## AC 2009-716: AN EXAMINATION OF ENGINEERING MATHEMATICS COURSES

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## An Examination of Engineering Mathematics Courses

## Introduction

There are many alternatives to deliver the mathematics content required for engineering accreditation and career success. These alternatives include four credit hour calculus courses, three credit calculus courses, statistics, differential equations, linear algebra, and multiple other options. This paper examines the literature and an empirical sample of program content to establish a theoretical benchmark for the reasons and potential effectiveness factors related to these options. The goal is to describe and categorize the mathematics requirements of varied programs and provide comparative information as a foundation for further study.

## Literature Review

The importance and impact of mathematics in the engineering curriculum has been the subject of many studies which have covered a wide spectrum of topical and curricular impact areas. For example, Ruane ${ }^{1}$ examined differential equations and its relation to calculus and curricular delivery reform focused on applications. Hampikian ${ }^{2}$ studied integration of pre-calculus with other engineering courses to improve retention and student success. Carpenter et al. ${ }^{3}$ studied the mathematical topics which were critical for chemical engineering and James and High ${ }^{4}$ summarized the literature related to freshman engineering mathematics.

A consistent theme of this literature is the need to improve retention and student success. Two examples typical of work specifically addressing this issue are the following. Monte and Hein ${ }^{5}$ studied the relationship of supporting engineering courses in improving student success in mathematics. Lavelle and Keltie ${ }^{6}$ examined intervention approaches in freshman calculus to improve retention of first year students. As demonstrated by these examples, much of the retention related activity is focused at effort outside the mathematics class room.

An important engineering mathematics area which has received limited attention is the study of what goes on inside the class room. More specifically, this includes what topics should be covered and in what depth should they be studied. This is a particularly important question for the engineering graduate who will work in the $21^{\text {st }}$ century. In addition, the authors are involved in the start up of a general engineering program which is focused on building broad interdisciplinary problem solving skills and supporting diverse concentrations in biomedical engineering, bioprocess engineering, industrial and systems engineering, and mechanical engineering. A key question involves the mathematics skills required by a broad-based engineering problem solver who is able to grow and adapt as technology changes. This paper contributes to the literature addressing this question. The next section presents a general foundation by examining a group of mathematics course sequences at a representative number of universities to assess whether curricular practice has identified a consistent viewpoint on general topical coverage.

## Overview of Current Mathematics Approaches

As a starting point for our curricular examination, we compared how various engineering programs were investing their mathematics credits. The universities discussed below were identified based on two criteria. First we started with universities in the ASEE listings of top volumes in total engineering graduates, female graduates, or minority graduates. Second, information on mathematics course sequences had to be available through search of their web sites. In studying mathematics content in engineering programs, an obvious classification scheme is to begin with the traditional calculus sequence. Using this as a starting point, based on three or four credit sequences, a number of options are possible. Tables 1 and 2 provide representative examples of ten engineering programs which have three and four credit calculus sequences respectively.

In interpreting the columns in these tables, engineering programs have generally defined calculus I and II to cover single variable differential and integral calculus and calculus III covers multivariable calculus. When a Calculus IV course is included in a program, Calculus III and IV typically have segmented differentiation and integration of multivariable equations. Complex variables are also included in calculus IV in these cases. Statistics describes a mathematical statistics course with calculus content to assure ABET (Accreditation Board for Engineering and Technology) inclusion in the mathematics area of the curriculum. DEQ / LA (differential equations / linear algebra) describes a course which combines study of differential equations and linear algebra together.

Table 1 Example Four Credit Calculus Options

| Program | Calc I | Calc II | Calc III | Calc IV | Statistics | Lin. Alg. | Diff. Eq. | DEQ /LA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Clemson | 4 | 4 | 4 |  | 3 |  | 4 |  |
| Auburn | 4 | 4 | 4 |  |  | 3 | 3 |  |
| Virginia | 4 | 4 |  |  | 3 |  | 4 |  |
| George Mason | 4 | 4 | 4 |  | 3 |  | 3 |  |
| NC State | 4 | 4 | 4 |  | 3 |  | 3 |  |
| Howard | 4 | 4 | 4 |  | 3 |  | 4 |  |
| Rose Hulman $^{\text {I }}$ | 5 | 5 |  |  | 3 |  |  |  |
| Wayne State $_{\text {NC A\&T }}^{4}$ | 4 | 4 | 4 | 3 |  |  |  | 4 |
| NCe | 3 | 3 | 4 |  |  |  | 3 |  |
| Rice | 3 | 3 |  |  |  |  | 3 |  |

1. Rose Human is on the quarter system and these are semester equivalents

Table 1 indicates that for four credit calculus programs.

- $60 \%$ require statistics, and it is consistently a three credit course.
- Calculus IV was only required by one program.
- Four programs include linear algebra, but this is a standalone course in only one of these schools. In the other three cases, it is combined with differential equations.
- Stand alone differential equations courses are either three or four credits and there does not appear to be a pattern in this.

Table 2 Example Three Credit Calculus Options

| Program | Calc I | Calc II | Calc III | Calc IV | Statistics | Lin. Alg. | Diff. Eq. | DEQ /LA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| UNC Charlotte | 3 | 3 | 3 |  | 3 |  | 3 |  |
| Miss. State | 3 | 3 | 3 | 3 |  |  | 3 |  |
| Oklahoma | 3 | 3 | 3 | 3 |  | 3 | 3 |  |
| Southern Meth. | 3 | 3 | 3 |  |  |  | 3 |  |
| Arizona | 3 | 3 | 4 |  |  |  | 3 |  |
| Baylor | 3 | 3 | 3 |  | 3 | 3 | 3 |  |
| Texas Tech | 3 | 3 | 3 |  | 3 |  | 3 |  |
| Arizona State | 3 | 3 | 3 |  |  | 3 | 3 |  |
| Virginia Tech ${ }^{2}$ | 3 | 3 | 3 |  |  | 2 | 3 |  |
| East Carolina | 3 | 3 | 3 |  | 3 |  |  | 4 |

1. Three credit numerical methods taught by engineering. 2. Program also has a two credit freshman class in vector geometry.

Table 2 indicates that for three credit calculus sequence programs:

- Are no more likely to have either statistics or linear algebra as standalone courses.
- $40 \%$ compared to $60 \%$ above require statistics and it is consistently a three credit course.
- Do not appear to be more likely to have a Calculus IV requirement (two programs here compared to one program above).
- Stand alone differential equations courses are uniformly three credits.

The next section examines the question of specific content details for engineering mathematics courses.

## Mathematics Content

When the authors started this research project, we expected to find a comprehensive body of literature examining pros and cons of issues such as the three versus four credit calculus sequence, embedded questions, assessment results, and similar topics. Unfortunately, this literature does not appear to exist and this will be a focus for additional work. However, several papers provide guidance in structuring research questions and research hypotheses related to content investigation. These papers may provide a context for future changes,

Klingbeil et al. ${ }^{7}$ studied an alternative curricular approach based on a funded NSF project. This curricular restructuring focused on preparation for topics in the calculus sequence. To accomplish this, they proposed a five-credit course to prepare for calculus. Topics in this course include: Algebraic manipulations, trigonometry, two-dimensional vectors and resolution, complex numbers, sinusoids and harmonics, systems of equations and matrices, basics of differentiation and integration, and linear differential equations with constant coefficients. Most important, it emphasizes that the course is taught by engineering faculty, includes significant application orientation, and involves use of MatLAB software.

Janowski et al. ${ }^{8}$ wrote on their efforts involving development of a course which eliminates calculus III and replaces it with a course covering first and second order ordinary differential
equations, multivariable calculus topics (gradient, divergence, and curl), and partial differential equations. This course involves problem-based and activity-based learning.

Carpenter et al. ${ }^{3}$ studied mathematics topics required for success in chemical engineering. They emphasized the importance of visualization and its importance in understanding differential, integral, and vector calculus, and conversion of coordinate systems. Of critical importance, this paper identified probability and statistics as the most common application of mathematics for the practicing chemical engineer.

## Conclusions and Future Directions

When we started this study, we hoped to identify some compelling logic to drive selection of a three or four credit calculus sequence but this is also an unexplored area. It is clear that significant work is needed to examine specific engineering topics and their effectiveness in building engineering skills in mathematics. In particular, information on depth of topical coverage and the relative effectiveness of alternative topical sequences is lacking.

The literature of innovation in the area of engineering mathematics points strongly to the concept of integration of commonly defined topics. Examples of this are the DEQ/ LA courses which already are entering the educational practice field. Another area of emphasis is the movement to integration of applications into mathematics courses. Although not the primary focus of this study, we found several examples of a lab or practicum component integrated into an engineering mathematics course.

As an important next step in this work to identify a clearer listing of mathematical topics, we plan to develop a list of example problems which can be used as embedded assessment questions and also relate these to topical coverage on the fundamentals of engineering exam. This can provide a basis for comparison of topical coverage and depth and also map to national standards such as the fundamentals of engineering exam.

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