



Playing Relieves Stress... Concentrations!

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Most, if not all of the Mechanics of Materials texts, have a section on Stress Concentrations. This increase in stress in an axial, torsional, or flexural member arise from a change in the cross-sectional area such as fillets, notches, holes, etc. as the load travels through the material. Most lessons on stress concentrations are limited to a few pictures from the text and an explanation of how to use the stress concentration figures to determine the stress concentration factor.

This paper will present the physical model (sheet of paper of varying shapes) used during a lesson on stress concentrations. The author will present the lesson and the use of the physical models (cut out shapes with discontinuities). The physical model allows each student to play/demonstrate to themselves the importance of the change in shape and the placement of holes and notches. Assessment of homework data will highlight the value of simple hands on activities within as many classes as possible. Each person in the session will participate in the use of this physical model.

Introduction

Mechanics of Materials is a critical course in most engineer's technical development, but especially civil and mechanical engineers. Up to this point in their education, all analysis assumes the body is rigid, but we all know that materials deform when loaded. The amount of deformation directly determines the size of members and many times the material itself. Visualizing the expected deformation associated with a member shape and dimensions for each type of loading is crucial when determining the deformation when a member is subject to combined loads. Therefore, physical specimens that can show the deformations or failure points are critical tools during each lesson. Rubber shapes in the form of rectangular beams, circular columns, and wide-flange shaped beams along with a rectangle foam beam can provide physical demonstrations in most lessons. However, when it comes to stress concentrations, many professors 35 years ago used a Polariscope and the Theory of Photoelasticity. When clear rubber shapes were twisted, pulled, or compressed, and a light shown through a polariscope using an overhead projector, the change in stress was easily seen by a fringe pattern of colored bands. Stress concentrations were easily seen as points where the gradient between color bands were very close and numerous.

These tools are not as readily available or used today especially with the advent of computer programs to study stress analysis. Therefore, students are not able to bend or pull the clear rubber shapes themselves. Computer analysis is available, but most students learn best by doing and experiencing. So another technique was needed. Enter the challenger – a single sheet of 8.5x11 paper. As it turns out, this simple tool once cut into shapes can demonstrate the importance of stress concentrations and directly demonstrate the effect on the material and member. This paper will provide the academic background for the lesson, its location within the course, the lesson, the use of the tool, and an example problem and homework using the stress concentration experimentally derived tables in most Mechanics of Materials texts. Even though the tables exist for axial, torsional, and flexural members, this paper will focus only on the axial loading case.

Course Outcome

A course description and course outcomes are available for each course within the civil engineering program. For the Mechanics of Materials course, the course description is:¹

Elastic properties of structural materials, **internal stresses** and strains, **principal stresses** and strains including Mohr's Circle, **axial**, torsion, flexural, shear, riveted and bolted joints, combined stresses, shear and moment diagrams, and beam deflections.

The lesson on stress concentrations provides coverage within the bolded areas of the course description. The key course objective is: Analyze elastic elements under axial loading.²

Lesson Objective

In this Mechanics of Materials course, stress concentrations are covered as a portion of lesson 11 after completion of axial deformation that follows axial and thermal loadings. For this portion of the lesson covering stress concentrations, there are two learning objectives:³

- a. Explain how discontinuities in a member cause stress concentrations.
- b. Use stress concentration factors to solve problems involving axial loading of members with discontinuities.

The reading assignment does well at explaining theoretically the concept, but students need to feel to understand. The author pulls content from students to put minimal content on the board to use for completing an example problem prior to a homework problem (Figure 1). The board notes shown are the actual practice session by the faculty member, in color, in his/her handwriting, and how he hopes the content will appear on the whiteboard during the class.

Model

Prior to completing Worksheet 11D or during completion of Worksheet 11D (Appendix I), the author uses the following model (Figure 2), a single sheet of paper with four shapes, to create physical models for each student. A student worker makes the copies and cuts out the shapes with scissors and a paper cutter. It takes less than 30 minutes for the student to cut 30-50 sheets with a cost per sheet from \$0.02-\$0.04 based on copying or printing costs. The hole is completed using a large three-hole punch. Shape 3 is cut using the green outlined shape since it is the more extreme and the others transition to Shape 4. The instructor passes out the four shapes at the beginning of the lesson and the students are asked not to play with (load) them until time to experiment (Figure 3).

Prior to the example (or could be incorporated into the example with each change in shape considered) each shape is tested. Prior to placing axial load on each shape with thumb and pointing finger on each hand, there is a discussion as to where the paper shape will break as well as placing the load with as much contact between the pointing finger and thumb (spread the stress at load application). Some students will use multiple fingers to apply the load. The students start with Shape 1 where they predict breakage in the middle section. As it usually turns

out, the shapes normally break at the point of application of the load (thumb, Figure 4). Most students do not evenly pull using as much contact between the pointing finger and thumb, but rotate their wrists to place the axial load; therefore, placing a stress concentration with the tip of their thumb at the point of load application.

Term: <u>FAM 15</u> Lesson: <u>11</u> Subject: <u>STRESS CONCENTRATIONS</u> 3/3	
<p><u>STRESS CONCENTRATIONS</u></p> <p>STRESSES AT DISCONTINUITIES OR ABRUPT GEOMETRY CHANGES EXCEED AVERAGE STRESS.</p> <p><u>LINES OF FORCE</u> vs. <u>Lines of STRESS</u></p>	<p>$K = \frac{\sigma_{max}}{\sigma_{avg}}$ DEPENDS ON GEOMETRY NOT THE MATERIAL</p> <p>STRESS CONCENTRATION FACTOR</p> <p>K DETERMINED IN LAB FOR COMMON GEOMETRIC CHANGES (HOLES, GROOVES, FILLETS, ETC.)</p> <p>DEPENDENT ON TYPE OF LOAD →</p> <p>AXIAL ; TORSION ; FLEXURAL</p> <p>p. 163 p. 238 p. 330</p>
<p><u>CRITICAL WHEN:</u></p> <p>- Brittle Materials: Always!!</p> <p>- Ductile Materials: cyclic loading</p> <p>↳ large static loads cause plastic deformations for Ductile Materials.</p>	<p><u>WS#11D</u> ACTUAL ≤ ALLOWABLE</p> <p>$\sigma_{ACT} \leq \sigma_{ALL} = \frac{\sigma_{TENS}}{FS} = \frac{\sigma_{TENS}}{FS}$</p> <p>$\sigma_{ACT} \leq \frac{26 \text{ ksi}}{2.0} = 13 \text{ ksi}$</p> <p><u>Check @ Neck</u></p> <p>$\frac{P}{A} = \sigma_{ACT} \leq 13 \text{ ksi}$</p> <p>$P \leq 13 \text{ ksi} \cdot A = 13 \text{ ksi} (3 \text{ in}^2 / 4)$</p> <p>$P \leq 39 \text{ k}$</p>
<p><u>Check @ Fillet (Paper)</u></p> <p>$K \sigma_{ACT} \leq \sigma_{ALL} = 13 \text{ ksi}$</p> <p><u>DET Fillet:</u></p> <p>$\frac{r}{h} = \frac{.42 \text{ in}}{3 \text{ in}} = .14$</p> <p>$\frac{W}{h} = \frac{6 \text{ in}}{3 \text{ in}} = 2.0$</p> <p>$K \approx 2.0$</p> <p>$\therefore 2.0 \left(\frac{P}{A}\right) \leq 13 \text{ ksi}$</p> <p>$\therefore P \leq \frac{13 \text{ ksi} (3 \text{ in}^2 / 4)}{2.0} = 19 \text{ k}$</p>	<p><u>Check @ Hole</u> $K \sigma_{ACT} \leq \sigma_{ALL} = 13 \text{ ksi}$</p> <p><u>DET - K hole:</u></p> <p>$\frac{2r}{W} = \frac{2(.1 \text{ in})}{1 \text{ in}} = .2 \Rightarrow K \approx 2.45$</p> <p>$\therefore 2.45 \left(\frac{P}{A}\right) \leq 13 \text{ ksi}$</p> <p>$P \leq \frac{13 \text{ ksi} (1 - .2)(4)}{2.45} = 25.5 \text{ ksi}$</p> <p>$\therefore P \leq 19 \text{ k}$ Ans.</p>

Figure 1. Board Notes for Stress Concentration Lesson

With this experiment and discussion as to why the shape broke at the fingertip (stress concentration), the students try to reload the remaining piece of Shape 1 with most still breaking

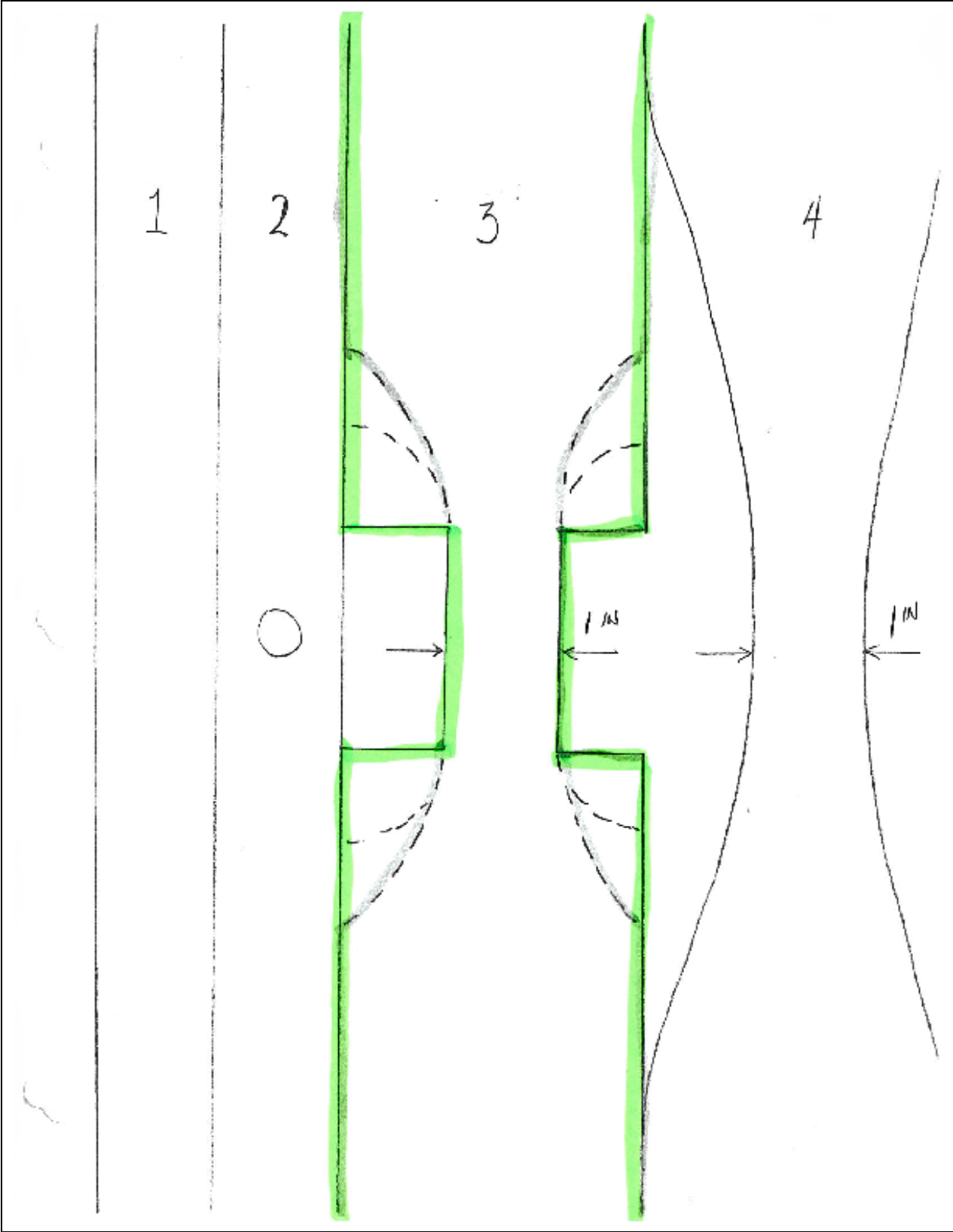


Figure 2. Stress Concentrations Model

at the fingertips. Many, except the very strong (arms and hands), cannot break the shape except at the load application point. Then the students move to Shape 2. Before they place a load, most students predict the break will occur at the hole. Further discussion moves most students to predict breakage from the outside toward the inside. The Professor asks the students to look closely at the hole as they gradually place pure axial load with no twisting of wrists and keeping the point of application of load using the fleshy portions of the two fingers on each hand and not the tips (Figure 5). Most quickly see more stretching of the inside of the hole which is the result of more stress than the outside edge similar to what is drawn on the first board of the board notes...the stress contours are much closer on the inside next to the hole versus the outside edge bringing the theory to life (Figure 6). The hole becomes elliptical with stretching of the bottom and top of hole. The load to break the shape is less than Shape 1.

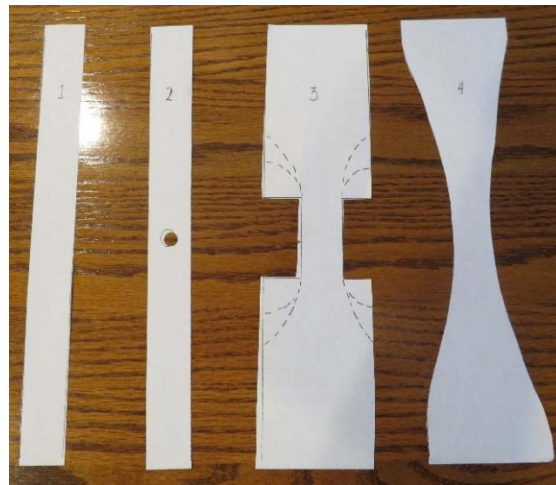


Figure 3. The Four Shapes

The third shape results in all students seeing the break in the middle, but only a little over half extrapolate the previous results to note the break at the corner where the abrupt change from wide to thinner width occurs (Figure 7). The stress contours are the closest at the inside corner. The students qualitatively feel the load is much less than Shape 2 that is less than Shape 1.



Figure 4. Shape 1 Broken at Finger Tips



Figure 5. Figure 2 Loaded



Figure 6. Stretching Inside of the Hole (Shape 2)

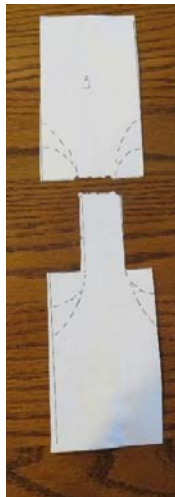


Figure 7. Shape 3 Broken at Abrupt Shape Change

The fourth shape is for fun. All students assume the shape will break in the middle, where it should. However, the gradual change in shape makes it very difficult to break the shape if only pure axial load is applied and no stress concentration at point of load application. Even those who could break Shape 1 in the middle normally cannot break Shape 4. Only some of the students with extremely strong hands and arms will be able to break Shape 4 (Shape 4 wins in most cases!), and still most who break the shape break it at the fingers due to the load with the tips of the fingers and a twist of the wrists. They can easily see that the gradual change from wide to thin made the shape as strong as the thinnest section without any apparent stress concentrations. The only difference between Shape 1 and Shape 4 is the smaller stress concentration at the application of the load (fingertips) based on the wider width (use of more fingers to load). However, if the student digs in with the fingertips to apply the load, the students note qualitatively that Shape 4 takes a larger load to break. Some will use the entire hand to apply the load (Figure 8).



Figure 8. Shape 4 Loaded

A key focus for the Worksheet (Appendix I)³ is spending time using the empirically developed charts, calculating the values to enter the charts, correctly using the charts, and then applying the stress concentration factor in the stress equation. Before using the charts⁴, a discussion as to the number of experimental tests required to create the textbook charts is impactful since the students only loaded one example (one point) on any given curve. Additionally, applying principles learned in previous lessons on axial loading, the load and cross sectional area are key values. Using the loading of the paper shapes, the correct direction to measure thicknesses and width for area calculations is highlighted to determine the cross section area for the fillet (ht of smaller section) or the hole ($w-2r$)(t). The ensuing discussion leads to the calculation to determine which curve to use within the fillet chart (w/h) and how to enter each chart, fillet (r/h) and hole (r/w). Applying the dimensional analysis to an experiment the students just completed assists in highlighting the key nuances of using the tables.

Homework

An example homework (Appendix II) uses a figure from an older version the course text⁵ and adjusts the size of the holes, thicknesses, and radii for each section of the specimen. The students are required to use the charts⁴ in their textbook since life-long learning is stressed using the textbooks assigned.

Results on Graded Activities

The impact of using the paper models to stimulate understanding on how to use the charts and the impact of the varying dimensions on student understanding is highlighted in Table 1. In the fall of 2015, the author was traveling and returned late the morning of Lesson 11 at 1 PM. It was during the lesson that he discovered that his lack of preparation led to no models being cut. There was no time to go backwards since the example was completed in class and the homework due the next lesson. The author did not see any red flags while completing the worksheet, but did see larger concept errors on using the table and equation in the homework (Table 1). Many students used the wrong dimensions for the area calculations and entered the table using the wrong dimensions. There was a clear decrease in performance on the homework in the year when the model was not used as part of the lesson content and worksheet completion. This simple axial load concept is not graded on an exam based on other higher priorities for each exam.

Table 1. Homework Results on Stress Concentration Problem

Term	Average	High	Low
Fall 2012	98.3	100	10
Fall 2013	90.3	100	33.3
Fall 2014	87.5	100	46.3
Fall 2015	65.6	93.1	6.76
Spring 2016	87.6	100	13.0
Fall 2016	85.2	100	47.0

Conclusion

There are experimental results noting the fact that many engineering students are visual (versus verbal), sensory (versus intuitive) and active (versus reflective) learners (Figure 8).^{6,7} Felder's research notes the importance of ensuring all students study at least some time in a preferred learning style. Using physical models and demonstrations are crucial to improve learning and understanding of concepts when students are visual, active, and sensory learners. Of course, many students have loaded structures before (intuitive), the content on the board and words used are verbal, and as the shapes break, the learner is reflecting on what they observed. Therefore, the student is using many complimentary learning styles in this lesson. However, the key is using the physical model to add the visual, sensory and active learning that is missing in many lessons.

During a simple lesson on stress concentrations, the professor uses a single sheet of paper cut into four shapes for each student in the class. This ensures each student can be an active learner through personal sensory and visual experimentation. The professor always cuts additional shapes in case a student tests a shape inappropriately as noted above and earlier than desired.

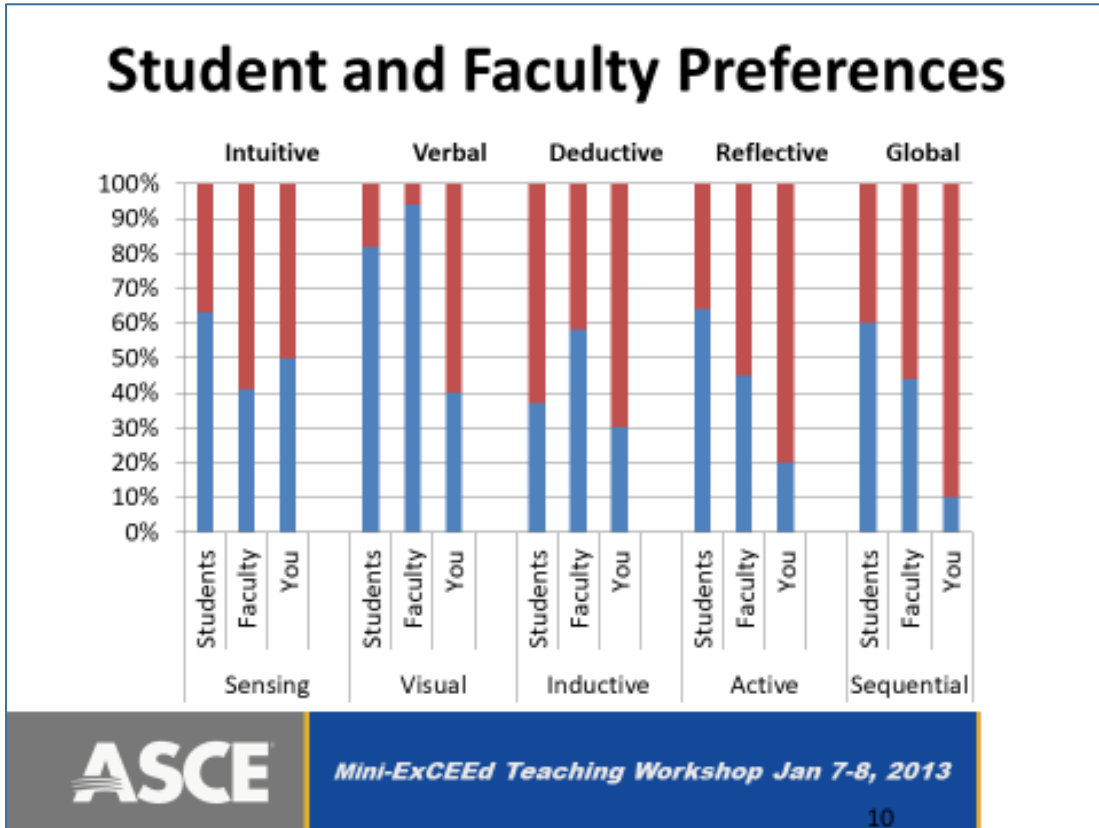


Figure 8. Comparison of Learning Styles for Engineering Students, Faculty, and Faculty at an Mini-ExCEED Teaching Workshop.

Even when the students load Shape 1 with their finger tips and through rotation of their wrists, the results further demonstrate the importance of understanding the effect of stress concentrations on loading since the shape breaks at the fingertips. This mistake in loading also highlights the importance of how the load is applied during actual experiments which occur in the lab portion of the course. The gradual movement from Shape 1 to Shape 4 allows for scaffolding of the learning leading to deeper understanding of what, where, and how stress concentrations are calculated to include which dimensions to use. The students are able to discuss qualitatively the change in loading to break the shape as the minimal cross sectional area remains the same, but the change in shape varies. The cost is minimal to create deeper understanding, the goal of each educator, right?

References

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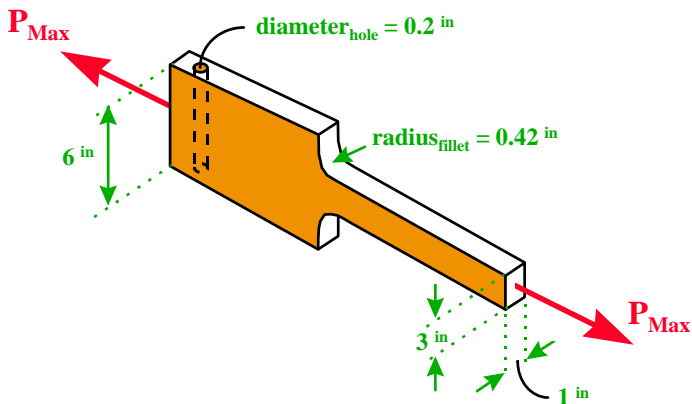
Appendix I

Stress Concentrations

Worksheet #11D

Determine the maximum axial load the structure shown below can support. Use a factor of safety with respect to the ultimate stress of 2.0 on this brittle material. The material is gray cast iron.

(ANS: $P = 18.75$ kips)



Appendix II

Example Homework

10c. (15 points) The steel bar has the dimensions shown in the figure. Determine the maximum axial force P that can be applied so as not to exceed an allowable tensile stress of $\sigma_{\text{allow}} = 170$ MPa.

