Work in Progress: Redesigning of a Computing Course in the Civil Engineering Curriculum

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Abstract

In the presented paper, the thought process and special considerations for redesigning of an introductory level computing course in the Civil Engineering and Construction Department at Georgia Southern University is included and discussed. This redesign exploits the special numerical, symbolic, and programming capabilities of Excel, MATLAB, and Mathcad software tools to determine the solution of solid and structural problems. Example problems related to other areas in the CE curriculum can also be developed as desired. In the provided samples problems, the effective utility of various loop structures, conditional statements, programming operators, special array operations, as well as user-defined functions is demonstrated. The redesigned computing course familiarizes the students with the full capabilities and special advantages of each of the three selected software tools. The students can effectively utilize the computing and programming skills mastered through taking this course to develop the solutions to a wide range of more advanced problems in various specialty areas in Civil Engineering.

I. Introduction

There has been a long-standing debate among engineering educators from various engineering fields regarding which software tools should be included in their respective departmental computing courses to maximize the benefit for the students. The details regarding some of these deliberations and considerations have also been documented in various publications. In the presented paper, a course redesign for a computing course offered in the Civil Engineering program at Georgia Southern University, involving the specific utilization of Excel, MATLAB, and Mathcad is discussed. In several earlier publications of the author[1-3], the utility of several computing tools including Excel and MATLAB in support of student projects and engineering courses was considered. In redesigning of the computing course described in this paper, prior positive experience and success of the author in effective utilization of Excel and MATLAB played a convincing role in the selection of these two specific tools. Involvement of the author in offering other higher-level engineering course such as the Introduction to Finite Elements, has also convinced him that Mathcad is also an important tool for civil engineering students and this software should also be included in the departmental computing course. This application software has proven to be especially valuable when dealing with problem solutions which involve manipulation of matrices and solving systems of equations. Additionally, equations created in the Mathcad environment appear in the same visual form as written on paper, making it easier for students to formulate and document their work. Moreover, formulation of problem solutions involving symbolic operations using Mathcad also seems to be easier for students to master, compared to obtaining alternate solutions utilizing other computing and programming tools.

The presented sample Excel spreadsheet, MATLAB script file, and the three Mathcad files included in the paper show the utility of the selected computing tools in support of several required and elective courses offered in the Civil Engineering curriculum. The example problems such as these could complement some of the topics covered in statics, dynamics, mechanics of materials, structural analysis, as well as in other more advanced courses such as finite elements, and structural dynamics. Example tasks performed using the selected software pieces could include: determination of centroid/center of mass/center of gravity of bodies, determination of internal reactions and corresponding developed stresses in beams, calculation of nodal and element forces in various structural members, and computations related to determining response of structures subjected to various dynamic loads. The five sample problem solutions included in the paper demonstrate the special utility of the three selected programming tools in performing several of the listed tasks.

II. Analysis of a Beam - Excel Example

The special capabilities of Excel in solving engineering problems are planned to be explored in the redesigned course first, before covering the utility of the other two selected computing tools. This decision was made due to the fact that the majority of students already have some basic familiarity with Excel, and therefore can probably better grasp the programming fundamentals using this basic tool first. One sample Excel spreadsheet for analyzing a beam is presented in this part of the paper to better illustrate the utility of this computing tool in the discussed computing course. The presented example problem is taken from an earlier publication of the author[1]. In this sample problem, an overhanging beam is subjected to a uniformly distributed load as shown in Fig. 1. The developed spreadsheet computes the shear, moment, slope, and deflection values along the length of the beam, and produces the corresponding plots for the beam using the presented algebraic Eqs. (1-8). When developing a spreadsheet such as this, the students are instructed to create proper identifying "names" for each of the cells containing the problem parameters (e.g., beam and loading parameters) so that formulas become easier to setup and manage. The content of one of the created Excel formulas for computing the deflection values is shown in the spreadsheet provided in Fig. 1. Note that in this formula an "IF" function is used for the sake of efficiency in calculating the deflection values, so that a single formula can be used for the entire beam. Similar formulas are also produced in the spreadsheet to yield the shear, moment, and slope values. Note that in the produced spreadsheet, the values of the beam and loading parameters can easily be modified to determine the solutions of a wide range of problems.

Region I (when $0 \leq x \leq L$):

$$V_1 = \frac{-w}{2L}(a^2 - L^2) - wx \tag{1}$$

$$M_1 = \frac{-wx^2}{2} - \frac{w}{2L}(a^2 - L^2)x \tag{2}$$

$$v_1' = \frac{-w}{24EIL}\left[4Lx^3 + 6(a^2 - L^2)x^2 - L^2(2a^2 - L^2)\right] \tag{3}$$

$$v_1 = \frac{-wx}{24EIL}\left[Lx^3 + 2(a^2 - L^2)x^2 - L^2(2a^2 - L^2)\right] \tag{4}$$

Region II (when $L \le x \le L+a$):

$$V_2 = w(L + a - x) \tag{5}$$

$$M_2 = \frac{-w(L + a - x)^2}{2} \tag{6}$$

$$v_2' = \frac{-w}{24EI}\left[-4(L + a - x)^3 - (L^3 - 4La^2 - 4a^3)\right] \tag{7}$$

$$v_2 = \frac{-w}{24EI}\left[(L + a - x)^4 - (L^3 - 4La^2 - 4a^3)x - (a^4 - L^4 + 4L^2a^2 + 4La^3)\right] \tag{8}$$

III. Analysis of a Beam - MATLAB Example

To familiarize the students with some of the commonly used programming fundamentals and constructs, MATLAB software was selected for utilization in the redesigned course. It is perceived that if students can master building these constructs utilizing this powerful and well-recognized computing tool, they can easily use their acquired skills to create similar constructs using other software tools. The students may be required to create such programming structures in their follow-up courses and/or future professional careers. One sample problem that could be used in the course for analyzing a beam using MATLAB is presented in Fig. 2. The details of the beam are provided in this figure, along with the prescribed details regarding how the students are expected to produce their script file. This exercise is similar to what the students had to perform in the previous problem using Excel. When creating the M-file for the problem, the students are required to utilize MATLAB library and user-defined functions, in addition to using various programming constructs such of various loops and conditional statements. Utilization of these functions and procedures are required to produce a table of values for shear, moment, slope, and deflection, and to additionally generate the corresponding requested plots in the desired format. MATLAB script file and the produced results for the presented problem are respectively provided in Figs. 3 & 4. If desired, the students can further be asked to incorporate additional programming lines in their script file to also compute various normal and shear stresses at specific locations on the beam.

=IF(@x<=L,-w*@x/(24*E*I*L)*(L*@x^3+2*(a^2-L^2)*@x^2-L^2*(^
2*a^2-L^2)),-w/(24*E*I)*((L+a-@x)^4-(L^3-4*L*a^2-4*a^3)*@x-(
a^4-L^4+4*L^2*a^2+4*L*a^3)))

	A	B	C	D	E	F	G	H	I
1	**Analysis of a Statically Determinate Beam**								
2	Loading Type: Distributed Force Acting Over the Entire Beam								
3	x (in.)	Shear (lb)	Moment (lb.in)	Slope (rad)	Deflection (in)				
4	0	1500	0	-1.655E-03	0.000E+00				
5	12	1300	16800	-1.620E-03	-1.972E-02				
6	24	1100	31200	-1.519E-03	-3.861E-02				
7	36	900	43200	-1.365E-03	-5.597E-02				
8	48	700	52800	-1.165E-03	-7.119E-02				
9	60	500	60000	-9.310E-04	-8.379E-02				
10	72	300	64800	-6.720E-04	-9.343E-02				
11	84	100	67200	-3.981E-04	-9.986E-02				
12	96	-100	67200	-1.192E-04	-1.030E-01				
13	108	-300	64800	1.548E-04	-1.027E-01				
14	120	-500	60000	4.138E-04	-9.931E-02				
15	132	-700	52800	6.480E-04	-9.291E-02				
16	144	-900	43200	8.474E-04	-8.390E-02				
17	156	-1100	31200	1.002E-03	-7.275E-02				
18	168	-1300	16800	1.102E-03	-6.006E-02				
19	180	-1500	0	1.138E-03	-4.655E-02				
20	192	-1700	-19200	1.099E-03	-3.305E-02				
21	204	-1900	-40800	9.757E-04	-2.051E-02				
22	216	-2100	-64800	7.581E-04	-1.001E-02				
23	228	-2300	-91200	4.361E-04	-2.736E-03				
24	240	-2500	-120000	0.000E+00	0.000E+00				
25	252	1800	-97200	-4.486E-04	-2.786E-03				
26	264	1600	-76800	-8.077E-04	-1.041E-02				
27	276	1400	-58800	-1.087E-03	-2.185E-02				
28	288	1200	-43200	-1.298E-03	-3.623E-02				
29	300	1000	-30000	-1.448E-03	-5.276E-02				
30	312	800	-19200	-1.549E-03	-7.079E-02				
31	324	600	-10800	-1.610E-03	-8.978E-02				
32	336	400	-4800	-1.642E-03	-1.093E-01				
33	348	200	-1200	-1.654E-03	-1.291E-01				
34	360	0	0	-1.655E-03	-1.490E-01				
36	**Beam and Load Parameters:**								
38	w (lb/in) =	16.667		L (in) =	240				
39	E (psi) =	2.90E+07		a (in) =	120				
40	I (in^4) =	100							

"Named" cells containing the beam & load parameters created for utilization in the Excel formulas used.

Fig. 1 Excel Example Spreadsheet for Analysis of a Beam

Analysis of Beam Using MATLAB

For the beam shown, the expressions for the deflection, slope, shear force, and bending moment are provided below. In these expressions: $E = 29 \times 10^6$ psi., $I = 1830$ in^4, $L = 20$ ft, $a = 6$ ft, $b = 14$ ft, and $P = 35000$ lb.
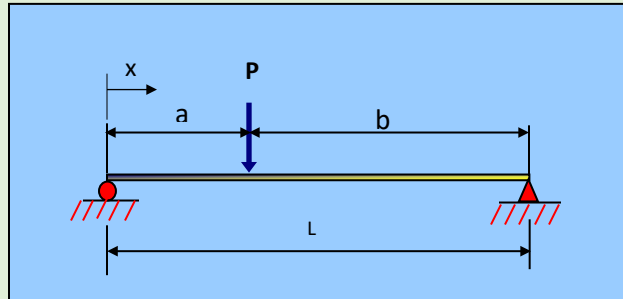
When $0 \le x \le a$:

$$v(x) = \frac{Pbx}{6EIL}(x^2 + b^2 - L^2)$$

$$\theta(x) = \frac{Pb}{6EIL}(3x^2 + b^2 - L^2)$$

$$V(x) = \frac{Pb}{L}$$

$$M(x) = \frac{Pb}{L}x$$

When $a \le x \le L$:

$$v(x) = \frac{Pa(L-x)}{6EIL}(x^2 - 2Lx + a^2)$$

$$\theta(x) = \frac{Pa}{6EIL}(-3x^2 + 6Lx - a^2 - 2L^2)$$

$$V(x) = -\frac{aP}{L}$$

$$M(x) = \frac{aP}{L}(L-x)$$

Develop a MATLAB script file to compute and plot the deflection (v), Slope (θ), shear force (V), and bending moment (M) for the beam in the manner described below.

➤ The main calling program should prompt the user to input the values for E, I, L, a, b, and P.

➤ The program should employ a user-defined function and a "for loop" to compute the values of deflection (v), Slope (θ), shear force (V), and bending moment (M) for values of x ranging from 0 to L using increments of 12 inches for x. Note that these values are expected to be returned to the main calling program.

➤ The values of x and the corresponding computed values of deflection, Slope, shear force, and bending moment are expected to be printed in a table format (with appropriate column headings) using "fprintf" statements placed in the main program.

➤ Plots of deflection, slope, shear force, and bending moment are expected to be produced using the "subplot" statements placed in the main program. To learn more about the "subplot" command, please refer to the provided instructor handout. Place all equations on the plot using the procedure discussed in the lab.

Fig. 2 MATLAB Example Assignment Statement for Analysis of a Beam

Main Script File:

```
E=input('Enter the value for E (psi)\n');
I=input('Enter the value for I (in^4)\n');
L=input('Enter the value for L (in)\n');
a=input('Enter the value for a (in)\n');
b=input('Enter the value for b (in)\n');
p=input('Enter the value for p (lb)\n');
```

Prompting the user to input the values for the beam and loading parameters.

```
L=L*12;a=a*12;b=b*12;

fprintf(' x(in)    Deflection(in)   Slope (radians)    Shear (lb)     moment (lb.in)\n')

[x,deflection,slope,shear,moment]=compute(E,I,L,a,b,p);
```

```
for k=1:1:L/12+1
    fprintf('%10.0f  %14.3e  %14.3e  %14.3f  %14.3f\n', ...
    x(k),deflection(k),slope(k),shear(k),moment(k))
end
```

Do loop used for providing the problem results in a table format.

```
subplot(2,2,1),plot(x,deflection),title('Deflection'),xlabel('x (in.)'),ylabel('Deflection (in)')
subplot(2,2,2),plot(x,shear),title('Shear'),xlabel('x (in.)'),ylabel('Shear (lb)')
subplot(2,2,3),plot(x,slope),title('Slope'),xlabel('x (in.)'),ylabel('Slope (radians))')
subplot(2,2,4),plot(x,moment),title('Moment'),xlabel('x (in.)'),ylabel('Moment (lb.in)')
```

Subplot command used for plotting the distributions of deflection, shear, slope, and moment along the length of the beam.

Created user-defined function for calculating the problem results.

Function m-file:

```
function[x,deflection,slope,shear,moment]=compute(E,I,L,a,b,p)
```

MATLAB Library function used for creating equally spaced of *x*.

```
x=linspace(0,L,L/12+1);
```

```
for k=1:1:L/12+1
    if x(k)<=a
        deflection(k)=(p*b*x(k))*(x(k)^2+b^2-L^2)/(6*E*I*L);
        slope(k)=(p*b)*(3*x(k)^2+b^2-L^2)/(6*E*I*L);
        shear(k)=p*b/L;
        moment(k)=(p*b/L)*x(k);
    else
        deflection(k)=(p*a*(L-x(k)))*(x(k)^2-2*L*x(k)+a^2)/(6*E*I*L);
        slope(k)=(p*a)*(-3*x(k)^2+6*L*x(k)-a^2-2*L^2)/(6*E*I*L);
        shear(k)=-a*p/L;
        moment(k)=(a*p/L)*(L-x(k));
    end
```

Do loop and with a nested conditional "if" statement used for computing the deflection, slope, shear, and moment values for the two regions of the beam.

Fig. 3 MATLAB Script File for Analyzing the Beam

```
Enter the value for E (psi)
29000000
Enter the value for I (in^4)
1830
Enter the value for L (in)
20
Enter the value for a (in)
6
Enter the value for b (in)
14
Enter the value for p (lb)
35000
 x(in)   Deflection(in)  Slope (radians)   Shear (lb)    moment (lb.in)
    0     0.000e+000      -2.260e-003       24500.000        0.000
   12    -2.699e-002      -2.227e-003       24500.000     294000.000
   24    -5.318e-002      -2.127e-003       24500.000     588000.000
   36    -7.778e-002      -1.961e-003       24500.000     882000.000
   48    -9.998e-002      -1.728e-003       24500.000    1176000.000
   60    -1.190e-001      -1.429e-003       24500.000    1470000.000
   72    -1.340e-001      -1.064e-003       24500.000    1764000.000
   84    -1.444e-001      -6.790e-004      -10500.000    1638000.000
   96    -1.504e-001      -3.229e-004      -10500.000    1512000.000
  108    -1.523e-001       4.748e-006      -10500.000    1386000.000
  120    -1.504e-001       3.039e-004      -10500.000    1260000.000
  132    -1.451e-001       5.746e-004      -10500.000    1134000.000
  144    -1.368e-001       8.167e-004      -10500.000    1008000.000
  156    -1.256e-001       1.030e-003      -10500.000     882000.000
  168    -1.121e-001       1.216e-003      -10500.000     756000.000
  180    -9.658e-002       1.372e-003      -10500.000     630000.000
  192    -7.932e-002       1.501e-003      -10500.000     504000.000
  204    -6.069e-002       1.600e-003      -10500.000     378000.000
  216    -4.103e-002       1.671e-003      -10500.000     252000.000
  228    -2.068e-002       1.714e-003      -10500.000     126000.000
  240     0.000e+000       1.728e-003      -10500.000        0.000
```
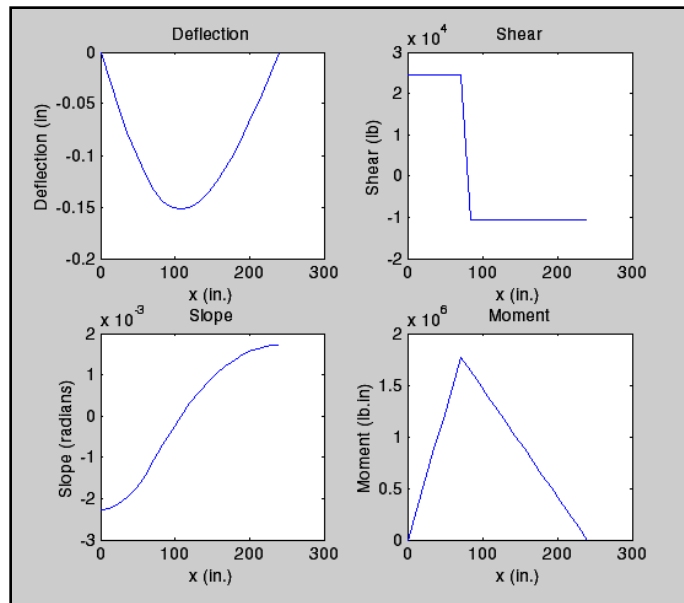

Fig. 4 MATLAB Results for the Analyzed Beam

IV. Determination of the Centroid of an Area - Mathcad Example 1

As already discussed in the introduction section of the paper, for producing the results of certain classes of engineering problems, such as the ones requiring symbolic operations and performing mathematical operations such as integration or differentiation, Mathcad is perhaps more suited to offer a simpler and more convenient solution. Additionally, as stated previously, equations produced in the Mathcad environment appear in the same visual form as written on the paper, making it easier for students to formulate the solutions of problems and document their work. It should additionally be stated that even though in the redesigned course, some of the special programming features of Mathcad are also expected to be briefly covered, but due to time consideration these capabilities will not be fully discussed.

One elementary example problem[4] involving the determination of the x and y coordinates of the centroid of an area using direct integration is provided in Fig. 5. The formulation shown in this figure is based on using a vertical rectangular differential element.

Determination of the Centroid of an Area

Problem Statement From:
Engineering Mechanics (Statics), 15th Edition, Hibbeler, R.C.,
Pearson Publishing, Inc.

Calculate the centroid of the shaded
area shown using direct integration.

$y = (4 - x^{\frac{1}{2}})^2$

16 ft

4 ft

4 ft

x-Coordinate:

$$x := \frac{\int_0^4 x \cdot \left(x - 8 \cdot x^{\frac{1}{2}} + 16\right) dx}{\int_0^4 \left(x - 8 \cdot x^{\frac{1}{2}} + 16\right) dx} = 1.60$$

y-Coordinate:

$$y := \frac{\int_0^4 \frac{1}{2} \cdot \left(4 - x^{\frac{1}{2}}\right)^2 \cdot \left(x - 8 \cdot x^{\frac{1}{2}} + 16\right) dx}{\int_0^4 \left(x - 8 \cdot x^{\frac{1}{2}} + 16\right) dx} = 4.15$$
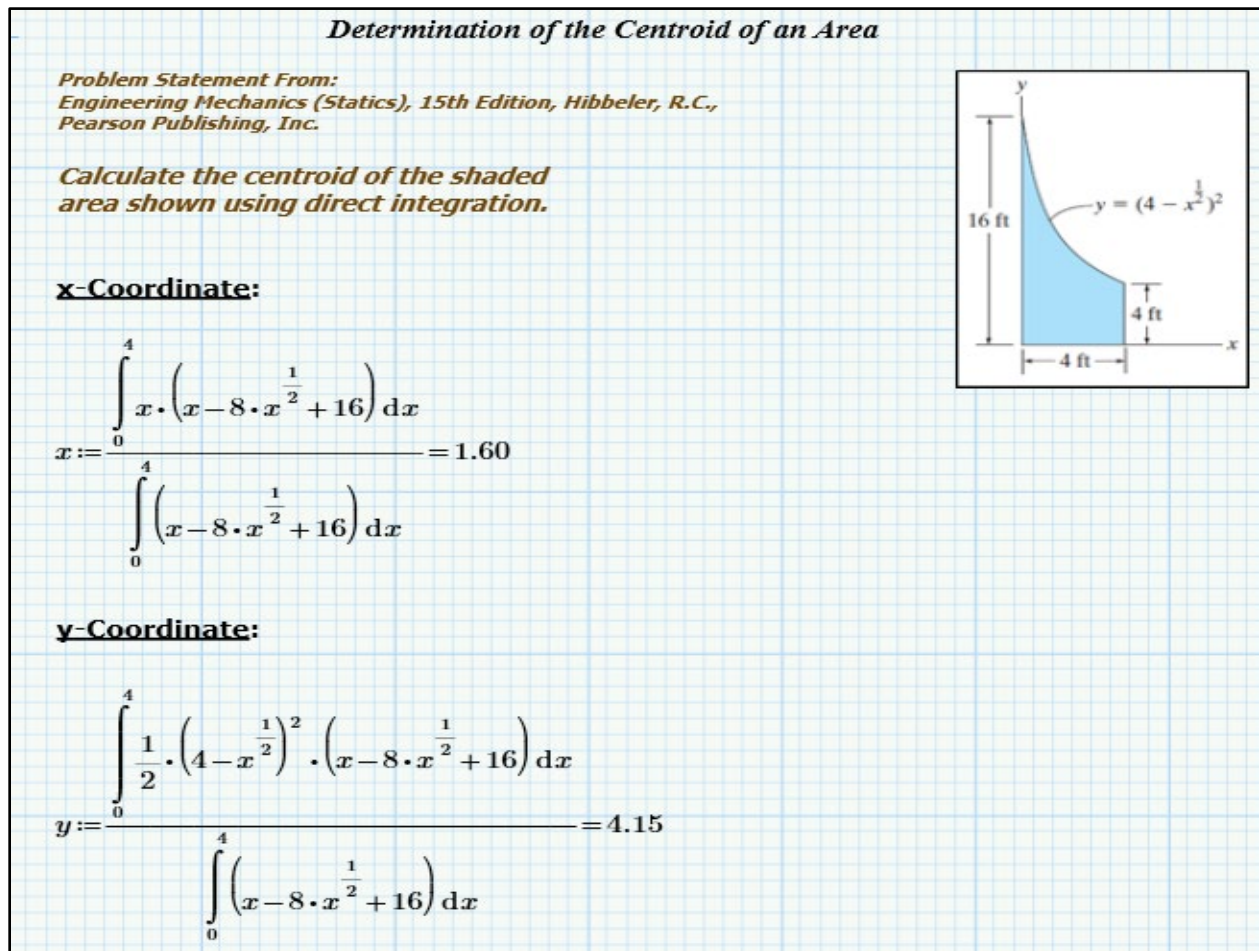
Fig. 5 Mathcad Results for Determining the Centroid of an Area

Determination of centroids of lines, areas, and volumes using direct integration is one of the topics covered in the Statics course required in the CE curriculum in the author's institution. Using Mathcad, the students can easily verify the validity of the solution of the problems they have developed in their mechanic's course.

V. Three-Dimensional Equilibrium - Mathcad Example 2

A second Mathcad example involving the equilibrium of a rigid body in a three-dimensional space is provided in Fig. 6. In this sample exercise[4], a bent rod is subjected to a 200 lb vertical force and a 30 lb-ft couple moment and supported by journal bearings at A, B, and C. In this problem, the students can be asked to determine the force reaction components at the support brackets. The solution of this problem involves the utilization of the force and moment equilibrium equations shown in Eqs. (9-14).

Problem Fig. Source:
Engineering Mechanics (Statics)
Hibbeler, R.C., 15th Edition, Pearson

(a)   (b)
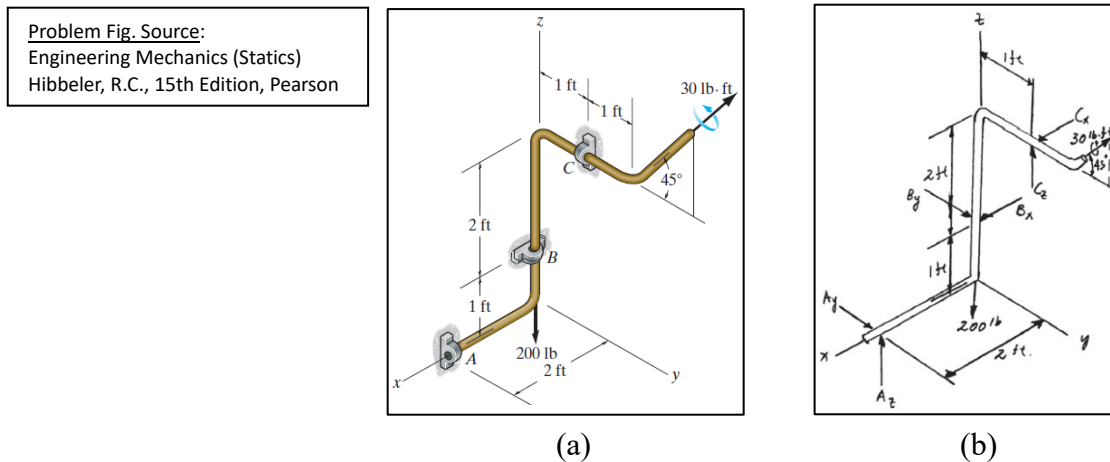
Fig. 6 (a) Bent Rod Subjected to a Force & a Moment, (b) Free-Body Diagram of the Bent Rod

$$\Sigma M_x = 0: \quad -B_y(1) + C_z(1) = 0 \tag{9}$$

$$\Sigma M_y = 0: \quad -A_z(2) + B_x(1) + C_x(3) + 30\cos 45° = 0 \tag{10}$$

$$\Sigma M_z = 0: \quad A_y(2) - C_x(1) + 30\sin 45° = 0 \tag{11}$$

$$\Sigma F_x = 0: \quad B_x + C_x = 0 \tag{12}$$

$$\Sigma F_y = 0: \quad A_y + B_y = 0 \tag{13}$$

$$\Sigma F_z = 0: \quad A_z + C_z - 200 = 0 \tag{14}$$

Manual solution of the presented set of linear equations in the form shown can pose a challenge since careful consideration has to be given to both the selection, as well as the order in which the selected set of equations should be considered in succession to ultimately lead to the solution of all force reaction components. Two convenient and easy solutions for this problem can be obtained in Mathcad by effectively solving a set of linear equations as shown in Fig. 7.

Three-Dimensional Equilibrium of a Rigid Body

Problem Statement From:
Engineering Mechanics (Statics), 15th Edition, Hibbeler, R.C.,
Pearson Publishing, Inc.

Given:

A bent rod is subjected to a force of F = 200 lb and a moment of M = 30 lb.ft as shown.

Determine:

The force reactions at the journal bearings at A, B, and C.

Setting up the equilibrium equations in the matrix form:

$$A := \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -2 & 1 & 0 & 3 & 0 \\ 2 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad B := \begin{bmatrix} 0 \\ -30 \cdot \sin(45°) \\ -30 \cdot \sin(45°) \\ 0 \\ 0 \\ 200 \end{bmatrix}$$

Journal Bearing Force Reactions:

Alternate solution using the Isolve function in MATHCAD:

$$\begin{bmatrix} A_y \\ A_z \\ B_x \\ B_y \\ C_x \\ C_z \end{bmatrix} := A^{-1} \cdot B = \begin{bmatrix} 168 \\ 368 \\ -358 \\ -168 \\ 358 \\ -168 \end{bmatrix} \qquad \begin{bmatrix} A_y \\ A_z \\ B_x \\ B_y \\ C_x \\ C_z \end{bmatrix} := \text{lsolve}(A, B) = \begin{bmatrix} 168 \\ 368 \\ -358 \\ -168 \\ 358 \\ -168 \end{bmatrix}$$

Fig. 7 Mathcad Results for the Bent Rod Example

VI. Plane Truss Example - Mathcad Example 3

Depending on the strength level and capability of students enrolled in the discussed computing course, other more involved problems can also be incorporated in the course. These problems can be selected from the textbooks of some of the advanced courses[5-6] offered in the Civil Engineering curriculum. One sample problem for analysis of a two-dimensional truss using the finite element approach is presented in Figs. 8 & 9. The problem statement and the stiffness matrices used in this example are taken from the text[5] used for the Introduction to Finite Elements course offered at the author's institution. In the presented problem, a truss is subjected to two forces as shown in Fig. 8, and displacements at node 3, as well as the element forces in each of the three truss members are expected to be solved for. Mathcad provides an easy to use and a convenient environment for solving engineering problems such as this, especially when these problems require solving sets of linear equations and matrix manipulations. The Mathcad environment additionally allows the users to create the problem solution in an organized and structured format, in a form that could easily be incorporated in professional reports. Note that units can also conveniently be added in these reports if desired. The incorporated units may additionally be altered with ease throughout the created Mathcad workspace.

VII. Summary & Conclusion

In the computing course redesign described in the presented paper, the utility of Excel, MATLAB, and Mathcad was described to further enhance the course and better arm the students with the programming skills they need to solve a variety of engineering problems. Specific course example problems from the selected three software pieces were also included in the paper to provide more details regarding the manner in which these tools could contribute to the problem-solving capabilities of the students. Through taking the discussed course, the students learn how to exploit the specific features of each of the described software tools to effectively solve problems in their specific field of interest within the Civil Engineering field. It is perceived that the combined utility of the three selected software tools can sufficiently prepare the students for their follow up courses and future careers. The assessment regarding the effectiveness of the selected software tools and methods used in the described course for solving engineering problems can be conducted through a student survey at the next opportunity when the author is assigned to teach the course.

Analysis of a Plane Truss
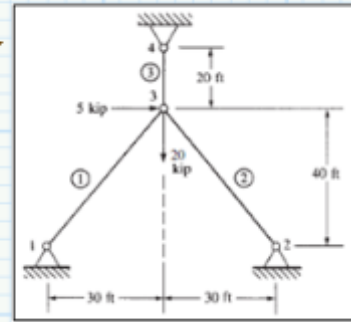
Given:

Young's Modulus:	$E := 30 \cdot 10^6$ *psi*	
Cross-sectional Area:	$A := 3$ *in*^{2}	
Applied Nodal Forces:	$F_{3x} := 5000$ *lbf*	$F_{3y} := -20000$ *lbf*
Initial Nodal Displacement:	$u_1 := 0 \quad v_1 := 0 \quad u_2 := 0 \quad v_2 := 0 \quad u_4 := 0 \quad v_4 := 0$	

Determine:

(1) Displacements of node 3.
(2) Forces in each element of the plane truss.

Element Direction Cosine and Sine Values: Length of Truss Member:

Element 1: $\qquad C_1 := \dfrac{3}{5} \qquad\qquad S_1 := \dfrac{4}{5} \qquad L_1 := \sqrt{30^2\ ft^2 + 40^2\ ft^2} = 600\ in$

Element 2: $\qquad C_2 := \dfrac{-3}{5} \qquad\qquad S_2 := \dfrac{4}{5} \qquad L_2 := \sqrt{30^2\ ft^2 + 40^2\ ft^2} = 600\ in$

Element 3: $\qquad C_3 := 0 \qquad\qquad S_3 := 1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad L_3 := 20\ ft = 240\ in$

Element Stiffness Matrices:

Element 1:

$$k_1 := \frac{(A \cdot E)}{L_1} \begin{bmatrix} C_1^2 & C_1 \cdot S_1 & 0 & 0 & -C_1^2 & -C_1 \cdot S_1 & 0 & 0 \\ C_1 \cdot S_1 & S_1^2 & 0 & 0 & -C_1 \cdot S_1 & -S_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -C_1^2 & -C_1 \cdot S_1 & 0 & 0 & C_1^2 & C_1 \cdot S_1 & 0 & 0 \\ -C_1 \cdot S_1 & -S_1^2 & 0 & 0 & C_1 \cdot S_1 & S_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 5.4 \cdot 10^4 & 7.2 \cdot 10^4 & 0 & 0 & -5.4 \cdot 10^4 & -7.2 \cdot 10^4 & 0 & 0 \\ 7.2 \cdot 10^4 & 9.6 \cdot 10^4 & 0 & 0 & -7.2 \cdot 10^4 & -9.6 \cdot 10^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.4 \cdot 10^4 & -7.2 \cdot 10^4 & 0 & 0 & 5.4 \cdot 10^4 & 7.2 \cdot 10^4 & 0 & 0 \\ -7.2 \cdot 10^4 & -9.6 \cdot 10^4 & 0 & 0 & 7.2 \cdot 10^4 & 9.6 \cdot 10^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{lbf}{in}$$

Element 2:

$$k_2 := \frac{(A \cdot E)}{L_2} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_2^2 & C_2 \cdot S_2 & -C_2^2 & -C_2 \cdot S_2 & 0 & 0 \\ 0 & 0 & C_2 \cdot S_2 & S_2^2 & -C_2 \cdot S_2 & -S_2^2 & 0 & 0 \\ 0 & 0 & -C_2^2 & -C_2 \cdot S_2 & C_2^2 & C_2 \cdot S_2 & 0 & 0 \\ 0 & 0 & -C_2 \cdot S_2 & -S_2^2 & C_2 \cdot S_2 & S_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.4 \cdot 10^4 & -7.2 \cdot 10^4 & -5.4 \cdot 10^4 & 7.2 \cdot 10^4 & 0 & 0 \\ 0 & 0 & -7.2 \cdot 10^4 & 9.6 \cdot 10^4 & 7.2 \cdot 10^4 & -9.6 \cdot 10^4 & 0 & 0 \\ 0 & 0 & -5.4 \cdot 10^4 & 7.2 \cdot 10^4 & 5.4 \cdot 10^4 & -7.2 \cdot 10^4 & 0 & 0 \\ 0 & 0 & 7.2 \cdot 10^4 & -9.6 \cdot 10^4 & -7.2 \cdot 10^4 & 9.6 \cdot 10^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{lbf}{in}$$

Fig. 8 Mathcad Results for the Analyzed Plane Truss Example

Element 3:

$$k_3 = \frac{(A \cdot E)}{L_3} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_3^2 & C_3 \cdot S_3 & -C_3^2 & -C_3 \cdot S_3 \\ 0 & 0 & 0 & 0 & C_3 \cdot S_3 & S_3^2 & -C_3 \cdot S_3 & -S_3^2 \\ 0 & 0 & 0 & 0 & -C_3^2 & -C_3 \cdot S_3 & C_3^2 & C_3 \cdot S_3 \\ 0 & 0 & 0 & 0 & -C_3 \cdot S_3 & -S_3^2 & C_3 \cdot S_3 & S_3^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.75 \cdot 10^5 & 0 & -3.75 \cdot 10^5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.75 \cdot 10^5 & 0 & 3.75 \cdot 10^5 \end{bmatrix} \frac{lbf}{in}$$

Global Stiffness Matrix:

$$K = k_1 + k_2 + k_3 = \begin{bmatrix} 5.4 \cdot 10^4 & 7.2 \cdot 10^4 & 0 & 0 & -5.4 \cdot 10^4 & -7.2 \cdot 10^4 & 0 & 0 \\ 7.2 \cdot 10^4 & 9.6 \cdot 10^4 & 0 & 0 & -7.2 \cdot 10^4 & -9.6 \cdot 10^4 & 0 & 0 \\ 0 & 0 & 5.4 \cdot 10^4 & -7.2 \cdot 10^4 & -5.4 \cdot 10^4 & 7.2 \cdot 10^4 & 0 & 0 \\ 0 & 0 & -7.2 \cdot 10^4 & 9.6 \cdot 10^4 & 7.2 \cdot 10^4 & -9.6 \cdot 10^4 & 0 & 0 \\ -5.4 \cdot 10^4 & -7.2 \cdot 10^4 & -5.4 \cdot 10^4 & 7.2 \cdot 10^4 & 1.08 \cdot 10^5 & 0 & 0 & 0 \\ -7.2 \cdot 10^4 & -9.6 \cdot 10^4 & 7.2 \cdot 10^4 & -9.6 \cdot 10^4 & 0 & 5.67 \cdot 10^5 & 0 & -3.75 \cdot 10^5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.75 \cdot 10^5 & 0 & 3.75 \cdot 10^5 \end{bmatrix} \frac{lbf}{in}$$

Displacements at Node 3:

$$\begin{bmatrix} u_3 \\ v_3 \end{bmatrix} := submatrix(K, 5, 6, 5, 6)^{-1} \cdot \begin{bmatrix} F_{3x} \\ F_{3y} \end{bmatrix} = \begin{bmatrix} 0.0463 \\ -0.0353 \end{bmatrix} in$$

Nodal Forces: **Reaction Forces:**

$$\begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \\ F_{4x} \\ F_{4y} \end{bmatrix} := K \cdot \begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} 39.68 \\ 52.91 \\ -5039.68 \\ 6719.58 \\ 5000 \\ -20000 \\ 0 \\ 13227.51 \end{bmatrix} lbf \qquad \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{4x} \\ F_{4y} \end{bmatrix} = \begin{bmatrix} 39.68 \\ 52.91 \\ -5039.68 \\ 6719.58 \\ 0 \\ 13227.51 \end{bmatrix} lbf$$

Element #1 Forces:

$$\begin{bmatrix} f_{1x} \\ f_{3x} \end{bmatrix} := \frac{(A \cdot E)}{L_1} \cdot \begin{bmatrix} C_1 & S_1 & -C_1 & -S_1 \\ -C_1 & -S_1 & C_1 & S_1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ v_1 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 66.14 \\ -66.14 \end{bmatrix} lbf$$

Element #2 Forces:

$$\begin{bmatrix} f_{2x} \\ f_{3x} \end{bmatrix} := \frac{(A \cdot E)}{L_2} \cdot \begin{bmatrix} C_2 & S_2 & -C_2 & -S_2 \\ -C_2 & -S_2 & C_2 & S_2 \end{bmatrix} \cdot \begin{bmatrix} u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} 8.40 \cdot 10^3 \\ -8.40 \cdot 10^3 \end{bmatrix} lbf$$

Element #3 Forces:

$$\begin{bmatrix} f_{3x} \\ f_{4x} \end{bmatrix} := \frac{(A \cdot E)}{L_3} \cdot \begin{bmatrix} C_3 & S_3 & -C_3 & -S_3 \\ -C_3 & -S_3 & C_3 & S_3 \end{bmatrix} \cdot \begin{bmatrix} u_3 \\ v_3 \\ u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} -13.23 \cdot 10^3 \\ 13.23 \cdot 10^3 \end{bmatrix} lbf$$

Fig. 9 Mathcad Results for the Analyzed Plane Truss Example (Cont.)

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