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Teaching/ Learning Modules for Structural Analysis

Abstract

A typical undergraduate Structural Analysis course of a civil engineering or civil engineering technology curriculum includes topics such as Moving Loads and Influence Lines for Trusses and Girders that often require several iterations, involving tedious, repetitive calculations to solve problems. But, only a limited number of examples can be presented in the classroom due to time constraint, despite the fact that such examples are necessary to reinforce important engineering concepts. To circumvent the situation, a logical option is to capitalize on the computer’s abilities to compute, display graphics, and interface with the user. The purpose of this paper is to present a computer-based problem-solving courseware that has been developed to complement traditional lecture-format delivery of the structural analysis course in order to enhance student learning. The courseware consists of interactive, web-based modules. Commercial symbolic-manipulation software (e.g., Mathcad) is utilized for the calculations performed in different modules. Examples to illustrate the computer program modules are also included in this paper. Besides allowing for faster solution of a problem, the tool is useful for experimentation with parameter changes as well as graphical visualization. The integration of Mathcad will enhance students’ problem-solving skills, as it will allow them to focus on analysis while the software performs routine calculations. Thus it will promote learning by discovery, instead of leaving the student in the role of a passive observer.

Introduction

With the objective of enhanced student learning, various instructional technology methods including computer-aided problem-solving modules have been integrated into the curriculum for civil engineering and civil engineering technology programs. More specifically, the effective incorporation of a variety of software packages for the teaching-learning process related to the structural analysis course has been addressed in several articles in recent years. Analysis of both statically determinate and statically indeterminate structures, by classical methods (slope-deflection and moment distribution) and stiffness method, using EXCEL, MATLAB and Mathcad, have been covered in those articles. However, it appears that such supplementary packages lack (in certain aspects) in coverage of the topic of influence lines for beams under moving loads, and thus an enhancement is worthwhile. The purpose of this paper is to present a simple and effective approach used by the author to facilitate both teaching and learning of this important topic of structural analysis incorporating the use of Mathcad software.

Moving Loads and Influence Lines

For analysis of structures subject to moving loads, the concept of influence lines is indispensable. An influence line is a graph of a response function of a structure as a function of the position of a downward unit load moving across the structure. The response function can be support reaction, shear or moment or deflection at a specific point in a member. The influence line can be
constructed by placing the unit load at a variable position $x$ on the member and then computing the value of reaction, shear, moment etc. as a function of $x$. The well-known Muller-Breslau principle can be used to facilitate drawing qualitative influence lines. Once the influence line of a function is established, the maximum effect caused by a live concentrated load is determined by multiplying the peak ordinate of the influence line by the magnitude of the load. To determine the maximum effect caused by a series of concentrated loads (such as wheel loads of a truck), a trial-and-error method can be used by placing each load at the specific point of interest. The critical loading is the one that causes the largest effect (shear, moment etc.) at the given location. The absolute maximum moment occurs under one of the concentrated loads in a beam, such that this critical load and the resultant of the concentrated load series are equidistant from the beam centerline. These basic concepts related to moving loads and influence lines can be found in any standard textbook on structural analysis\textsuperscript{11,12,13}.

**Advantages of Mathcad**

*Mathcad*, an industry-standard calculation software, is used because it is as versatile and powerful as programming languages, yet it is as easy to learn as a spreadsheet. Additionally, it is linked to the Internet and other applications one uses everyday.

In *Mathcad*, an expression or an equation looks the same way as one would see it in a textbook, and there is no difficult syntax to learn. Aside from looking the usual way, the expressions can be evaluated or the equations can be used to solve just about any mathematics problem one can think of. Text can be placed anywhere around the equations to document one’s work. *Mathcad’s* two- and three-dimensional plots can be used to represent equations graphically. In addition, graphics taken from another Windows application can also be used for illustration purpose. *Mathcad* incorporates Microsoft’s OLE 2 object linking and embedding standard to work with other applications. Through a combination of equations, text, and graphics in a single worksheet, keeping track of the most complex calculations becomes easy. An actual record of one’s work is obtained by printing the worksheet exactly as it appears on the screen.

**Program Features**

The program developed by the author will require input data pertaining to the geometry of the problem, and the loading. More specifically, the following information is required as input data: beam span; number, magnitudes and locations of concentrated loads; location of the specific point of interest where maximum shear or maximum moment is desired; and number of divisions for plotting the influence lines. Based on the input data, calculations are carried out in the following steps:

1. Calculate the influence line ordinates (at equal intervals along the length of the beam) for shear at a specific point in the beam
2. Calculate the influence line ordinates (at equal intervals along the length of the beam) for moment at a specific point in the beam
3. Calculate the shear at a specific point in the beam by placing each load at that location in turn; the largest value represents the maximum shear at that point.
4. Calculate the moment at a specific point in the beam by placing each load at that location in turn; the largest value represents the maximum moment at that point.

5. Calculate the maximum moment under each load by placing each of the concentrated loads and their resultant equidistant from the beam. The largest value represents the absolute maximum moment in the beam occurring under one of those loads.

**Student Assignment and Assessment of Performance**

The author provided an abridged version of the program (limited to loads moving from right to left) to his class. A list of variables used in the program and the program code are given in the Appendix. The students were asked to modify the program such that loads moving from left to right can be solved. The students were allowed to work in groups of 3 or 4. After the modifications were done, students had to validate their program using two problems of known solutions. A quiz was given to the class to test their knowledge of Mathcad programming as well as concept of influence lines.

To further extend the efficacy of the program developed, it can be placed on course web site to allow the students full access to this useful supplementary educational material.

**Example Problems**

Two example problems with their solutions obtained by using the program are given below. For any of these problems, one or more input data change would translate to change in the influence line ordinate values, and hence the maximum values of shear and moment as well. Any number of combinations of input data is possible and students can see the effects of these changes instantaneously. Moreover, with further additions to the program, it would be feasible to include other types of beams (e.g., beams with overhangs).
**Example 1:** Determine the maximum positive shear created at point C on the beam shown due to the wheel loads of the moving truck. (Reference 11, Example 6-18)

**Input Data:**

Length of beam: \( L := 20 \text{ ft} \)

Wheel loads: Number \( n := 4 \)

Magnitudes: \( P := (4000 \ 9000 \ 15000 \ 10000 \ 0 \ 0)^T \cdot \text{lb} \)

Locations (measured from the leftmost load): \( d := (0 \ 3 \ 9 \ 15 \ 0 \ 0)^T \cdot \text{ft} \)

Number of divisions: \( \text{div} := 12 \)

Distance to beam section C from left end of beam: \( a := 10 \text{ ft} \)
Solution:

Shear at C under different positions of concentrated load series:

\[
\begin{array}{c|c}
 x_j & V_x_j \\
\hline
0 \text{ ft} & 0 \\
0.167 & -0.008 \\
0.333 & -0.017 \\
0.5 & -0.025 \\
0.667 & -0.033 \\
0.833 & -0.042 \\
1 & -0.05 \\
1.167 & -0.058 \\
1.333 & -0.067 \\
1.5 & -0.075 \\
1.667 & -0.083 \\
1.833 & -0.092 \\
2 & -0.1 \\
2.167 & -0.108 \\
2.333 & -0.117 \\
2.5 & -0.125
\end{array}
\]

Influence Line Diagram for shear at C

Maximum shear at C: \[\max(V_C) = 7500 \text{ lb}\]
Example 2: Determine (a) the maximum moment at C and (b) the absolute maximum moment in the simply-supported beam shown below. (Reference 11, Example 6-21)

Input Data:

Length of beam: \( L := 30 \cdot \text{ft} \)

Wheel loads: Number \( n := 3 \)

Magnitudes: \( P := (2000 \ 1500 \ 1000 \ 0 \ 0 \ 0) \cdot \text{lb} \)

Locations (measured from the leftmost load): \( d := (0 \ 10 \ 15 \ 0 \ 0 \ 0) \cdot \text{ft} \)

Number of divisions: \( \text{div} := 12 \)

Distance to beam section C from left end of beam: \( a := 10 \cdot \text{ft} \)
Solution:

\[ x_j = \begin{array}{c|c}
0 \text{ ft} & 0 \\
0.25 & 0.167 \\
0.5 & 0.333 \\
0.75 & 0.5 \\
1 & 0.667 \\
1.25 & 0.833 \\
1.5 & 1 \\
1.75 & 1.167 \\
2 & 1.333 \\
2.25 & 1.5 \\
2.5 & 1.667 \\
2.75 & 1.833 \\
3 & 2 \\
3.25 & 2.167 \\
3.5 & 2.333 \\
3.75 & 2.5 \\
\end{array} \]

\[ Mx_j = \begin{array}{c|c}
0 \text{ ft} & 0 \\
0.167 & 0.333 \\
0.5 & 0.5 \\
0.667 & 0.833 \\
1 & 1 \\
1.167 & 1.333 \\
1.5 & 1.667 \\
1.833 & 2 \\
2.167 & 2.333 \\
2.5 & 2.5 \\
\end{array} \]

Moment at C under various positions of concentrated load series:

\[
MC = \begin{pmatrix} 20000 \\ 15000 \\ 11666.667 \end{pmatrix} \text{ lb ft}
\]

Maximum moment at C: \( \max(MC) = 20000 \text{ lb ft} \)

Maximum moment in the beam under i-th load: \( M_{max} = \begin{pmatrix} 20416.667 \\ 21666.667 \\ 17604.167 \end{pmatrix} \text{ lb ft} \)

Absolute maximum moment in the beam: \( \max(M_{max}) = 21666.667 \text{ lb ft} \)

Student Response

As mentioned before, the student assignments were group activities. The intent was to encourage cooperative learning. In general, students were quite receptive to the use of Mathcad, although they had no prior exposure to the software. The author had to familiarize the students with the essential features of Mathcad, before they were given the assignment. As part of the course, a two-hour-per-week computational laboratory makes it possible for the author to teach
the basics of this software. Eighteen students answered a survey which is summarized in Table 1.

Table 1. Summary of Student Surveys

<table>
<thead>
<tr>
<th>Statement</th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Not Sure</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>The use of <em>Mathcad</em> for Influence Lines was worthwhile and should be continued.</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>The use of <em>Mathcad</em> helped me learn the topic and increased my problem-solving skills.</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>Learning to use <em>Mathcad</em> was difficult, time-consuming and/or frustrating.</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>The programming part made me think more about the concept behind the topic.</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td><em>Mathcad</em> should be incorporated into Structural Analysis course for other topics as well.</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

From the survey, it appears that majority of the students are in favor of using the software for this topic (as well as others), despite the learning curve associated with new software. They also have acknowledged enhanced learning. Although no specific feedback information as to teamwork experience was asked in the survey, informal inquiry with the students has, however, revealed a positive response from the students.

Conclusions

For most part, the suggested approach to complement the traditional lecturing provides a better insight in the subject matter, in addition to making a convenient checking procedure readily available. The students can instantaneously solve complex problems involving different loading conditions, and also examine what-if scenarios by changing one or more parameters as input data (a manual solution for such a problem would be very tedious and time consuming). Also, the students acquire enhanced problem-solving skills, as they are engaged in, not just using the *Mathcad* software, but also in writing the programming code.
Appendix

Mathcad Program for Influence Lines for Beams
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Input Variables:

- **L**: Beam span
- **n**: Number of concentrated loads
- **P_i**: i-th concentrated load
- **d_i**: Distance to i-th concentrated load, from the left end
- **div**: Number of divisions in the beam length for plots of influence lines
- **a**: Distance to beam section C from left end of beam

Output Variables:

- **VC**: Shear at any given location C in a beam due to a series of moving loads
- **MC**: Moment at any given location C in a beam due to a series of moving loads
- **max(VC)**: Maximum shear at any given location C in a beam due to a series of moving loads
- **max(MC)**: Maximum moment at any given location C in a beam due to a series of moving loads
- **M_{max_i}**: Maximum moment in a beam under i-th load of the load series
- **max(M_{max})**: Absolute maximum moment in a beam due to a series of moving loads
1. Influence lines for shear and moment at any given location $C$ in a beam.

Provide plotting information:

Number of points: \( \text{pts} := \text{div} + 1 \)

Interval between points: \( \text{int} := \frac{L}{\text{div}} \)

\( j := 1 \ldots \text{pts} \)

\( x_j := (j - 1) \cdot \text{int} \)

Influence line ordinates for shear at $C$:

\[ V_x = \begin{cases} -\frac{x_j}{L} & \text{if } x_j \leq a \\ 1 - \frac{x_j}{L} & \text{if } x_j \geq a \wedge x_j \leq L \end{cases} \]

Influence line ordinates for moment at $C$:

\[ M_x = \begin{cases} x_j \left( 1 - \frac{a}{L} \right) & \text{if } x_j \leq a \\ a \cdot \left( 1 - \frac{x_j}{L} \right) & \text{if } x_j \geq a \wedge x_j \leq L \end{cases} \]

2. Resultant of load series – magnitude and location

A. Loads moving from right to left:

Resultant of wheel loads:

- Magnitude: \( R := \sum_{i=1}^{n} P_i \)
- Location: \( \overline{x} := \frac{\sum_{i=1}^{n} (P_i \cdot d_i)}{R} \)
3. **Maximum positive shear at any given location C in a beam due to a series of moving loads**

\[
k := 1..n
\]

\[
V_{C_k} := \begin{cases}
i & \text{if } k = 1 \\
j & \text{if } k > 1 \\
1 & \text{otherwise}
\end{cases}
\]

\[
k := 1..n
\]

\[
V_{C_k} := \begin{cases}
i & \text{if } k = 1 \\
j & \text{if } k > 1 \\
1 & \text{otherwise}
\end{cases}
\]

\[
\begin{align*}
v_{ij} & := \begin{cases}
\frac{a}{L} & \text{if } d_i \leq a \\
0 & \text{if } d_i > a
\end{cases} \\
\text{arm} & := \begin{cases}
\frac{a - (d_k - d_j)}{L} & \text{if } (d_k - d_j) < a \\
0 & \text{if } (d_k - d_j) \geq a
\end{cases}
\end{align*}
\]

4. **Maximum moment at any given location C in a beam due to a series of moving loads**

\[
y := a \left(1 - \frac{a}{L}\right)
\]

\[
L_{\text{slope}} := 1 - \frac{a}{L} \quad R_{\text{slope}} := \frac{a}{L}
\]

\[
k := 1..n
\]
\[ \text{MC}_k := \begin{cases} 
  i & \leftarrow 1 \\
  j & \leftarrow k \\
  \text{sum} & \leftarrow 0 \\
  \text{while } i \leq n \quad \text{if } k = 1 \\
  \text{break} & \quad \text{if } d_i > L - a \\
  \text{ordinate} & \leftarrow \left[ y - (R\text{slope} \cdot d_i) \right] \\
  \text{sum} & \leftarrow \text{sum} + \left( P_i : \text{ordinate} \right) \\
  i & \leftarrow i + 1 \\
  \text{while } j \leq n \quad \text{if } k > 1 \\
  \text{ordinate}_R & \leftarrow \left[ L - a - (d_j - d_k) \right] \cdot R\text{slope} \\
  \text{Term}_1 & \leftarrow P_j : \text{ordinate}_R \\
  \text{sum} & \leftarrow \text{sum} + \text{Term}_1 \\
  \text{while } i \leq k - 1 \\
  \text{break} & \quad \text{if } d_i > a \\
  \text{dist} & \leftarrow \begin{cases} 
    (d_k - d_j) & \text{if } (d_k - d_j) < a \\
    a & \text{if } (d_k - d_j) \geq a 
  \end{cases} \\
  \text{ordinate}_L & \leftarrow (a - \text{dist}) \cdot L\text{slope} \\
  \text{Term}_2 & \leftarrow P_i : \text{ordinate}_L \\
  \text{sum} & \leftarrow \text{sum} + \text{Term}_2 \\
  i & \leftarrow i + 1 \\
  j & \leftarrow j + 1 \\
  \text{sum} 
\end{cases} \]

5. Absolute maximum moment in a beam due to a series of moving loads

\[ i := 1 \ldots n \]
\[ \text{Rdist}_i := d_i - \text{xbar} \]
\[ R := \frac{L + \text{Rdist}_i}{2} \]
\[ A_y_i := \frac{\text{Ay}_i}{2L} \]

\[ M_i := \begin{cases} 
  0.5 \cdot A_y_i \cdot \left( L + \text{Rdist}_i \right) & \text{if } i = 1 \\
  0.5 \cdot A_y_i \cdot \left( L + \text{Rdist}_i \right) - \sum_{j=1}^{i-1} \left[ P_j \cdot (d_i - d_j) \right] & \text{otherwise} 
\end{cases} \]