Benefiting Professional Practice using Engineering Mathematics:
A Project-Based Learning Approach

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Abstract

Traditionally in the engineering profession, engineers are taught to read a problem, draw the diaphragm, identify the applicable engineering equations and solve for the numerical solution. While the methodology addresses the importance of understanding the problem, the process of solving leads to a single numerical value. As a result, the solution is case specific. Any change or modification of the initial variables as is common in engineering practice is at considerable effort and expense.

Using a more general mathematical approach, the applicable equations can be developed and manipulated symbolically in terms of unknown variables to develop a class of solutions (or system of equations). The resulting family of solutions is then reduced to a simple mathematical form, which can then be solved directly for the specific numerical solution. The focus of this paper is to present the student assessments of a sophomore level engineering statics course focusing on a mathematical technique used to develop generalized solutions to engineering problems. A fundamental approach is discussed which improves student understanding of the concepts in applied engineering mechanics. As part of the course, a steel truss bridge project is integrated into the curriculum.

I. Introduction

Engineers are technical problem solvers. From a historical prospective of the mid 20th century and after, engineers have been trained to be number "crunchers" due to significant changes in engineering education and technology as a result of the post World War II era[1-4]. From high school math and science courses through college engineering courses, engineers have been "molded" to crunch numbers. Here is a problem with all the associated numerical information. Now, solve for the solution.

The practice of number crunching has not only been ingrained in our engineering youth but also in our technology. Computers and now calculators have been developed which can rapidly crunch numbers[5]. In terms of analyses, numerical based methods such as difference methods and finite element methods have been developed to approximate differential equations. Such

solutions, even if the exact differential equations are known, generate only an approximate solution. And in the case of finite element analyses, the solutions are not conservative.

In engineering practice, number crunching has become routine. However, solutions are generated and constantly modified to meet unforeseen changes. After the solution has been calculated, modifying it is often done at considerable time and expense depending on the complexity of the problem and the dependency of the variable to other related system variables. It would be beneficial to teach engineers to develop general solutions which can be more routinely modified due to changes in constraints of variables or boundary conditions. Such solution strategies can be developed by solving problems symbolically in terms of variables rather than numerically.

One of the advantages of this problem solving technique is that students have an opportunity to develop equations in a pure mathematical form based on variables, devoid of arithmetic until the final step is performed. This gives students an opportunity to concentrate on the basic mathematical relationships between engineering variables[6].

II. Background

Engineers have traditionally used mathematics to study the behavior of nature for the purposes of human benefit. Over the last century, calculators, computers, computer languages and software have been developed for the purposes of rapid number crunching. However, questions remain regarding the influence of teaching using this approach. What is the impact of teaching number crunching to young engineers? Must engineers rely on a computer or calculator?

In an effort to get engineering students to think in terms of engineering mathematics rather than number crunching, problems were solved in an engineering statics course with a minimal amount of mathematical arithmetic and a practical hands-on project was introduced designed to demonstrate the benefits of this approach. Problems were solved in terms of variables rather than numerical values so that students would better conceptualize the material rather than thinking in terms of magnitude. Approximately 50 percent of all the problems solved in class and on homework, quizzes and tests were required to be solved in terms of variables. The remaining 50 percent were solved using the number crunching procedure, which is in general more familiar to the students. By using a 50/50 approach, the author hoped that students would come to understand and appreciate the benefit of solving problems in terms of variables.

The class project involved a preliminary hands-on inspection of a local steel truss bridge. College Street Bridge is a four span, steel, truss structure which crosses the Barren River in Bowling Green, Kentucky. An elevation view of the structure is shown in Figures 2 and 3. Spans 1 through 3 are through trusses, and span 4 is a pony truss. The historic bridge was built in 1915 and presently serves as a pedestrian bridge.

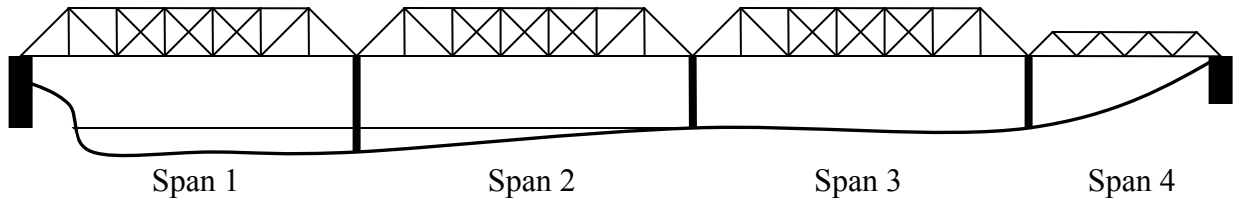Span 1   Span 2   Span 3   Span 4

Figure 2. Elevation of College Street Bridge

Figure 3. College Street Bridge

The purpose of the project was to give students an opportunity to work hands-on on a real engineering structure and to develop generalized equations for the dead load forces in selected members and to solve for these forces. Students were required to perform a preliminary inspection of the truss superstructure for spans 1 through 3 to obtain basic bridge geometry information. This information (since plans of the structure no longer exist) included the length of truss members (lower cord), distance between panel points, length of the vertical and diagonal members, traverse distance between the parallel trusses, and the roadway width. Member properties including size and shape as well as cross-sectional area were also found to determine dead loads.

III. Introducing Symbolic Problem Solving to the Classroom

The following equilibrium problem was presented in class as shown in Figure 1. Points A and C are pinned, and there is a hinge at point B.
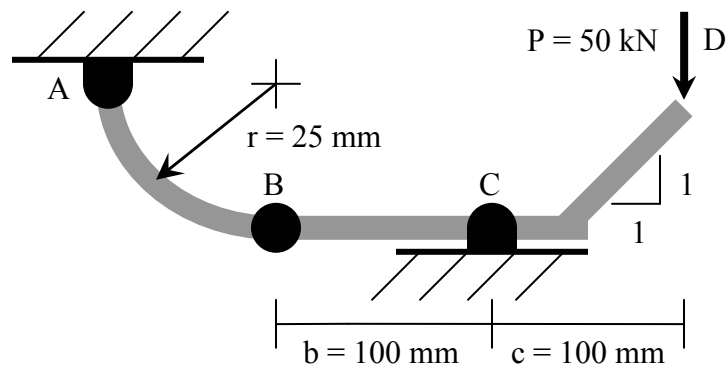
Figure 1. A two-dimensional statics problem

The students then proceeded to solve for the reactions at A and C. A question was then posed to the students, "What if b = 75 mm and P = 75 kN?" Students went back and re-engineered the problem to determine the reactions. Another question was posed to the students, "What if b = 85 mm?" Students became frustrated and started to rebel. "You said that b = 100 mm originally, then b = 75 mm. Well, what is the value of b?" I then explained the realities of engineering practice. Problems are never clearly defined. You may be given a problem and required to solve for the solution only to have to go back and re-work the problem many times often days, weeks or months after you have already done the calculation.

Having exposed the students to the less glamorous side of engineering practice, I then asked the students to solve symbolically in terms of variables b, c, r, and P for the reactions. The following solution was obtained.

$$F_A = \sqrt{2}\,\frac{c}{b}\,P \qquad\qquad \theta_A = \tan^{-1}(1) = 45°$$

$$F_C = \sqrt{1 + 2\frac{c}{b}P + 2\frac{c^2}{b^2}P^2} \qquad\qquad \theta_C = \tan^{-1}\left[-\left(1 + \frac{b}{c}\right)\right]$$

I then posed the following question, "Is solving this problem teaching engineering theory or application?" 83 percent of the students believed this to be theory, while the remaining believed this to be application. It appears that the students perceived engineering theory as working with variables even though the problem was an application of rigid body equilibrium. The students' perception of engineering application seems to be linked with dealing with physical numbers. Numbers have magnitude and, therefore, they have relative meaning. Variables do not have relative meaning unless they are compared in a specific context relative to each other. In a discussion with the students about theory versus application, many of the students viewed theory in terms of deriving formulas based on variables. While this explains why students thought that the above problem was engineering theory rather than application, the students did not realize that a variable can be thought of as a number with an unspecified magnitude.

All subjects throughout the semester were taught using the proposed methodology from centroids of areas and volumes to shear and moment diagrams for beams to the methods of joints and sections for truss analysis.

One of the challenges in teaching this methodology was the textbook. The textbook used for the course was "Vector Mechanics for Engineers: Statics" by Beer et al. (2004). In general, engineering textbooks encourage number crunching as the primary methodology for problem solving. Read the problem, set-up the equations with the numerical values and solve. While this method and the proposed method may seem similar, they are fundamentally different. The use of variables in engineering textbooks is primarily limited to proofs of engineering equations. Engineering textbooks do provide some example problems using variables such as centroids of a distributed force on a beam or centroids of an area as defined by a given function or functions. However, sample problems like these are limited.

For the purpose of overcoming the challenge of the textbook and to demonstrate the practical significance of the proposed methodology, a bridge project was introduced into the course. The results of the project are presented in the following section.

IV. Bridge Project

Results of the project are presented in Tables 1 and 2. See Figure 4 for truss nomenclature. Table 1 gives the schematic cross-section, cross-sectional area, and length of all of the truss members. This information including the self weight of the deck, beams, floorbeams and bracing was used in the calculation of the dead loads applied to the trusses.
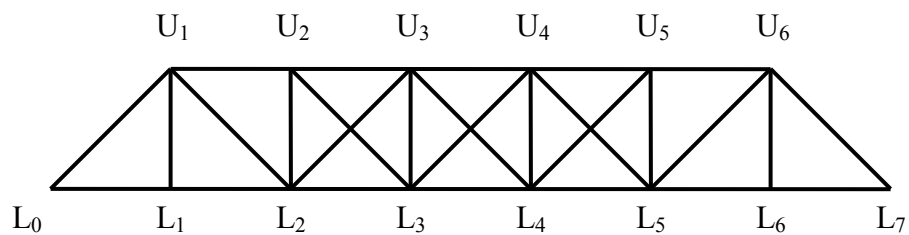

Figure 4. Truss Nomenclature for Spans 1 through 3.

From Table 2, the top cord members L_0-U_1 and U_1-U_2 bear a significant amount of compressive force, where as member U_1-L_1 is essentially a zero force member. Diagonal member U_1-L_2 and bottom chord member L_1-L_2 are tension members. Forcing the students to calculate the member force in terms of variables and as a specific quantity (in kilonewtons) gave the students the opportunity to compare member forces not just in terms of numbers but in terms of equations. The effect of changing the height, h, of the truss or the length from panel point to panel point, l, can readily be seen (Table 2). For instance (assuming no change in the loads P, P_{L1} and P_{U1}), decreasing the height of the truss by 50 percent will increase the force in the bottom and top chords by 200 percent. In addition, the effect of changing the loads, P, P_{L1} and P_{U1}, can also be taken into account.

Table 1. Truss Member Geometry for Spans 1, 2, and 3.

Member	Schematic Cross-Section	Cross-Sectional Area (mm^2)	Length (m)
L_0-U_1, U_1-U_2, U_2-U_3, U_3-U_4, U_4-U_5, U_5-U_6, U_6-U_7		14,030	L_0-U_1, U_6-L_7 = 8.00 All others: 5.18
U_2-L_2, U_3-$L3$, U_4-L_4, U_5-L_5		4,350	6.10
U_1-L_2, U_6-L_5, U_2-L_3, U_5-L_4, L_0-L_1, L_1-L_2, L_5-L_6, L_6-L_7,		3,870	U_1-L_2, U_6-L_5, U_2-L_3, U_5-L_4 = 8.00 All others: 5.18
L_2-L_3, L_3-L_4, L_4-L_5		7,740	5.18
U_1-L_1, U_6-L_6		1,855	6.10
L_2-U_3, L_5-U_4		792	8.00
L_3-U_4, L_4-U_3		1,555	8.00

Table 2. Selected Member Forces

Member	Schematic Cross-Section	Force[+] (in terms of variables)	Force[†] (kN)
L_0-U_1		$-\dfrac{\sqrt{l^2+h^2}}{h}\,P$	-246
U_1-U_2		$-\dfrac{1}{h}\left(2P - P_{L1} - P_{U1}\right)$	-265
U_1-L_1		P_{L1}	8
U_1-L_2		$\dfrac{\sqrt{l^2+h^2}}{h}\left(P - P_{L1} - P_{U1}\right)$	164
L_1-L_2		$\dfrac{1}{h}\,P$	159

[+] h = height of the truss.
l = horizontal length from one panel point to the next.
P_{L1} = tributary dead load forces from the deck, stringers, floorbeam, and truss applied at L1.
P_{U1} = tributary dead load from the bracing and truss applied at U1.
P = half the sum of all tributary dead loads from all panel points.
[†] Positive values denote tension, and negative values denote compression.

V. Student Assessments and Evaluation

Students were given a detailed survey at the end of the course. The students were asked, "How do you feel about solving problems in a more generalized manner in terms of variables? Did this feeling change over the course of the semester?" The response was that students "in the beginning did not like solving the problem this way, because the outcome was to develop a series of equations (rather than a specific numerical quantity)." Dealing with numbers, the students felt more comfortable since they have relative meaning as opposed to variables which have an unspecified magnitude. By the end of the semester, the students felt "more receptive to the concept of developing a 'family of solutions' based on generalized equations (rather than a single numerical quantity)." The remaining results are presented in Figures 5 and 6.

From statement 1 of Figure 5, 66.7 percent of the students either agreed or strongly agreed that upon reading the problem, they tended to quickly write the equations, plug the numbers in and solve. 33.3 percent of the students either disagreed or strongly disagreed with the statement preferring to take additional time to visualize the problem before proceeding. According to statement 2, 73.3 percent of the students agreed or strongly agreed that they made mistakes resulting from number crunching when solving problems, as compared to 26.7 percent who

disagreed or strongly disagreed. From statement 2, there is a tendency by a large majority of the students to rush through the numerical calculations often at the detriment of making errors. According to statement 3, 46.7 percent of the students either agreed or strongly agreed with liking the concept of setting up generalized equations based on variables and then solving for the solution. However, 53.3 percent disagreed preferring to solving the problem in a more numerical fashion. A similar almost equally dived class response was found regarding statement 4 where 40 percent of the students tended to "tune out," 13.3 percent were neutral, and 46.7 percent did not "tune out" when engineering problems were solved by developing generalized equations in terms of variables. Statement 5 indicates a strong preference by the students to learn about engineering applications as opposed to theory.
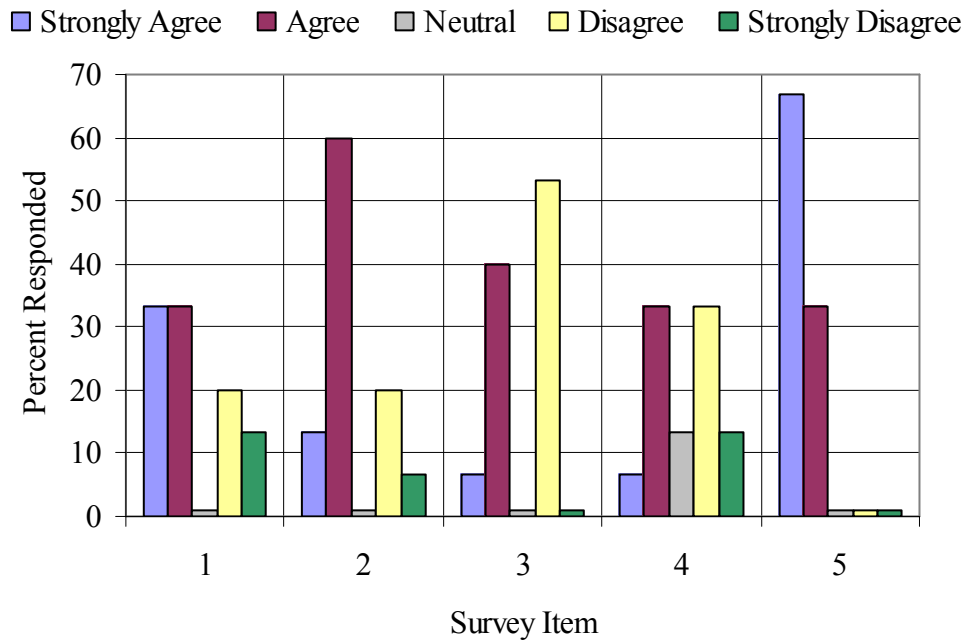

1. When I read a problem, I want to immediately write the equations, "chuck" the numbers in, and solve.
2. I sometimes make mistakes partially resulting from a number crunching approach to solving problems.
3. I like the concept of solving a problem by setting up the generalized equations consisting of variables and then solving for the numerical value by substituting all the numbers in.
4. If given a problem in class where the geometry and constraints are in terms of variables, I tend to "tune-out" since this is more abstract.
5. I prefer to learning about engineering applications.

Figure 5. Student Survey – Part I.

Statement 6 in Figure 6 reiterates the preference of the students to number crunch problems. Despite this, 40 percent of the students in statement 7 agreed that they preferred solving a problem by setting up and manipulating the generalized equations in terms of variables. However, 60 percent either disagreed or strongly disagreed.

In statement 8, 53.3 percent of the students agreed on preferring to solve a problem for a specific numerical value, and 46.7 percent of the students strongly agreed. Clearly, students feel a definite need to determine a physical number for the solution. However, in statement 9, 40 percent of the students preferred to develop a "family of solutions." While this seem contrary to statement 8 as one would expect the number to be far less than 40 percent, ultimately these students wanted to narrow the "family of solutions" down to a specific numerical value for the answer.
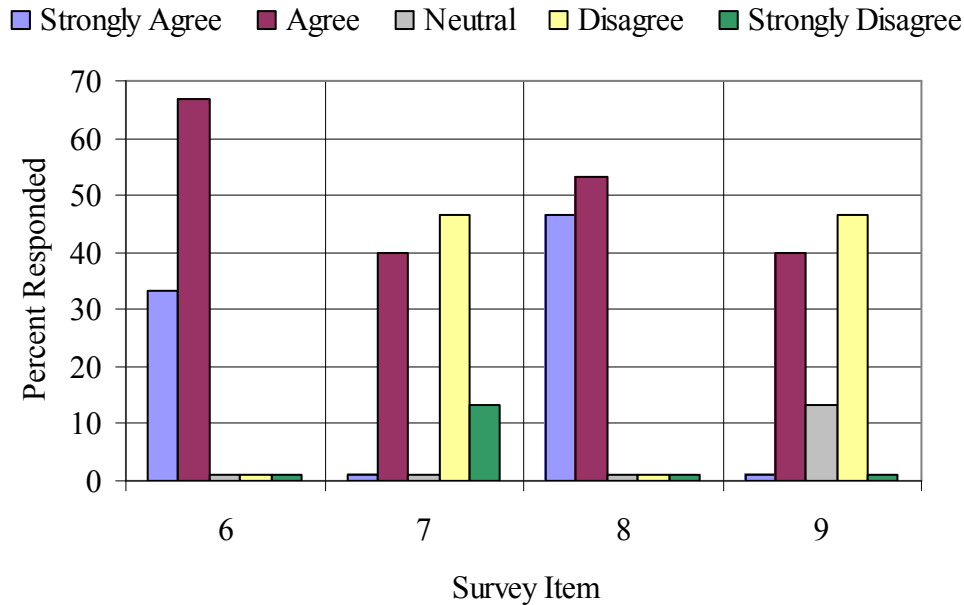
6. I prefer to number crunch when solving problems. (Just give me the numbers, and I will solve the problem.)
7. I prefer to solve a problem by setting up and manipulating generalized equations consisting of variables and then to solve for the solution by substituting in all the numbers.
8. I prefer to solve a problem for a specific numerical value.
9. I prefer to solve a problem by developing a "family of solutions" then to determine a specific value based on the numerical values of the variables.

Figure 6. Student Survey – Part II.

Despite the initial, inherent opposition of the students to solve problems by developing generalized equations, students found a number of advantages to this procedure. The generalized equations may be visually checked in a timely manner. Also, changes or modifications in the value of the variables may be performed quicker than the traditional approach. It is the author's opinion that students are not born thinking in terms of numbers but taught to think in terms of numbers. Greater emphasize needs to be place on the significance of variables as representing numbers and less on their magnitudes.

IV. Summary and Conclusions

In an engineering statics course, the students were taught to solve problems by setting up and manipulating generalized equations using variables. The proposed methodology departs from the traditional approach, which emphasized number crunching techniques. Students performed a preliminary inspection of a historical truss bridge and performed a dead load analysis to demonstrate the practical significance of the proposed methodology.

To introduce the proposed methodology, a two dimensional rigid body problem was presented in class. When the problem geometry was specified in terms of variables rather than numbers, students incorrectly perceived the problem to be engineering theory rather than application. Students perceived engineering applications as working with numbers, and they perceived engineering theory as working with variables.

The course assessments and evaluation indicated a strong preference by the students to solve problems using the traditional approach despite being taught the proposed approach. While students over the course of the semester developed an appreciation for the proposed methodology, the students were still inclined to use the traditional approach. While this rigorous numerical approach does enforce learning, it does not in the author's opinion leave a lasting impression on the fundamental theories of applied engineering. Students and industry alike would be better served if engineering students were taught to think less in terms of numbers and arithmetic and more about the equations they are trying to solve.

Acknowledgements

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