Engineering Faculty Perspectives on Student Mathematical Maturity

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Abstract

Mathematics coursework causes significant engineering student attrition. Many students drop out of engineering before even taking their first engineering course due to failing a prerequisite mathematics course. If the mathematics prerequisites fail so many engineering students, it is prudent to understand what exactly those students ought to be gaining by taking these courses. When asked what their students gain from the math course sequence, many engineering faculty members respond that it is not technical mastery, but “mathematical maturity” that matters. We conducted a qualitative thematic analysis of 27 interviews with engineering faculty members from 11 disciplines who taught engineering courses that list part of the core engineering mathematics sequence as a direct prerequisite. We examine which mathematical skills, habits, and attitudes constitute “mathematical maturity” for engineering students according to these engineering faculty members. We constructed an initial coding scheme from literature on mathematical epistemology, mathematical competencies, and symbol sense, with additional codes allowed to emerge during coding by two researchers.

Some of the findings of this study are presented here. 1) Faculty emphasize that students forget much mathematical content before encountering its applications in engineering courses. Many blame the fact that the engineering application of mathematical content may not come for years after the math course, and the engineering curriculum provides little reinforcement of math skills in the intervening semesters. This issue is particularly acute for complex numbers. 2) Engineering faculty are profoundly ignorant of what is currently being taught in mathematics classrooms. Many confess that they don’t know what is being taught at their own university, in the prerequisites for their own classes. Mismatched expectations may result. 3) Faculty repeatedly stressed that “mathematics is the language of engineering”, but don’t see their students holding the same view. Faculty find their students’ ability to use mathematics for the communication of precise, intricate ideas inadequate. 4) Faculty observe that students have excessive expectations of the certainty of mathematical knowledge. Faculty see students use excessive decimal digits, react with frustration to rough order-of-magnitude estimation or when presented with imperfect models. Faculty state that novice students seem to expect problem solving to not involve any kind of uncertainty, experimentation, or failure.

These results shed more light on the alignment of the current standard mathematics curriculum with the needs of the engineering students and faculty. This project exists in the context of a larger project examining mathematical education for engineering students and adoption of literature-supported curricula and pedagogy.

Introduction

Mathematics coursework causes significant engineering student attrition (Froyd 2005). Many students drop out of the major due to poor performance in prerequisite mathematics coursework before taking their first engineering course (Hoit 1998). As these fundamental mathematics prerequisites result in the loss of so many engineering students, it is prudent to understand what exactly those students are intended to gain by taking these courses.

In our informal discussions with engineering faculty at a large Midwestern research university, faculty members insisted that the fundamental skills gained by engineering students in the calculus classroom lay not in the specific learning objectives offered by that coursework, like the mastery of integration by partial fractions, but in the "mathematical maturity" gained through the experiences of these courses. This has also been found in the literature (Ferguson 2013). In this paper, we will report on the results of a study investigating what constitutes "mathematical maturity" according to engineering faculty, as well as the barriers their students experience in the pursuit of its development.

Methods

Research Question

This study investigates two research questions.

1) What do engineering faculty mean by the phrase "mathematical maturity" when discussing their engineering students?
2) What barriers to developing mathematical maturity exist in the current engineering curriculum ?

Interview Analysis

This study's data was a set of interviews with 27 engineering faculty members about their experience teaching core engineering classes and the mathematical abilities of their students. Faculty were selected to participate in the study if they had taught an engineering course that required any course from the Calculus Sequence (Calculus I, Calculus II, Calculus III, Linear Algebra, and Differential Equations) as either a direct prerequisite or as a corequisite. These interviews were approved by our university board governing human subjects research.

Interviews with faculty were semi-structured, with an initial interview protocol but with room to ask off-script questions to further explore the views of the subject. Because "mathematical maturity" is not well-defined in the literature, our interview questions were designed to investigate some of the possible definitions of this vague terminology. Interviews lasted about one hour and were transcribed verbatim. To analyze the data from faculty interviews, a team of two researchers conducted a thematic analysis (Braun 2006). The unit of analysis was pieces of participant's speech between two statements by the interviewer. We constructed an initial a-priori codebook from literature regarding documented immature epistemologies (Schoenfeld 1992), the KOM competencies (Niss 2011), and Symbol Sense (Arcavi 2005). Each unit of analysis was labeled with a code from the codebook or a new code was created to describe it. The two researchers each coded three interviews independently, then conferred and compared their results. This was used to refine the codebook and adjust code definitions. An inter-rater reliability of 81% was reached, indicating acceptable but not excellent agreement. We present four of those themes here: two on the nature of "mathematical maturity" itself, and two on some of the possible barriers to its development.

Results

Curriculum Structure

Participants emphasized that students forget many mathematical concepts before encountering those concepts' application in engineering courses. Many blamed the fact that the engineering application of mathematical content may not come for years after the mathematics course, and the engineering curriculum provides little reinforcement of many technical mathematics skills in the intervening semesters.

"*Most of them took it 4-5 semesters ago. If you haven't used it in the meantime, you might as well not have taken it. I operate as if they haven't taken it"*--Computer Science

"*In my senior class, by that point many of them had not taken a pure math class in some time and they were a little bit resentful when I made them do real math.*" --Materials Engineering

There are some disciplinary differences worth pointing out. Electrical engineering majors need complex numbers early on, but other majors do not use them until controls theory courses in the junior or senior year. By this time, many students have forgotten the relevant techniques they had learned in their mathematics courses.

"*You've gotta be able to integrate by parts. I do the standard stuff, chain rule and product rule and quotient rule, by parts, very important. Then I do complex numbers. I have to do that because I know they don't remember it."*--Agricultural Engineering

Curricular Knowledge

Professors insisted that the point of sending their students through the calculus sequence was to develop mathematical maturity, but they did not appear to know how mathematical maturity is developed. Many professors confessed they had little knowledge about the specific content included in their institution's calculus sequences.

"*I haven't seen the calculus curriculum at this institution or any in many years."*--Physics

Some did not even know what courses were prerequisites for their own classes, let alone these courses' names or content. As a solution, some faculty members did not enforce prerequisites.

"*Don't ask me. I don't make the prereqs. In fact i don't enforce them to be honest. Somebody decided this."* --Materials Science

Such ignorance may lead to a mismatch between what professors expect students to know and what students actually know. It can also have serious consequences for how professors teach their classes. One computer science professor shared the following anecdote.

"*We sent a survey to the CS faculty to see if they used the calculus. One of the stinger questions was epsilon delta proofs. Some people not only thought they depended on epsilon-delta proofs, but believed that calc was still teaching it. It was like one third believed this. I knew from talking to math people that those were long gone."*--Computer Science

<u>Mathematical Communication</u>

Participants expressed that mathematical communication skills were a signpost that students had become mathematically mature. This belief was expressed through students' beliefs about mathematical communication and students' abilities with mathematical communication. Participants repeatedly stressed that "mathematics is the language of engineering," but they felt that their students did not believe that communication was important. The participants found that their students were generally unable to use mathematics for the communication of precise, intricate ideas.

"*The degree to which a student can use the correct language of mathematics. Talks about solutions in the proper way and the approach for finding solutions in the proper way. Telltale signs is language in describing math."*--Civil Engineering

Some participants attributed this failure of mathematical communication to disciplinary differences between engineering and mathematics and to pedagogical choices.

"*Mathematicians see it differently. if you're interested in using math as the language of communication analysis of physics concepts, it's a different perspective."*--Physics

"*Math should be a technical language for storytelling. I don't think the math department shares that with them.*"--Bioengineering

Finally, participants elaborated that students were hesitant to explain their answers or comment on their solutions once they had reached the end of the mathematical process. Students neglected critical elements in the communication process like the interpretation and reflection steps of mathematical modelling. The use of mathematics to justify engineering decision-making is of great importance to practicing engineers (Gainsburg 2012), and justification requires communicating the results of these interpretation and reflection steps.

"*Once they have the answer, I wish i did this more, but getting students to comment on the result. Ok, here's the result, give me some comments. Tell me why you expect the terms up her or down* *here, or at least rationalize why this makes sense, or take some limits of the question."*--Mechanical Engineering

Certain Expectations

Participants observed that students have excessive expectations of the certainty of mathematical knowledge. This general observation was conveyed through three observations about student behaviors:

1. Students use excessive decimal digits
2. Students react with frustration to rough order-of-magnitude estimation or when presented with imperfect models, and
3. Students are reluctant to deviate from known solution paths. Faculty stated that novice students expect problem solving to be free of uncertainty, experimentation, or failure.

Overvaluing Numerical Certainty

According to participants, students are unwilling to accept answers that are not of high mathematical precision, despite the fact that high precision is often not possible in a real engineering context. One agricultural engineering faculty member commented on his struggle to change his students' perspective on this issue:

"*Engineers think they have to measure everything to 99% accurate. But we don't have that. If you can find an* R^2 *of 0.6 we jump for joy. 2% error doesn't matter. This is one thing we have to convey to our students.* --Agricultural Engineering

Participants also observed that many students have wildly inflated expectations of how unambiguous many engineering predictions using mathematics can be. In real engineering practice, estimating a parameter to within an order of magnitude may be all the design process requires. A chemical engineering professor argued:

"*When I call, say, 10/11 and call it .9, some students say it's technically .909, but it's close enough to .9 for me to not care. So that's I guess more of an engineering maturity as opposed to mathematical, but there is a mathematical side to it."* --Chemical Engineering

Another chemical engineering professor who teaches a junior-year course similarly commented,

"*They're torn between 'I know that I always…have to give six decimals places', but then 'I've also got this extrapolation that I did', to maybe look at the price of my raw material 3 or 4 years from now, and they struggle with…reconciling those two viewpoints.* "--Chemical Engineering

These comments reveal that participants see a difference between the certainty expected in mathematics coursework and the precision or certainty expected of mathematical knowledge in engineering contexts. Engineering knowledge needs to take the precision of measurements, models, and procedures into account when determining the certainty of mathematical computations and knowledge. Many participants indicated that the acceptance of mathematical uncertainty is one component of mathematical maturity; however, it is clear that the standard sequence does not typically result in this behavior. As a consequence, the engineering faculty is tasked with breaking even those students who have passed the calculus sequence of this expectation of certainty.

Imperfect Objects and Imperfect Models

Participants emphasized that students also reject imperfect objects and uncertain models. Faculty members stated that as a consequence of the pure math approach present in much of the traditional calculus sequence students perceive the mathematical world as very clean and perfect, and it is "up to engineering to introduce the imperfection" (e.g. imperfectly circular train wheels causing shock forces). One professor commented:

"*The mindset of 'real engineers' is different, we have to capture natural artifacts and be practical about it. It's a corn root. What's the diameter? Well if it were cylindrical that would be easy!" --Agricultural Engineering*

Practicing engineers value the contextual value of uncertain-but-still-useful information (Gainsburg 2012). However, students discount the value of imperfect information. One participant elaborated that taking uncertainty into consideration is an important skill for students to attain:

"*This idea of knowing things at different levels. Anyone who's looked at a weather forecast knows it's from a model. It's wrong, but we watch anyway. Because we have judgment. Because with judgment, you can use a model and still get information and make good decisions.*"
--Bioengineering

Orderly Approaches, Certain Methods

Many students adhere closely to fixed, recipe-like thinking, and faculty members consider that approach to be a problem. This epistemological belief of Orderly Process has been documented by Schoenfeld (Schoenfeld 1992). Students are inflexible and reluctant to deviate from classic "paradigm problems" they already know how to solve. Faculty members stressed that students are reluctant to try mathematical approaches they are not sure will work. Many claimed this belief makes students less effective in their engineering courses.

"*My style is to be a role model. Occasionally sloppy. That gives you an idea of how to start to attack. It emboldens you to view math as a grab-bag of different stuff, just piece it together and try it out and see if it gives something meaningful. If it doesn't, then fine." --Computer Science*

This participant defined "rigor" as the mathematics which covers all exceptions and corner cases. However, for his engineering students, it is maladaptive to constantly seek the most polished and rigorous form of a mathematical idea. He went on to say:

"*Here's a controversial one: math comes across as very slick. When you're being told you see them after 200-300 years of brilliant minds have sanded off all the edges and it's the most round* *shiny pebble it could be. It wasn't like that when it started. I encounter that that shapes an expectation that everything that comes down the pike is completely polished and rigorous that covers all the corner cases. I think it would be helpful for the students to let go of that and not overvalue that rigor for more that it's worth*" --Computer Science

Conclusions and Discussion

Many of the student attributes observed by these faculty members reflect the conclusions reached in mathematical epistemology literature such as Schoenfeld (Schoenfeld 1992).

The engineering community cannot blame mathematics departments for the lack of longitudinal reinforcement for mathematics in the engineering curriculum. This is not an issue unique to mathematics prerequisites; in the words of one nuclear engineering faculty member: "*Probably more than half of our students will not remember it. But that's probably true for almost anything that's upper level. Average or below average student, are they going to remember the material from some course? Probably not*."

The faculty also confessed that they have limited knowledge of prerequisite courses in their own discipline. Multiple professors wished for more collaboration between mathematics and engineering, but in the words of an Agricultural Engineering professor, "*I love teaching my courses. But do I ever talk to the math professor who teaches the courses before me? Never. We should. But everyone does their own thing. The math department isn't going to change anything because some dude from Agricultural Engineering tells them to change."* Another professor commented that just an hour every couple of weeks of a systems professor observing a differential equations course could make a difference, but there were always other priorities. Not all students know that prerequisites are not strictly enforced at their particular department or institution, which puts the disadvantage of the prerequisite structure on students with low knowledge of how college works. (O'Connor 2015).

It is unsurprising that the mathematical communication skills of students are so poor. Since mathematics assessments rarely ask for explanation or justification, students do not associate such activities with mathematics (Schoenfeld 1992). Note that mathematicians list mathematical communicating competency as an important goal (Niss 2014). The interventions to alter these mathematical attitudes require changes in assessment structure, such as the inclusion of writing assignments (Habre 2002). While engineering faculty expressed a belief that mathematics coursework did not teach students to use mathematics as a language of communication, we cannot verify that claim with this data. Future studies could investigate whether the participants' perceptions align with the enacted learning objectives of mathematics courses.

The expectation of certainty is not surprising, given how students are assessed in high school and early college (Schoenfeld 1992). Being off by 2% may not be a big deal in the practice of real engineering, but it is certainly enough to be marked wrong on an exam. Many phenomena encountered in the early years of engineering education, such as projectile motion, are simple and idealized enough to permit perfect, accurate answers. Due to the high speed and accuracy required on exams, there is no room for exploration or mistakes in problem solving. This attitude becomes more deeply ingrained in students during the mathematics-heavy freshman year. (Trautwein & Ludtke 2007)

These mathematical behaviors are among those considered necessary in the development of so-called “mathematical maturity.” However, the present mathematical curriculum was not designed with this contemporary ideation of mathematical maturity, as the curriculum has changed little since the 1940s. As a consequence of the availability of computational tools, the expectations of what is considered most vital in mathematics for future engineers has changed though the standard calculus sequence has not.

Engineering mathematics reform efforts are appearing around the United States to address some of these issues. Increased collaboration between engineering and mathematics departments may help students get the most out of their courses.

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