An EXCEL Tool for Teaching Theis Method
of Estimating Aquifer Parameters

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Abstract

Hydraulic conductivity and storage coefficient of aquifers are often estimated from non-equilibrium well pumping test data using Theis method. This method requires matching a type-curve with time-drawdown data curve from a pumping well. Typically, this is done by using a type-curve plotted on a graph paper and the data curve plotted on another transparent graph paper, and superimposing and sliding the transparent one over the other to get the best possible match. This is a tedious approach. Though commercial software package programs are available to perform this analysis, they are not transparent enough for instructional purposes. This paper presents an EXCEL tool to solve Theis equation. This method requires only basic EXCEL skills such as data entry and handling plotting features of EXCEL effectively, for example plotting the data in log scales, changing color and transparency of plot area, and moving the plot. Macro programming is not required. Moreover, all operations in this method could be done within EXCEL. Most of the junior and senior students are fairly conversant using EXCEL. This will be an effective educational tool to teach and demonstrate Theis method to students. The proposed EXCEL method can also be used for different aquifer types i.e., confined, unconfined, and leaky with appropriate tabulated type-curve data.

Keywords: Theis method, type-curve, curve matching, EXCEL, aquifers, teaching tool

1. Introduction

Aquifer well pumping test method is a widely adopted method to determine the aquifer parameters in groundwater hydrology. The detailed description about determining aquifer properties from pumping test data can be found in many groundwater hydrology text books[1, 2]. Equilibrium equation known as Thiem[3] equation can be used to determine the hydraulic conductivity or the transmissivity of confined aquifer from the fully penetrating well pumping test. Equilibrium equation also can be developed for the unconfined aquifer with Dupuit[4] assumptions. However, equilibrium equations are valid only for steady flow conditions. Under this condition, the aquifer is in equilibrium in regard to flow to the well being pumped. The aquifer may take several days to reach the equilibrium condition for any constant pumping rate.

When the pumping test is conducted in an unsteady flow (non-equilibrium) condition which is very common in practical situation, the solution for the governing equation or applicable differential equation for radial flow is a challenging task (Equation 1).

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$

$\qquad$ Eq.1

Where; h is head, r is radial distance to the observation well from the pumping well, S is the storage coefficient, T is the transmissivity, and t is the time since beginning of pumping.

1.1 Theis equation

The solution for the governing equation of unsteady flow (Equation 1) was derived based on the analogy between groundwater flow and heat conduction by Theis[5] (Equation 2a, b, c). The equation is known as Theis equation or nonequilibrium equation.

$$s = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}du}{u} = \frac{Q}{4\pi T}W(u)$$

$\qquad$ Eq. 2(a)

$$s = \frac{Q}{4\pi T}\left[-0.5772 - lnu + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \cdots\right]$$

$\qquad$ Eq. 2(b)

$$u = \frac{r^2 S}{4Tt}$$

$\qquad$ Eq. 3(c)

where s is drawdown in the pumping well, Q is constant well discharge, and $W(u)$ is well function.

Theis nonequilibrium equation allows determining the storage coefficient(S), and the transmissivity (T) of the aquifer. The detailed descriptions about Theis equation and its limitations can be found in Todd and Mays[2].

1.2 Methods of solving Theis equation

Though Theis equation is widely recognized, solving it is a challenging task. Methods have been proposed by Theis[5], Cooper and Jacob[6], and Chow[7] to solve Theis equation. Out of these, Theis method involves curve matching. The other two are approximate methods.

1.3 Theis method

Theis method is one of the widely applied methods in groundwater hydrology to determine S and T[1, 2]. However, this method requires curve matching of a type-curve and time-drawdown data from a pumping well test. Typically, this is done using a type-curve (W(u) Vs u) plotted on a logarithmic graph paper and the drawdown(s) data curve plotted on another transparent logarithmic graph paper (s Vs r^2/t) with the same scale, and superimposing and sliding the transparent one over the other to get the best possible match while keeping the coordinate axes of two plots parallel. Obviously this is a tedious approach. Based on the coordinates of a matching point from both plots (W(u), u, s, and r^2/t) in the matched position, the values of S and T can be determined using Theis nonequilibrium equation. The detailed descriptions about the manual type-curve matching can be found in many text books including Fetter[1] and Todd and Mays[2].

Due to the tedious nature of the conventional type-curve matching method, several commercial software programs have been developed for curve matching and are available (Infinite Extent by Scientific software group, AQTESOLVE by HydroSOLVE, Inc). Cleveland[8] described a method of type-curve matching using a computer spreadsheet. In this method, he provides formulas to create axis shifting for curve matching. It involves a trial and error determination of shift parameters. Another method using EXCEL and PowerPoint has been posted by Boll[9]. However, using these tools for teaching of type-curve matching is not easy because of the special requirements and the "black box" nature of those tools.

This paper presents an EXCEL tool to solve Theis equation. The objective of this paper is to demonstrate type-curve-matching with only basic EXCEL skills such as data entry and handling plotting features of EXCEL effectively. This can be an effective and easy-to-adopt teaching tool in class room to demonstrate the type-curve matching to the students. This tool doesn't require macro programming or parameter determination and additional numerical calculation. Moreover, all operations in this method could be done within EXCEL itself.

2. Type-Curve Matching Using EXCEL

Procedure for the EXCEL type-curve-matching is described in this section. Systematic instructions to perform type-curves matching follow. The pump test data used is of a confined aquifer and is taken from the textbook by Todd and Mays[2] problem 4.4.5, pg. 201. This demonstration is based on Microsoft Office EXCEL 2007 version. It should be noted that some of the EXCEL features may slightly differ for other versions of it.

2.1 Enter data and create plots

- Tabulate the well function data (W(u) and u) in EXCEL spreadsheet (Fig. 1). This needs to be done only once for the given type-curve. Appropriate well function need to be used for other type of aquifers.

- Tabulate the aquifer well pumping test data in EXCEL i.e., drawdown data with time, and create a column r^2/t; where r is the radial distance of the observation well from pumping well (Fig. 1).

- Create the EXCEL plot of type-curve i.e., W(u) Vs u in log-log scale. To create the plot, highlight the entire type-curve data set i.e., W(u), u. Then, select the insert menu, and select the "Scatter" button and from the pull down menu select the "scatter with straight lines and markers" graph (to convert the X and Y axis to logarithmic scale, right click on the axes then select format axis, after that select the axis option and select the logarithmic scale base 10). Moreover, fix the major unit and minor unit values to 10; and assign fixed values for minimum and maximum values for both axes (i.e., for confined aquifer type-curve, vertical axis minimum 1×10^{-5}, maximum 1×10^{2}; and horizontal axis minimum 1×10^{-15}, maximum 1×10^{1}).

- Add the grid lines and axis titles for both axes.

- Format the chart area, gridline, data series, and axes for clear visualization (Fig. 1).

u	W(u)
1.00E-15	33.96
2.00E-15	33.27
3.00E-15	32.86
4.00E-15	32.58
5.00E-15	32.35
6.00E-15	32.17
7.00E-15	32.02
8.00E-15	31.88
9.00E-15	31.76
1.00E-14	31.66
2.00E-14	30.97
3.00E-14	30.56
4.00E-14	30.27
5.00E-14	30.05
6.00E-14	29.87
7.00E-14	29.71
8.00E-14	29.58
9.00E-14	29.46
1.00E-13	29.36
2.00E-13	28.66
3.00E-13	28.26
4.00E-13	27.97
5.00E-13	27.75
6.00E-13	27.56
7.00E-13	27.41
8.00E-13	27.28
9.00E-13	27.16
1.00E-12	27.05
2.00E-12	26.36
3.00E-12	25.96
4.00E-12	25.67
5.00E-12	25.44
6.00E-12	25.26
7.00E-12	25.11
8.00E-12	24.97
9.00E-12	24.86
1.00E-11	24.75
2.00E-11	24.06
3.00E-11	23.65
4.00E-11	23.36
5.00E-11	23.14
6.00E-11	22.96
7.00E-11	22.81
8.00E-11	22.67
9.00E-11	22.55
1.00E-10	22.45
2.00E-10	21.76
3.00E-10	21.35
4.00E-10	21.06

Time, t/min	r²/t	Drawdown, s/ft
1	40000.00	0.05
2	20000.00	0.22
3	13333.33	0.40
4	10000.00	0.56
5	8000.00	0.70
7	5714.29	0.94
10	4000.00	1.20
20	2000.00	1.80
40	1000.00	2.50
100	400.00	3.40
300	133.33	4.50
1000	40.00	5.60
4000	10.00	7.00

r = 200 ft
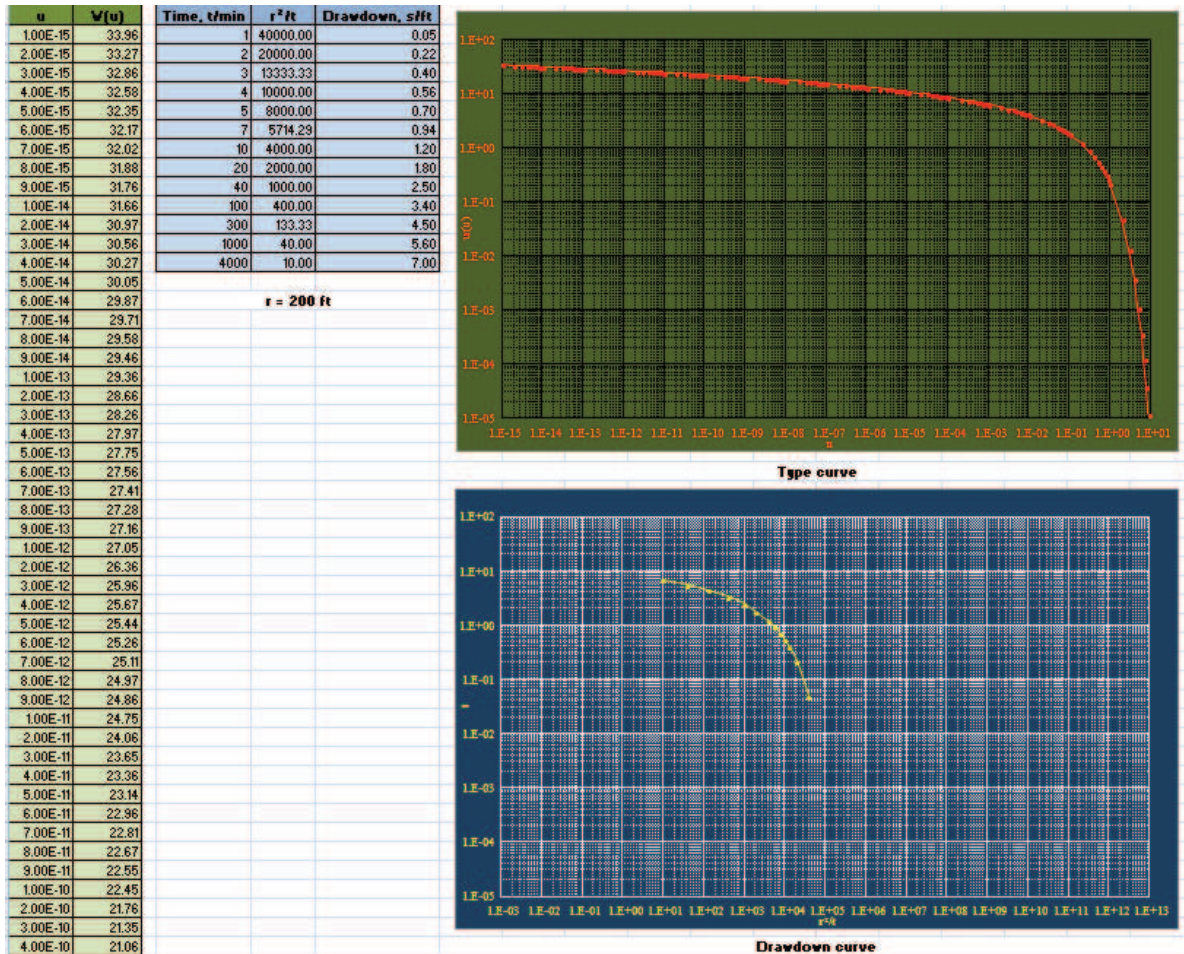
Type curve

Drawdown curve

Figure 1: Tabulated type-curve data (not a complete set), drawdown data, plotted type-curve and drawdown curve in Microsoft Office EXCEL 2007

- Plot the drawdown curve with the same size and scale as the type-curve. This can be performed by making a copy of the type-curve, and change the date series to drawdown data. However, both plots should have same number of log cycles in vertical and horizontal axis to preserve the same size and scale. For this particular problem drawdown data is between 0.05 and 7, and r^2/t is between 10 and 40000. The vertical axis (drawdown) range fall within the range of W(u), so there is no need to change the axis range of vertical axis. However, horizontal axis range (r^2/t) is not within the range of horizontal axis range of type-curve i.e., u. In this situation, the horizontal axis range needs to be changed in the drawdown curve. For this problem the horizontal axis range has been set from 1×10^{-3} to 1×10^{13} (sixteen log cycles, same as the type-curve). At this point, make sure that both curves have the same number of log cycles for horizontal and vertical axis. If not, appropriate minimum and maximum values need to be fixed. It is necessary to format the chart area, axes, gridlines, and data series for both plots so that they can be easily differentiated.

2.2 Curve matching

In order to perform the curve matching, one of the plots should be transparent. In this demonstration, the transparency is set to 75% for the drawdown curve. Then, chart area of the drawdown curve need to be moved over the type-curve while keeping the coordinate axes of two plots parallel until get the best match (Fig. 2).
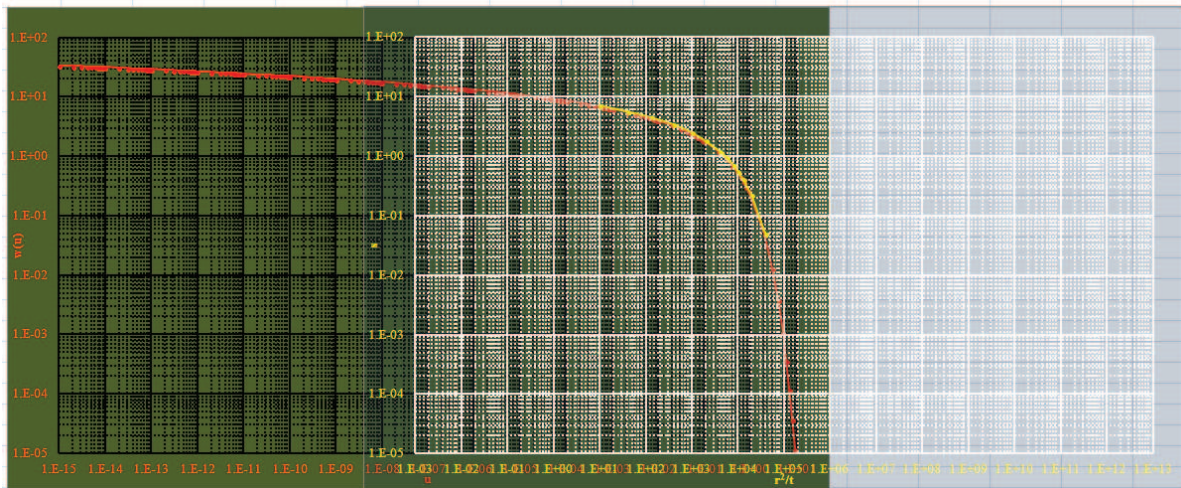

Figure 2: Curve matching of entire type-curve and drawdown plot

2.3 Enlarge the plots

Though curve matching is obtained (Fig. 2), it is difficult to get the required parameters to calculate aquifer properties. The curve matching of drawdown curve was performed with entire type-curve but only a portion of the type-curve is required. It is difficult to determine which portion of the type-curve is required for curve matching. The curve matching is performed with entire type-curve to identify the matching portion. Then, matching portion is enlarged to obtain required parameters. In this example, the matching was obtained for the vertical axis range of 10 to 0.01(three log cycles) for both type-curve and drawdown curve. The corresponding horizontal axis for type-curve range from 1×10^{-5} to 1×10^{1} (six log cycles). The horizontal axis of drawdown curve range from 1×10^{0} to 1×10^{6} (six log cycles). It is necessary to keep the same number of log cycles for both axes to obtain the same size and scale for the plots. Once the required range of values are determined for curve matching, and to keep the same number of log cycle for both axis, the axis minimum and maximum values need to be assigned for axes. Thereafter, curve matching need to be performed again using enlarged plots to get the best matching (Fig. 3). Once the best matching is achieved, record from both plots the coordinates of any matching point i.e., u, W(u), s, and r^{2}/t to determine the aquifer parameters using Theis equation.
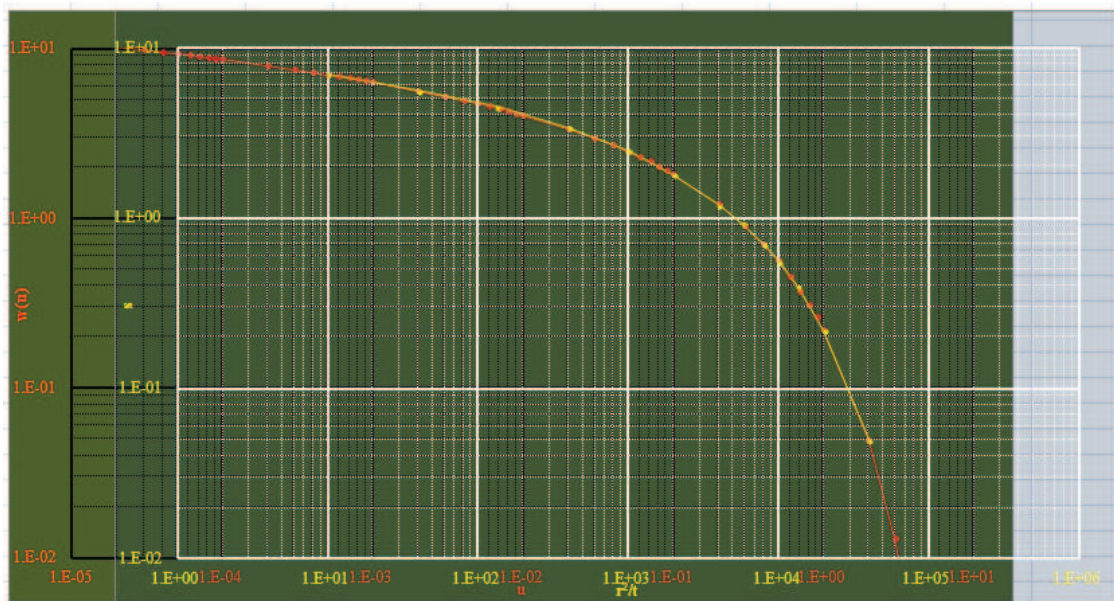
Figure 3: Curve matching of enlarged type-curve and drawdown plots

3. An Instructional Tool for Teaching Theis Graphical Method of Estimating Aquifer Parameters

Theis method of estimating aquifer parameters is a major topic in groundwater and/or hydrogeology courses. The method requires matching a type-curve to a data generated curve as described in the previous section. Teaching Theis method is often cumbersome because of the difficulty of showing the curve matching using graph sheets, one transparent and one opaque. We have demonstrated a method of accomplishing good curve matching using exclusively Microsoft EXCEL software which is generally available in most of the class room computers and/or computer clusters in universities. Moreover, the target student group of this tool (junior, senior, and graduate students) is fairly conversant using EXCEL. Once the students understand the fundamental concept of the curve matching in Theis method, it can be easily performed using this tool. Since the entire method can be performed exclusively in EXCEL, the instructor may demonstrate the use of this tool following the step-by-step procedure described in the previous section by projecting the laptop screenshots on to the large screen in the class room. In addition, the instructor may assign a set of problems on Theis curve-matching and encourage the students to use this tool for solving the problems. This tool was used in the Spring 2013 Groundwater and Seepage class at North Dakota State University. It was well received by students, and they used this method to solve their class assignments successfully.

4. Conclusion

We demonstrated a type-curve matching procedure using EXCEL to determine the aquifer parameters from pumping test data. The entire procedure can be done in EXCEL without any micro programming, and parameter determination. We tested it in a groundwater course. This can be an effective tool for teaching the type-curve matching. Though this paper illustrated the

tool to solve the Theis equation for a confined aquifer, it can also be adapted for different types of aquifers such as unconfined aquifers and leaky aquifers by accommodating corresponding type-curves.

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