learning and visual learning are not new topics. Such efforts have been actively developed over the years, as early as 300BC. Counting boards, abacus, clay and wooden tablets were used to convey, preserve, and share complex ideas across communities. The rapid advancement of teaching technology has never stopped and as we advance and build our knowledge, the tools and methods of teaching must also align, adapt, and grow.

Development of visual aids is important because it enhances our ability to reach and teach every student within our diverse community; by breaching the communication boundary through vivid visualization. There are many words which can be used to describe concepts and phenomenon. Sometimes the meaning may be lost due to differences in interpretation of language between the student and the teacher which may manifest due to cultural and ethnic differences. Visual aids allow the student to develop their own interpretation of the concept, using their internal language and understanding, which then can be guided by us the instructors. For instance, in Statics, the use of traditional methods of drawing isometric free-body diagrams and sketches representing vector mathematics concepts can be challenging; especially while modeling the physical world using vector mathematics in 3D space.

In this work, we create and implement supplemental course content to aid our students in developing their spatial understanding of the following concepts: Vector definition, addition, subtraction, as well as dot and cross products. This work will explore how interactive 3D viewports may be used within the classroom with focus on topics that are fundamental for the understanding of the mathematical analysis process in engineering. Produced content will be evaluated by instructors and teaching assistants during the office hours to assess effectiveness and develop methods of implementation within LMS (Learning Management System). The results obtained during this study will serve as a basis for further utilization of Interactive viewport technology to assess its potential of enhancing other courses.

Motivation and background of the work

A common challenge that the Mechanics department faces are in early fundamental courses that are often taught during the first two years where mathematics and physics are combined and applied. Examples of these courses include statics, strength of materials, and dynamics. The students are expected to learn fundamental vector mathematics during pre-engineering curriculum in mathematics classes such as *MATH 230: Calculus and Vector Analysis* which is a 4-credit course and includes detailed coverage of vector mathematics including vector definition, and operations such as the dot and cross products. These concepts are presented in both 2D (Two Dimensional) and 3D (Three Dimensional) space; and are notoriously difficult for the students to understand [1]. Some students will be able to perform the analysis and manipulate the equations describing the geometry concepts by applying general algorithmic solutions without the ability to visualize them. They are struggling to understand how geometric equations are represented within a physical space and this becomes even more difficult when we start to consider them in 3D. More advanced operations such as projections, dot and cross products may also be quite difficult to visualize, plot and predict. This ability to model complex systems using an FBD (Free Body Diagram) and system of equations is the enabling skillset for successful mastery of fundamental mechanics courses. Students are often required to visualize and represent an abstract mathematical model of reality in terms of vectors; enabling them to develop analytical representations and make informed decisions based on predicted outcomes.

The 2D space is often easy to represent and can be plotted using cartesian coordinate axes on paper/PowerPoint/chalkboard. This approach has been utilized repeatedly in classes throughout the curriculum. The resultant skewing of the image is natural, we can quickly adjust to these effects by observing the rectangular shape of the paper/screen which aids us while trying to interpret the graph. On the other hand, the representation of 3D space is limited because it is typically drawn on a 2D surface with implication of a 3rd axis projected into and out of the board. The interpretation of the projection of the space within the student's mind can often lead to a conflicting perspective of the vantage point. While these approaches work well for students who have already developed the visualize, plot, and predict skill sets; the initial hurdle of learning these skills can be challenging for many first- and second-year engineering students.

Traditional educational content delivery methods in these courses include textbooks, power-point presentations and in-person lecturing aided with chalk and board. Recent advances and availability of technology [2] have enabled the content to be delivered using digital medium in form of web accessible pages. Our class courses are often supplemented with content which students may access independently on LMS (Learning Management Systems) such as Canvas. Most faculty use LMS as hub for sharing lecture notes, videos, and supplemental information; often in a passive learning modality and one-way communicative approach. On these webpages, text may be supplemented with audio descriptions, images, videos, downloadable content, and other embeddable medium. This content can be used by the students during class to follow along the lecture, after class during independent study and office hours.

In addition to images and video, 3D interactive viewports may be integrated within the curriculum [3] to promote understanding and motivation. 3D technology has been demonstrated and used to teach topics of medicine [3] [4] [5], geometry [6], chemistry [5], especially where complex relationships and positioning within 3D space become hard to describe [5] and represent within 2D medium. Interactive viewports embedded within webpages provide the students with the means to explore 3D spaces while using the 2D content delivery platform such

as a phone, a screen or through AR (Augmented Reality) and immersive VR (Virtual Reality) headsets. The 3D content's perspective is no longer limited to a single vantage point, allowing interactions of panning, and manipulating the observer's perspective. This provides the means for the student to interact with the content in a natural way; similar to handling a physical object in one's hand. Interestingly, in one study of anatomy class students, interactive 3D visualizations were preferred when compared to physical models [4].

In addition to traditional supplemental content such as images and video; other advanced medium may be embedded within the webpages. For example, a traditional printed medium may also contain links to webpages, though use of printed QR-code the supplemental material can be accessed on user's device by taking a photo. A benefit of this work is to add supplemental digital content that can enable students learn independently and in an asynchronous manner providing an active learning activity to reinforce concepts taught during the lecture.

Methods

Interactive 3D viewports

3D content must be authored to accommodate our diverse community and their needs. Thus, the importance of visual appearance was considered. The colors must be of high contrast and easy to distinguish for people with visual impairments. The color palette was checked using gray scale filters and the dark gray color background was chosen to resemble the traditional blackboard. In addition, the embedded viewports are accompanied by written descriptions of the content to be displayed, assuring compatibility with assistive technologies.

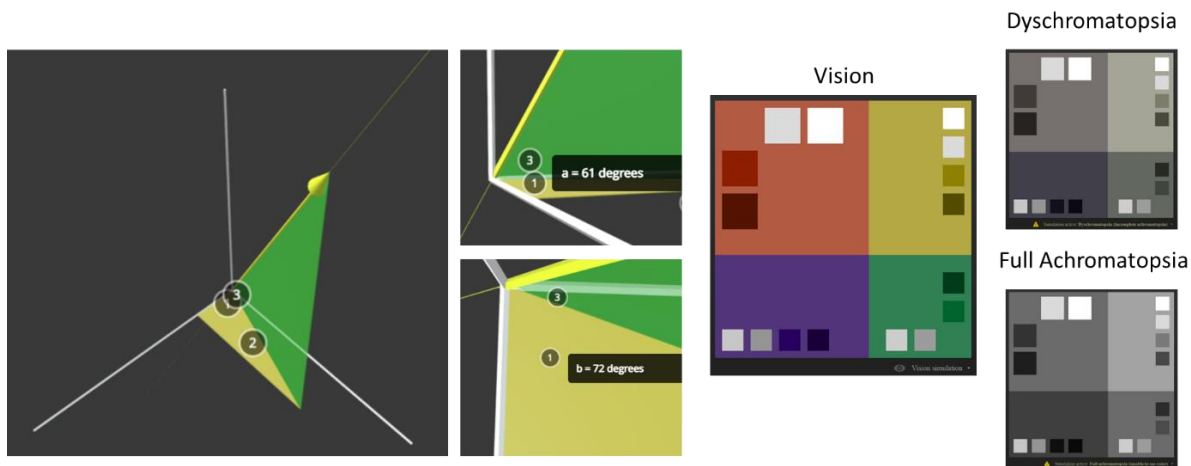


Figure 1 – Color selection

Background - #393939, Red - #B35A43, Yellow - #B3A843, Purple - #51347A, Green - #308053

Content preparation

When creating 3D models for presentation of educational content, it's important to establish clear learning objectives. Testing your experience to determine how will a student interact with your content [5] is an important step. Simply recreating 2D figures found in literature examples may not always work the same within 3D space. Surrounding context choice is also very important and affects learning and recall of information [7]. In these examples, simple shapes and single tone background are chosen to avoid unnecessary distractions. Additional details such as coordinate axes are not labeled, leaving this task for the student while they draw the free-body diagram. The position of points within the space are defined by using right-handed coordinate system and $\langle i, j, k \rangle$ unit vector components.

Embedding of Interactive 3D viewports

Models were created using Blender 3D modeling software and hosted on Sketchfab content hosting and delivery platform. The 3D viewports of models were then embedded within Canvas page similarly to the process of embedding a YouTube video. After embedding, a borderless window with controls appears within the context of the page which allows interaction. In order to provide easy access to this material for other educators a web page containing links and supplemental content was created (*sites.psu.edu/vlib*). This public webpage will index 3D educational experiences in addition to other supplemental material such as printable handouts and tutorials.

Interactive viewports for vector mathematics

While there are many variations of problems and fundamental concepts which are essential to understanding vector mathematics in mechanics. Six types of problems were created as shown in Figure 2. It is assumed that the students have already taken vector mathematics courses and will use this material as a refresher; helping them develop spatial thinking and understanding. These viewports were intended to be viewed either on a cellphone or computer screen where the user interacts with the models by zooming in/out and rotating the space with their fingers or mouse. In addition, these experiences can be viewed using VR/XR headsets.

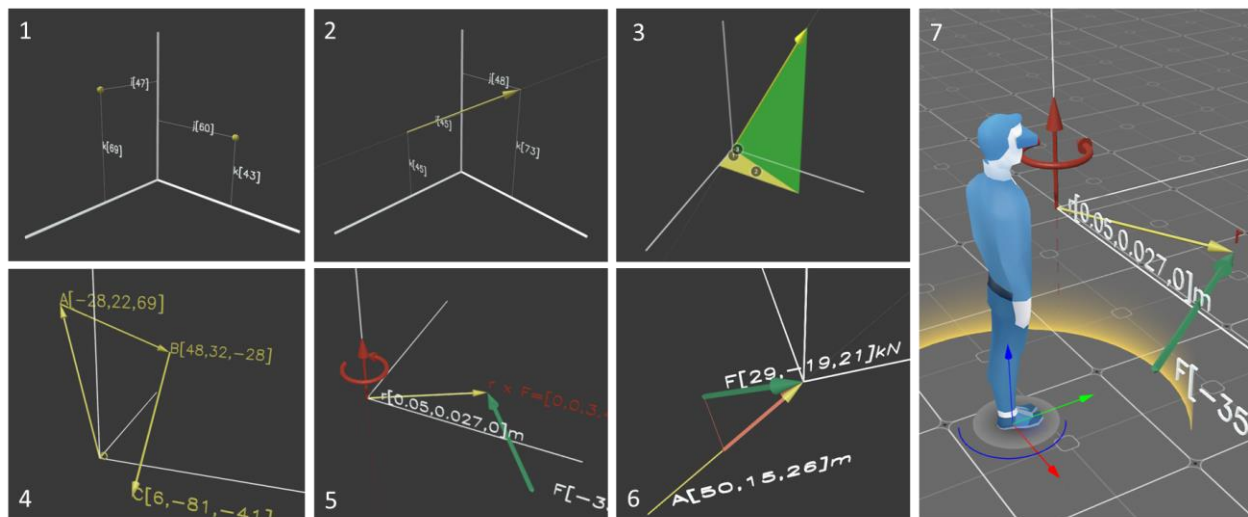


Figure 2 – Example use of interactive viewports

(1) Distance between two points. (2) Defining a vector. (3) In-plane vector projection. (4) Tail to tail vector addition. (5) Defining a moment (Cross Product). (6) Vector projection (Dot Product). (7) Perspective use of content within VR/AR space.

Assessment of effectiveness planning

Initial testing of the technology tools was conducted during EMCH 211 (Statics) office hours by the instructor of class section consisting of 100 students. An invitation for voluntary participation was encouraged and 4 students participated in this preliminary study. Initial assessment included interactive 3D visualization of vector mathematics problems and a moment couple example shown by the instructor on a computer tablet while explaining the topics. The qualitative assessment of student's satisfaction and perceived benefit was conducted via an informal conversation. This allowed improving the current implementation of content by

adjusting the 3D model features, colors and methods of delivery before exposure to a wider audience.

Currently, the students are provided with worksheets containing six types of vector mathematics practice problems. The worksheets contain printed QR code links which lead to the interactive viewports to be viewed on student's personal device. In addition, students can use a VR headset to view the 3D model immersivity. These worksheets are attached as supplement A through G.

Results

Preliminary Findings

Initial student reactions were positive. It is possible that the observations of student's interactions may be influenced by prior experience with other technologies which use 3D graphics and their level of current spatial understanding. One student was able to relate their experience in computer-aided drafting (CAD) software to manipulate the scene and simplify the 3D vector problem into a set of 2D problems by considering projections along coordinate hyperplanes. Another student mentioned interaction similarity to video game experience. Students who observed the models through VR headsets moved around the space by crouching and looking around the virtual space containing vector model. Some students also mentioned that the 2D visuals as presented in class and textbook figures made the vector problem appear more difficult to solve and easier after observing it within an immersive 3D view; noting that it's easy to see the geometrical constraints that lead to simplification of the 3D vector problem.

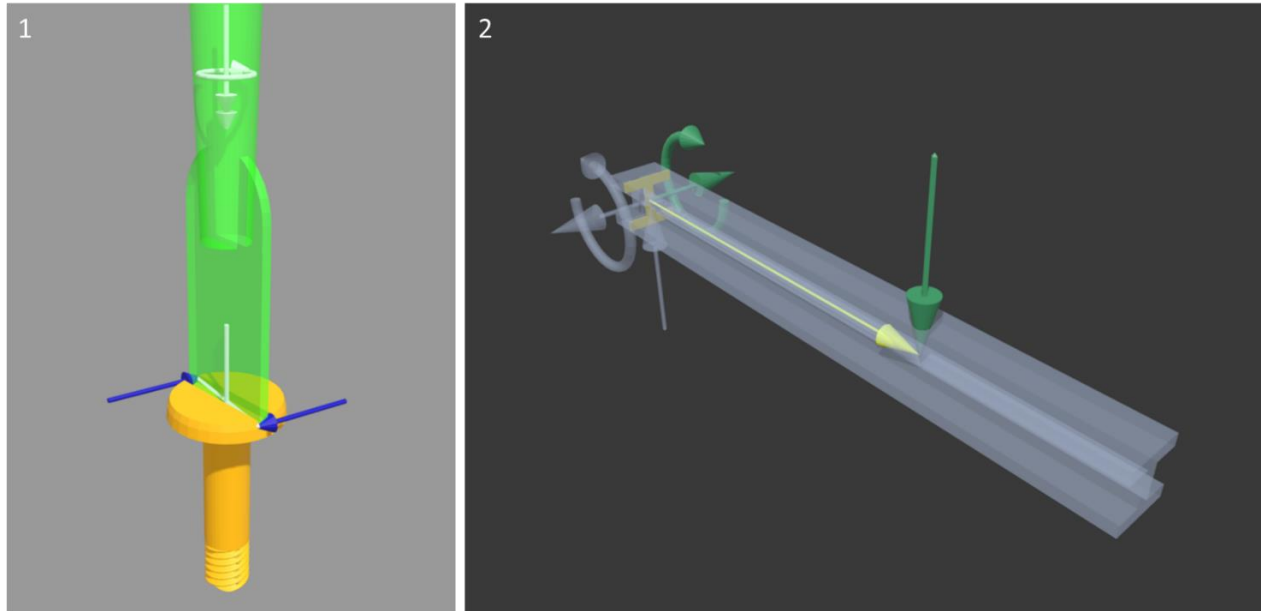
One apparent result was that unguided experiences remains an issue. Some initial feedback indicated the need for user manuals and guides to aid the students during the experience rather than expecting them to maneuver on their own. Students also asked for ability to interact with and change the scene; beyond the ability to observe the 3D models. While the initial results are positive, further assessment is necessary to determine the generalizable impact on engineering students and beyond.

Further development

Fundamental operations such as vector definition and addition are easily explained through the visualization; while advanced operations such as cross and dot products are harder to portray and may need further breakdown; showing multiple steps to increase their clarity. This can be achieved through animation by revealing portions of the 3D model sequentially. In addition, object motion may be recorded within a simulation environment, yielding more accurate representations of concepts in physics and beyond. Lastly, it is possible that some complex problems may benefit from additional stimulation through use of auditory cues and haptic feedback; providing a more immersive experience. Additional examples describing the moment due to a force couple during installation of a fastener and simple beam loading scenario were created bridging the gap between theoretical mathematics and its application.

As this is a Work-In-Progress study, we are interested to see how traditional modalities of 3D representation on (flat paper/board/screen) compared to 3D interactive viewport experiences while learning vector mathematics. We would like to see what the student's perception of this technology use within the classroom is, are there any unexpected side effects and hurdles? How does it affect their motivation to learn and engage with the class content? We would also like to see the technique's effectiveness when applied in classes with a very large student body, where personal attention students receive is often limited. In addition, we would like to explore the use

of technology during supervised and independent study. Lastly, we want to see how the interactive viewports could be used on the projection screen during the lecture, expanding instructors ability to explain spatial relationships; while enabling the students engage with the content during the lecture on their own devices.



*Figure 3 – Use of vector mathematics within statics class
(1) Moment due to a force couple. (2) Simple beam loading scenario.*

For future assessments, the goal is to implement these improved models into other statics course sections as both authors teach mechanics courses throughout the year. Some considerations for implementing traditional and non-traditional methods may be by having one section taught using traditional approaches and another with access to interactive viewports, VR/AR experiences. A quantitative assessment may be conducted using a common quiz between the two sections on several problem sets: 1) determination of components, 2) dot-, 3) cross-products, 4) moment of a couple, and a 5) 3D Rigid Body equilibrium problem. Further qualitative assessment of students' satisfaction and self-motivation to learn will also be explored and compared. Lastly, spatial understanding evaluation could be conducted at the beginning and the end of the semester to measure the teaching tool's effectiveness.

Significance

Development of educational content delivery methods

Continued development and improvement of educational tools and content is vital for all stakeholders in the education system, across all levels, from primary schools to universities and during continued workforce development after graduation [8]. Sometimes, communication barriers due to cultural differences or limited language skills may result in misunderstanding of the material being taught. Visual aids can help the student develop their own internal understanding of the subject while learning new information [7]; which is then guided by the instructor. By utilizing the latest technology in combination with traditional tools we can provide a more inclusive environment for everyone. Fostering a student-centric environment which promotes self-motivation to learn.

Considering the students motivation

Students are often required to learn a vast amount of information which we the educators consider fundamental; enabling background required to master the advanced topics. Sometimes the students do not understand why they are learning the given topic and assume they will not use the skills gained in the future; beyond passing the class. It's difficult to find additional time to cover the significance/relevance of the topic in detail during class, because it would take away valuable time required to explain the main topic of the subject.

One solution to address this problem is to develop supporting content which conveys the social significance and relevance of the subject though indirect methods such as contextual ques. Similar approaches are already used within the visual aids found in textbooks; where each chapter often has an introduction showing how and where the topics can be applied and why they matter. This is often explained though written word and supplemented with passive content consumption experiences such as pictures and videos. By offering interactive content which students can control and engage with; we can help the student maintain attention and increase interest. It's been shown that context within immersive experiences such as VR positively affects information encoding and recall if chosen properly [7]. Through a conscious choice of context and 3D model features; topic's application and social impact can be highlighted and explained, while promoting students' information retention and recall.

Interactive viewports and immersive VR experiences are active forms of content consumption. Students can manipulate the scene and engage with the content while receiving continuous real-time feedback. This may be most beneficial in classes with a large number of students, where students may receive limited personalized feedback from the instructor.

Developing accessible content

This work examines methods, practices, and considerations while producing and publishing supplemental course content enriched by 3D visualizations. By offering this content through multiple platforms such as interactive viewports on 2D screens and immersive experiences though virtual reality headsets; we enable more students to benefit from the supplemental experiences. By creating publicly accessible virtual libraries (*sites.psu.edu/vlib*) the produced content becomes easily deployable at scale for use by a broader community of educators at little to no additional cost. Lastly, the described methods can be adopted beyond the scope of explaining vector mathematics, any subject where spatial information is required could benefit from additional supplemental material powered by 3D visualization.

Supplement A: Interactive vector mathematics worksheet:

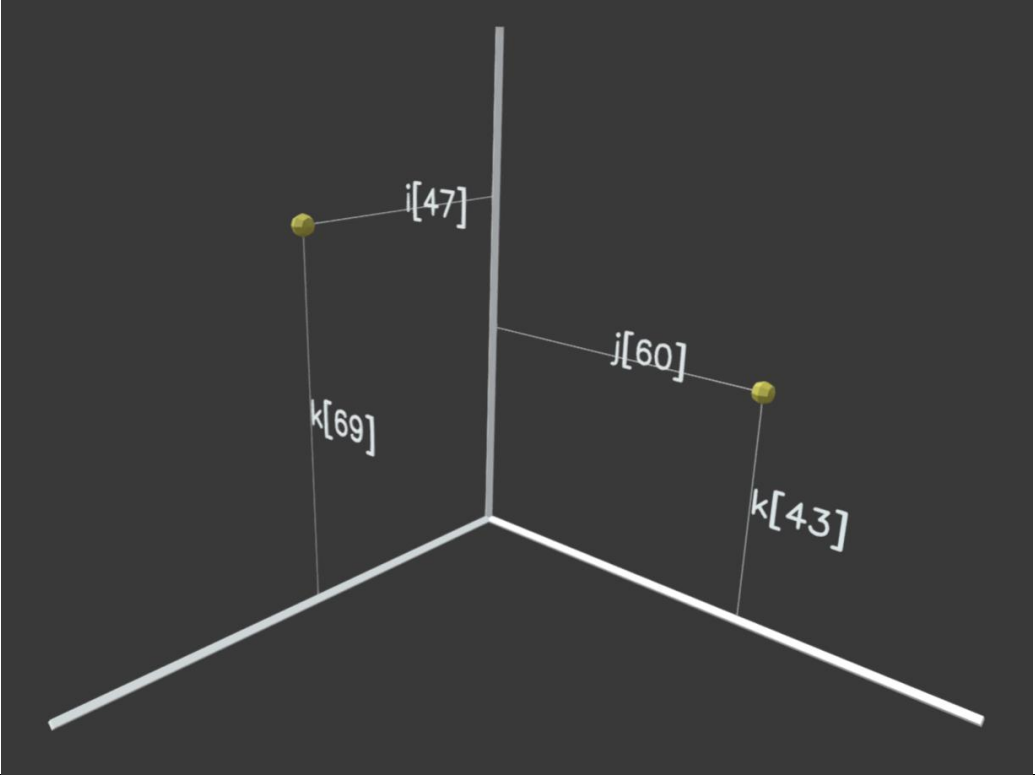



<https://sites.psu.edu/vlib/> ESM Virtual 3D Library: Catalogue of educational 3D content

Please scan the QR code using your phone or tablet's photo app to explore the interactive experience.

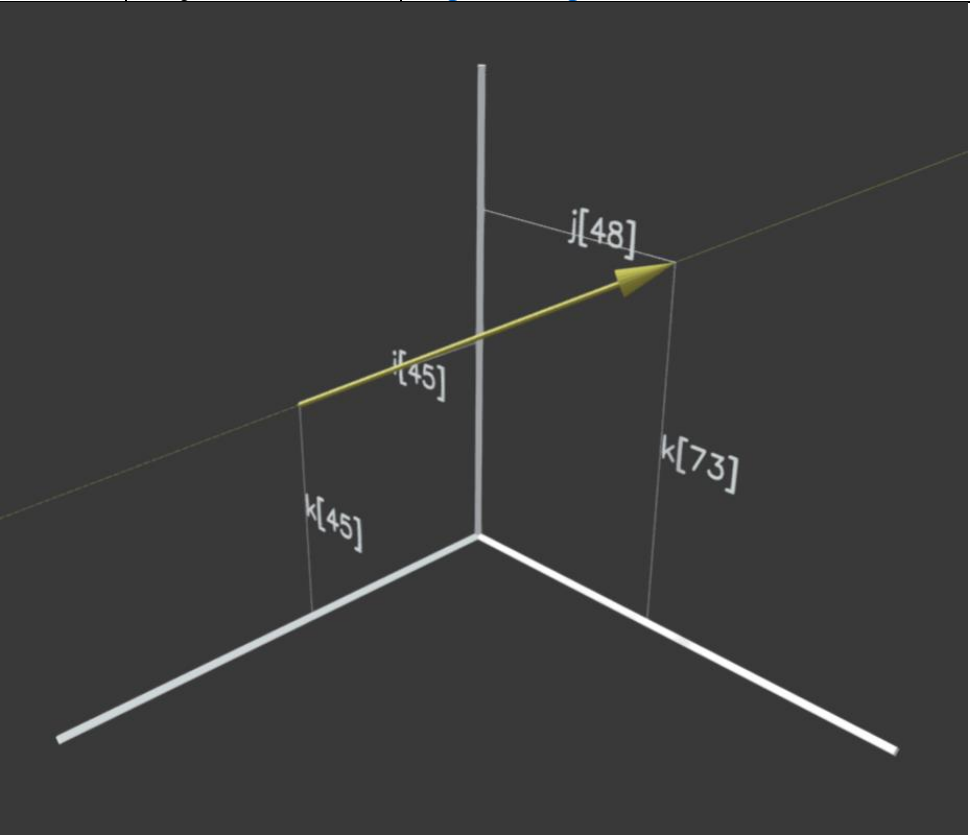

	Interactive viewport description	Preview	Live example
1	<p>Distance between two points.</p> <p>Calculate distance between two points in: A – (47,0,69) B – (0,60,43)</p>		 Link
2	<p>Defining a vector</p> <p>Define a vector between two points: A – (45,0,45) B – (0,48,73)</p>		 Link
3	<p>In-plane vector projection</p> <p>Define a vector based on in plane projection: (1) A – 72 deg (angle between x-axis and vector projection) (2) Length – 65 mm (vector projection length on x-y plane) (3) B – 61 deg (in plane angle)</p>		 Link
4	<p>Tail to tail vector addition</p> <p>Add vectors: A – (-28,22,69) B – (48,32,-28) C – (6,-81,-41)</p>		 Link
5	<p>Defining a moment, (Cross Product)</p> <p>Calculate a moment due to force F given the moment arm vector r: r – (0.05,0.027,0) m F – (-35,50,0) kN</p>		 Link
6	<p>Vector projections, (Dot Product)</p> <p>Calculate the projection of vector F along A: F – (29,-19,21) kN A – (50,15,26) m</p>		 Link

Supplement B:
Interactive vs Traditional vector mathematics worksheet:

	Year:	Major:	https://sites.psu.edu/vlib/ ESM Virtual 3D Library
			
1	<p>Distance between two points.</p> <p>Calculate distance between two points in: A – (47,0,69) B – (0,60,43)</p> <p>Work Area:</p>	<p>Draw Free Body diagram:</p>	 Link
	<p>Describe your experience:</p>		<p>Interactive: Yes / No</p>

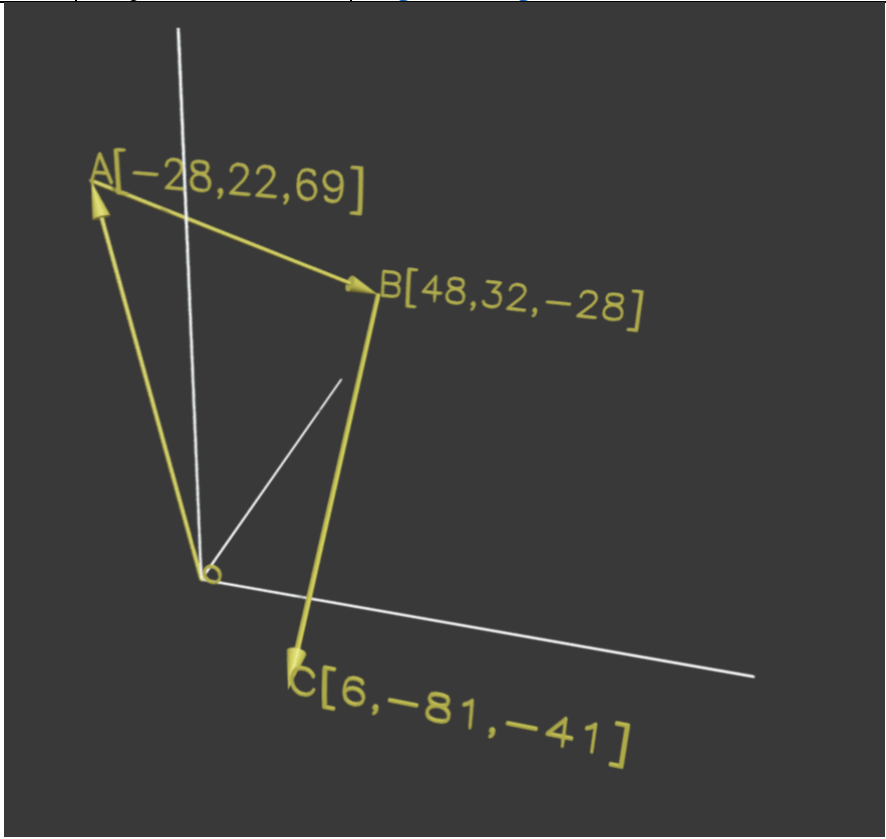

Supplement C:

Interactive vs Traditional vector mathematics worksheet:

Year:	Major:	https://sites.psu.edu/vlib/ ESM Virtual 3D Library	
			
2	<p>Defining a vector</p> <p>Define a vector between two points: A – (45,0,45) B – (0,48,73)</p> <p>Work Area:</p>	<p>Draw Free Body diagram:</p>	 Link
<p>Describe your experience:</p>			<p>Interactive: Yes / No</p>

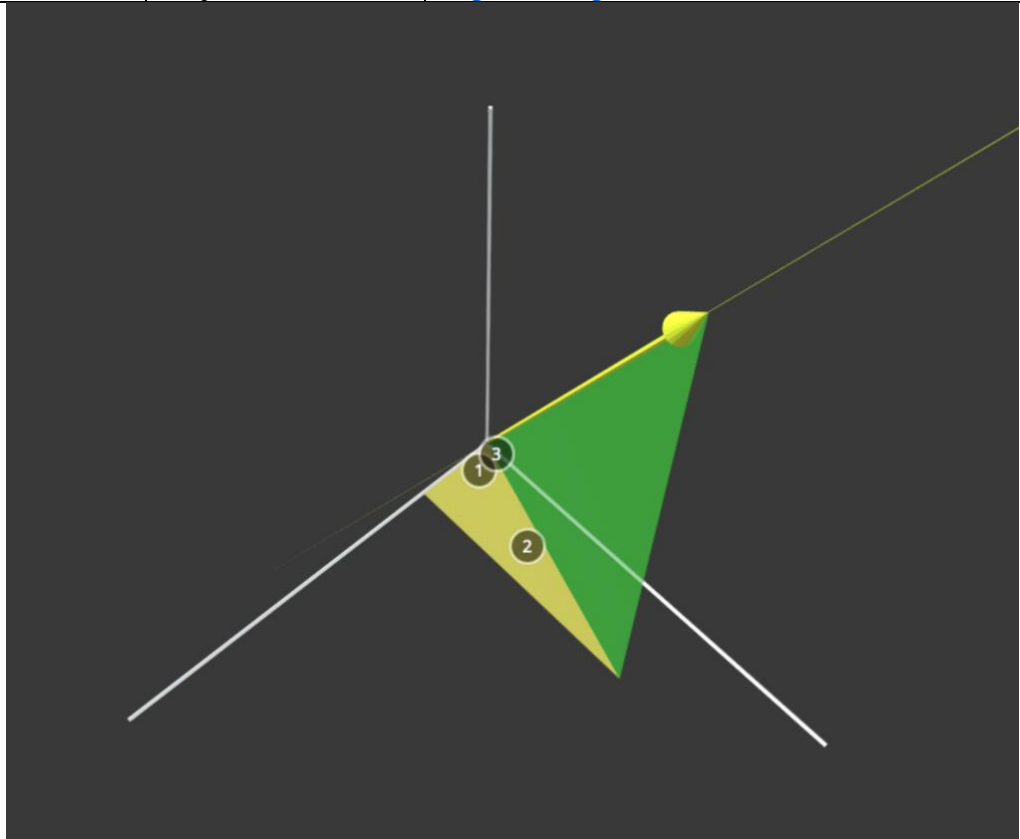

Supplement D:

Interactive vs Traditional vector mathematics worksheet:

Year:	Major:	https://sites.psu.edu/vlib/ ESM Virtual 3D Library	
			
4	<p>Tail to tail vector addition</p> <p>Add vectors: A – (-28,22,69) B – (48,32,-28) C – (6,-81,-41)</p> <p>Work Area:</p>	<p>Draw Free Body diagram:</p>	 Link
<p>Describe your experience:</p>			<p>Interactive: Yes / No</p>

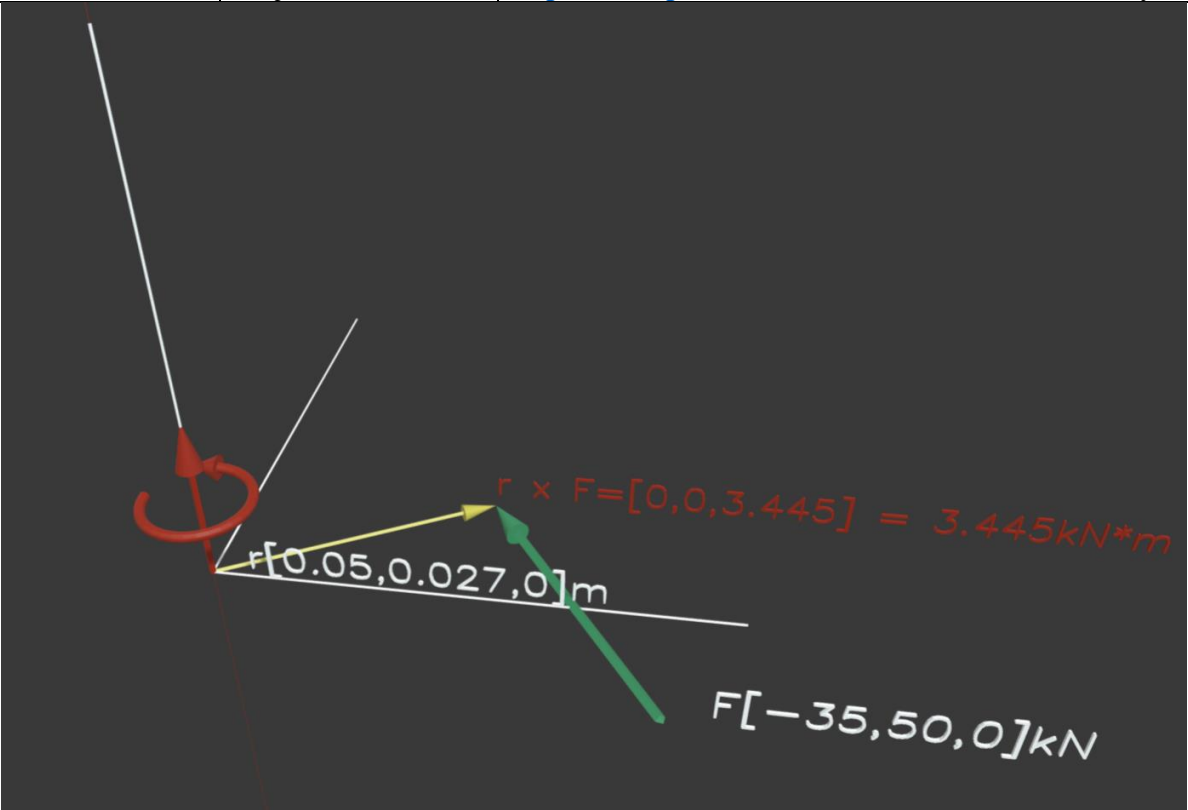

Supplement E:

Interactive vs Traditional vector mathematics worksheet:

Year:	Major:	https://sites.psu.edu/vlib/ ESM Virtual 3D Library	
			
3	<p>In-plane vector projection</p> <p>Define a vector based on in plane projection:</p> <p>(1) A – 72 deg (angle between x-axis and vector projection)</p> <p>(2) Length – 65 mm (vector projection length on x-y plane)</p> <p>(3) B – 61 deg (in plane angle)</p> <p>Work Area:</p>	<p>Draw Free Body diagram:</p>	 <p>Link</p>
<p>Describe your experience:</p>			<p>Interactive: Yes / No</p>

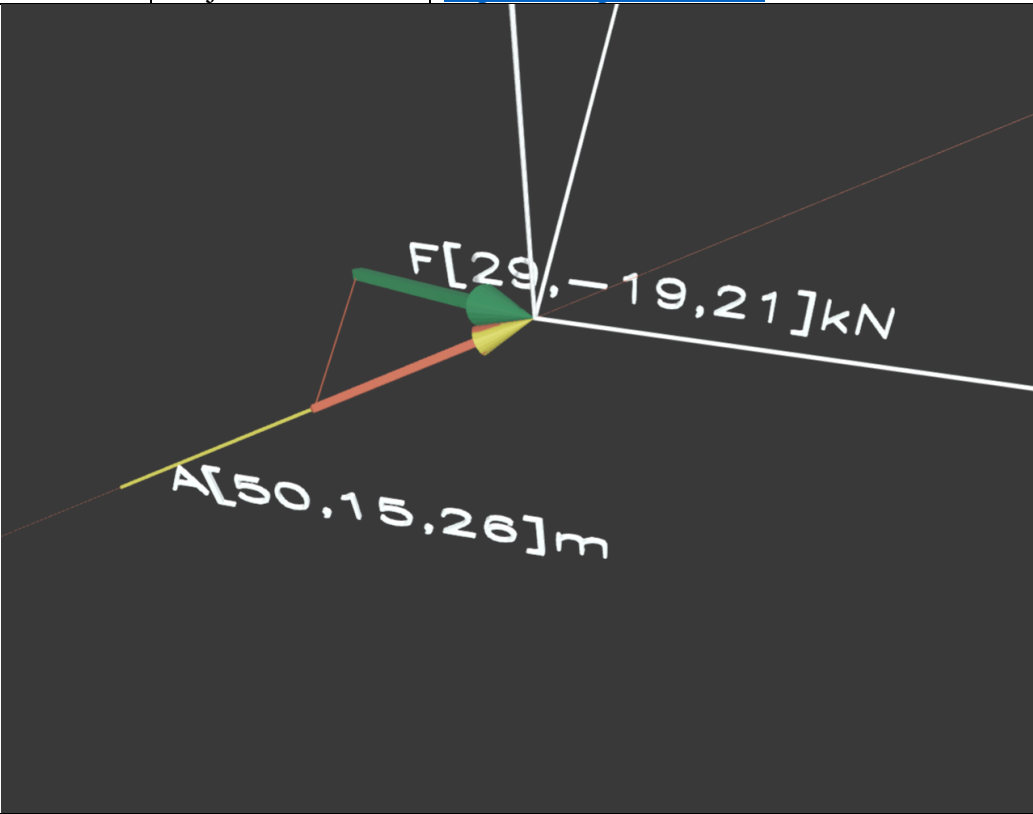

Supplement F:

Interactive vs Traditional vector mathematics worksheet:

Year:	Major:	https://sites.psu.edu/vlib/ ESM Virtual 3D Library	
			
5	<p>Defining a moment, (Cross Product)</p> <p>Calculate a moment due to force F given the moment arm vector r: $\mathbf{r} = (0.05, 0.027, 0) \text{ m}$ $\mathbf{F} = (-35, 50, 0) \text{ kN}$</p> <p>Work Area:</p>	<p>Draw Free Body diagram:</p>	 <p>Link</p>
<p>Describe your experience:</p>			<p>Interactive: Yes / No</p>

Supplement G:

Interactive vs Traditional vector mathematics worksheet:

	Year:	Major:	https://sites.psu.edu/vlib/ ESM Virtual 3D Library	
				
6	<p>Vector projections, (Dot Product)</p> <p>Calculate the projection of vector F along A: $\mathbf{F} = (29, -19, 21) \text{ kN}$ $\mathbf{A} = (50, 15, 26) \text{ m}$</p> <p>Work Area:</p>		<p>Draw Free Body diagram:</p>	 Link
	<p>Describe your experience:</p>			<p>Interactive: Yes / No</p>

Bibliography

- [1] J. D. Will, P. E. Johnson and C. Graunke, "Virtual reality for 3D visualization in a statics course," in *ASEE Annual Conference and Exposition, Conference Proceedings*, 2005.
- [2] C. Grant and C. Mairn, "3D, virtual, augmented, extended, mixed reality, and extended content forms: The technology and the challenges," *Information Services Use*, vol. 40, no. 3, pp. 225-230, 2020.
- [3] J. J. Azkue, "Embedding interactive, three-dimensional content in portable document format to deliver Gross Anatomy Information and Knowledge," *Clinical Anatomy*, vol. 34, no. 6, pp. 919-933, 2021.
- [4] J.-E. Ha and D.-Y. Choi, "Educational effect of 3D applications as a teaching aid for anatomical practice for dental hygiene students," *Anatomy Cell Biology*, vol. 52, no. 4, p. 414, 2019.
- [5] C. P. Fabris, A. J. Rathner, A. Y. Fong and C. P. Sevigny, "Virtual reality in higher education," *International Journal of Innovation in Science and Mathematics Education*, vol. 27, no. 8, 2019.
- [6] M. Flores-Bascuñana, P. D. Diago, R. Villena-Taranilla and D. F. Yáñez, "On augmented reality for the learning of 3D-geometric contents: A preliminary exploratory study with 6-grade primary students," *Education Sciences*, vol. 10, no. 1, p. 4, 2019.
- [7] "Enhancing learning and retention with distinctive virtual reality environments and mental context reinstatement," *npj Science of Learning*, no. 7, p. 31, 2022.
- [8] Committee on the Options for a National Plan for Smart Manufacturing, *Workshop on Education, Training, and Workforce Needs*, Washington: National Academies of Science, 2023.
- [9] E. Scott, A. Soria and M. Campo, "Adaptive 3D virtual learning environments—a review of the literature," *IEEE Transactions on Learning Technologies*, vol. 10, no. 3, pp. 262-276, 2017.
- [10] O. Ha, "Development of a low-cost immersive virtual reality solution for STEM classroom instruction: A case in Engineering Statics," in *2nd International Workshop on Artificial Intelligence and Education (WAIE 2020)*, New York, NY, USA, 2020.