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Building bridges between mathematics and engineering: identifying modeling practices through Differential Equations and Simulation

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Building Bridges between Mathematics and Engineering:

Identifying Modeling Practices through Differential Equations and Simulation

Abstract

This article aims to share the study done over 2 years in a private university in the northeast of Mexico. Its objective is to study the modeling practices used in a specific course, Dynamic Systems, with students of Industrial Engineering taking a minor in Systems Engineering. This study should lead to the incorporation of activities based on their practices in the Differential Equation (DE) courses for engineers. It is emphasized the importance of the introduction of Systemic Thinking and System Dynamics by means of using a specific software. The contributions and advantages are studied with a qualitative paradigm. It is assumed that by introducing holistic thinking, the global engineer is educated to better meet the needs of the 21st century. The results provide evidence of the advantages of this course design since students perceive them as useful for their understanding of the main concept of DE. They also like having another representation of the DE to better understand the relations of equations and the use of this tool in complex problems which resemble those they will find in their work setting.

Introduction

Since the world is changing so quickly, it seems necessary to rethink all the working schemes that have been used for a long time. The 21st century has brought different demands because we live in a connected, global world, which has the need to build teams with people from different cultures. Therefore, this is why we have to rethink education from different points of view. This paper aims to share an effort being made from the Mathematics Education perspective, especially in the effort of rethinking or reformulating the intentions of math education. This should lead to provide training in key skills that the 21st century citizen requires. Thus, it is important to review some key ideas previously pursued on substantially modifying the teaching of mathematics in the classroom by introducing the use of technology and simulation, as well as by incorporating the development of social, communication, and teamwork skills in a math course.

From an international perspective, studies such as the report of the Program for International Student Assessment (PISA) [1] states the need to train people with the future skills such as mathematical literacy. PISA [1] defines mathematical literacy as the capacity to identify, to understand, and to engage in mathematics and to make well-founded judgments about the role that mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen. Subsequently, more specific studies [2] [3] aimed at a very specific population, future engineers, have made explicit the prevailing need for the basic individual education taking into account the fact that they should develop generic skills that complement and reinforce the disciplinary skills. In particular, we highlight part of a report [2] which aims to develop the

global dimension in shaping the future engineer and highlights the need and importance of these skills in several areas.

Generic Skills from [2]:

1. ***holistic thinking**, critical enquiry, analysis and reflection*
2. *active learning and practical application*
3. *self-awareness and empathy*
4. *strong communication and listening skills*

Hence, the need to develop **holistic thinking** as an important skill for students and future citizens of the 21st century appears explicitly. Based on this request –to train students from basic education in this area- we decided to explore this part. Holistic thinking is also related to Systems Thinking (ST), which is why we propose to think how to include, in engineering education, some of the abilities or skills from ST, and from the math education perspective. The report [2] explicitly mentions the work done by Senge [4] and motivated by this fact this paper aims to show the advantages and benefits of incorporating systems thinking in a math class. It is hoped that through this, it can be stated that the wealth of integrating the two seemingly disjoint in two different disciplines (Systems Thinking and Mathematics). The present work shows the results of the design of an innovative course of Differential Equations (DE), by means of using modeling and computer simulation, to have an active learning environment [4]. This course has been taught for some years now in a private university in the northeast of Mexico and seeks to make evident for the students the use and practical applications of this mathematical object (DE) in several areas, especially in those of physical nature, as well as those of social nature. The study presented here is analyzed from a qualitative paradigm.

First, by means of a qualitative approximation, meaning using: interviews and collegiate discussions with an expert (Industrial Engineer with minor in Electronics), observations in a specialty course (System Dynamics), analysis of textbooks, the technologies and languages more used. All this, led us to identify the key aspects of modeling which tend to be of interest for the future engineer. Second, the discussion and the collaborative design of modeling activities for engineers and math teachers (DE) will be treated in detail. These have allowed us to set a series of activities for the students to model the reality that surrounds them or those which will eventually become part of their everyday working life. All the above considers two aspects: the key ideas of the mathematical part concerning DE, and above all, the modeling practices of future value for the 21st century engineer (Phase 1). Finally, as a third point, the results of an institutional survey are shown. These actions set the frame for the qualitative paradigm; also, an opinion survey was done with the engineering students enrolled in a DE course. The survey dealt with the qualitative paradigm of the implementation of activities throughout a year (2014). It is mainly intended, to validate the contributions of this approach, to teaching DE through modeling and simulation of practical applications with fundamental modeling practices from the engineering point of view (Phase 2). The results provide evidence of the advantages of this course design in regards to the

students' perception as they see them useful for their understanding of the main concept of DE, and above all, the use of this tool in problems and settings which bring them closer to their future work setting. We consider these could be the basis for future research. We start the article by providing a general view of the global environment in which the present engineer will perform.

1 SYSTEMS THINKING APPROACH

As seen in the last section, some international reports and studies emphasize that "Education needs to prepare students for life-long learning in a globalized society which enables them to cope with and adapt to this complexity, uncertainty and vulnerability" [2]. Holistic thinking within the system perspective framework seems to be a way to afford it. Bourn and Neil state that "The nature of engineering is changing. […]Traditional boundaries between disciplines in science (chemistry, biology and physics) and disciplines within engineering (civil, mechanical, electrical, etc.) are breaking down. […]. Holistic thinking not only requires understanding complexities within engineering systems but also the relationship between engineering systems and their context". These authors [2] also state "Systems thinking is, in essence, the ability to see a problem or situation holistically, from a multitude of perspectives and understand the relationships, interconnections and complexity between the different parts that make up the whole. Peter Senge [5] is the most representative of this school and perhaps best known for his book The Fifth Discipline. In his work, he tries to show the need of seeing the world considering a fifth component or aspect (in addition to analytical, graphical, numerical and verbal into a modeling perspective, more details in section 5). In this paper, we develop the importance of incorporating a different way for looking the phenomena of real life in the math class. This brings direction to the student and future citizen: developing skills, which in the overall were poorly worked in schools, in particular, in a math class. It is important to explain what a system is. Meadows [6] proposes that "a system isn't just any old collection of things. A system is an interconnected set of elements that is organized coherently in a way that achieves something. A system consists of three kinds of things: elements, interconnections, and a function or purpose ". It is important to emphasize that a model is more than the sum of its parts. In a previous study done in 1990, Richmond [7] mentions only 5 skills to promote systems thinking, whereas in a more modern classification published in 2010, the author considers only 4 of them. Structural thinking (not mentioned explicitly) is about the difference between stocks and flows (rates of changes and magnitudes). He further notes that these skills would be part of the systems thinking skills and also relates them to critical thinking. It is important to notice that systems thinking and critical thinking are mentioned as 2 important generic skills to be developed by an engineer (future citizen) of the 21st century. It is precisely these two skills that are central for us to study in this work. Richmond [8] proposes in 2010 eight thinking skills (related also to critical thinking) that are fundamental to develop system thinking in students.

In this paper, we share Richmond's point of view on Systemic Thinking, but as a math educator researcher, we wonder how to help or contribute to develop this kind of skills in a math course. We want to retake the work done in the Differential Equations Course [5], and want to continue

working in this direction taking mainly into account the problematic present in this math subject in the last years. We would like to further discuss it.

2 TEACHING AND LEARNING IN MATH EDUCATION: A DIFFERENTIAL EQUATIONS COURSE

All over the world, and specifically in Mexican universities, the teaching of differential equations predominantly focuses on analytical methods rather than on qualitative and numerical methods. This has been reported in spite of the wealth of both approaches in the teaching of DE [9, 10]. This, and other developments, has been evidenced in the community of mathematics education for over 20 years [10], yet, some changes have been reported in the daily classroom activities. While successful innovative proposals have been documented on teaching DE (especially internationally) over the past few years [11,12], and some other research on the subject has been published, only few changes can be observed in classrooms and academic programs at various universities nationwide, particularly in the area of engineering. This proposal aims to acknowledge the importance of the changes in different registers (algebraic, numeric and graphic; "rule of three") [4, 10], the modeling approach [13], and the effective use of technology in the teaching/learning process of DEs. We also incorporated a fourth register, the verbal (word problems, "rule of four", [14]), and in the last years, we have recognized the importance for dealing with "reality" through a physical experiment (such as building an electric circuit) or a simulated "reality" through the use of simulators. We proposed in this study the necessity to integrate a 5^{th} register ("¿rule of five?" [5]), while working in a math course. We consider that the introduction of the fifth register in the DE Course could be the way to introduce the generic, the dynamic, the scientific, and the 10,000-meter thinking that Richmond proposes. The fifth register could also be helpful to promote another kind of perspective and way to regard the problems and contexts studied in this course. In the next section, we want to explain more about this specific technique from the System Dynamics viewpoint.

3 SYSTEM DYNAMICS & MATH COURSES

Many authors in literature consider that introducing system dynamics concepts is very natural in mathematics. System Dynamics (SD) models explore possible futures and ask "what if" questions. Therefore, we want to emphasize that the idea of introducing SD in a math course is not new, but it has been studied little in recent research, at least in the Math Education Community. In this section, we want to recuperate some important works in this direction.

Bourguet & Pérez [15] establish in their work that System Dynamics models and differential equations are two effective representations to express changes of things through time. System Dynamics uses symbolic and graphical representations, as well as computer simulation models to represent and understand the dynamics of a situation. This latter approach is also found in disciplines such as ecology, electrical engineering, chemical engineering, among others but these

disciplines use differential equations as their representation tool. Thus it seems to be useful to understand how to pass from one representation to another. [15, 16].
Fisher [14], also remarks that the use of differential equations can be expanded formally in the development of models in a Calculus class. Generally, differential equations are given very little time in most introductory Calculus classes. Unfortunately, this delays or (for many) eliminates the study of some of the most interesting applications in high school mathematics.
As simple as these experiences may seem, most students have not had concrete experiences in a math class, with the attendant vocabulary and reinforced connections that are so important for interpreting the equations and word problems that are found in the courses [14, 15].
A vocabulary using a systems perspective can be developed using an intuitive set of exercises that most students find easy to understand. Using "characteristic behavior-over-time" in addition to "rate of change" to describe the standard linear, quadratic, exponential, and periodic functions is reinforced repeatedly.
Fisher [17, 18] herself, recognizes that this is not a very high level use of system dynamics, but it connects system dynamics to the traditional curriculum smoothly, providing a leverage point for expanding analysis of applications and functions via the system perspective in future exercises. For example, this approach permits that students have the opportunity to apply both growth and decay components to the same problem (something that is noticeably absent in most textbook problems before Calculus level). They can also combine functions within the same problem, applying for example, exponential growth and linear decay. According to Fisher and the evidence observed over the years, it seems to be easier for students to explain simulation models they have created, since the structure of the model is more closely connected to the application components than traditional symbolic representations of problems. That is one reason why we want to include this fifth component in math courses. Just as Fisher, we also believe that this representation permits that once students have become accustomed to representing problems, generally presented in their texts, they become comfortable using lessons that introduce problems that would have been beyond the scope of the course, via traditional symbolic, numeric, or graphical expressions. […]

4 RESEARCH QUESTIONS

At the beginning of this study, we had some initial questions:

Which are the most important uses of the mathematical objects used by engineers ~~use~~?

In this paper, we are especially interested in the community of Industrial Engineering students with a minor in Systems Engineering, more specifically in the use and meaning related to the DE math object, so our first thoughts could be stated as follows:

Phase 0 (background of the study)

We are interested in identifying what the most important uses of Differential Equations are for Industrial Engineers with a minor in Systems Engineering, specifically in their specialty courses (Dynamic Systems –DS- Course).

With the known background, with this study we would like to discover the following:

Question 1. **How can math professors retake some important ideas of the DS approach to promote generic skills, additional to the mathematical ones, to better educate the new global engineer of the 21st century?**

Question 2. **How does the introduction of DS-based activities in a DE course affect our students, the future engineers?**

We want to describe the methodology that helped us provide some answers to these questions.

5 METHODOLOGY

The methodology for this study was divided in 3 phases:

Phase 0: Qualitative Analysis to answer question 0 from January-December 2013.

Phase 1: Qualitative Analysis to answer question 1 from October 2013-January 2014.

Phase 2: Qualitative analysis to answer question 2 from April- May 2014 and from November-December 2014.

5.1 PHASE 0

***Interviews and collegiate discussions with an expert:** We interviewed an industrial engineer with the following profile: he holds a PhD in Artificial Intelligence, a master's degree in Science in Electrical Engineering, and a bachelor's degree in Industrial Engineering in Electronics. He has 15 years of experience working in the field and the same number of years in teaching experience. It is worthwhile mentioning, that the product of this work [19], was the basis of the design of activities over a year of discussions, with a weekly 3-hour meeting for at least 32 weeks a year (a total of 96 hours). During this time it was discussed what the two apparently disjoint disciplines – System Thinking and Mathematics- and two different courses -System Dynamics and Differential Equations- have in common.

***Class observation** (System Dynamics and Differential Equations)

Over a period of 16 weeks, a course at the undergraduate level, System Dynamics, taught by an expert during the 2013 spring term was audited (7^{th} semester). Several annotations were made over the semester concerning participation, the class itself and the games included. This course is usually taught to students of two majors, Industrial Engineering with minor in Systems Engineering –IIS-, and, Business Engineering and Information Technologies -INT. The expert also

audited a Differential Equations Course during 16 weeks over the spring term (taken by students in their 3rd or 4th semester), which is taught by a math education researcher. This course is usually taught to students of 24 majors (all the engineering majors including IIS and INT; see more details in [19]).

***Analysis of textbooks**

The course textbooks for the Systems Dynamics course written by Senge [5] and Sterman [20] were analyzed. The first textbook is considered a classic textbook, which is the best contribution to the discipline of System Dynamics. The book explains the main dynamic systems in business and management. The second book, made the fundamentals of System Dynamics accessible to everyone in the 90s, with the aim of using systemic thinking. It is worthwhile mentioning the similarities with Blanchard, Devaney and Hall [10] as well as the work done by Fisher [17, 18].

***Technologies and language used**

The software and main tool used in this subject was Vensim [21] in its demo-for-education version, PLE. This is part of a broader range of software such as iThink/Stella. Its characteristics are described below. ST carries with it an icon-based lexicon called "stocks and flows". This language facilitates cross-disciplinary thinking, and hence, implementation of a "horizontal" perspective. The most commonly used model building blocks in Vensim are explained with details in Appendix 1 and other important works such as Fisher [14, 17, 18]. With the vocabulary and rate-of-change concepts previously emphasized, it is not difficult to expand problems to include a wider scope.

5.2 OPPORTUNITIES AND CHALLENGES BETWEEN MATH & DYNAMICS SYSTEMS

In **Phase 0** of this study, two professors (a math professor/math education researcher and an Industrial Engineering professor) worked together [19] to diagnose the interaction between two courses (Differential Equations & Systems Dynamics) during the 2013 spring semester, conducting 2-hour sessions over 13 weeks. Observations and notes were shared during a year (January-December 2013) in order to identify common areas of opportunity in the interaction of the two disciplines [16]. These areas are shown below [Further information in 19]. Both courses Differential Equations (Math) and Dynamic Systems are interested in:

1. Modeling phenomena of diverse nature (social and physics).
2. Using modeling software.
3. Doing qualitative analysis as a means of understanding phenomena.
4. Solving the mathematical model in question.
5. Using numerical methods to solve DE that represent the phenomena

We want to recover in the next sections these categories to study how the students accept (or not) our proposal (phase 1) about how introduce the DS perspective in a math course: a DE course (phase 2).

6 DESIGNING MODELLING ACTIVITIES CONNECTED WITH THE STUDENTS' FUTURE PROFESSIONAL LIFE, SUPPORTED WITH COMPUTER SOFTWARE AND VISUALIZATION EFFORTS

Test over the 2014 Fall Term

Session 1, Part A. Total Time: 90 minutes; time for this part A: 45 minutes. Week 14/16.

The Vensim software is introduced, and so are the philosophy of Systemic Thinking and System Dynamic during the first 45 minutes of the session. During this time, it is discussed a mathematical model previously developed in class (week 5) with the filling of a water tank. It is shown how to model the system of a water tank that is filled and emptied in function of its incoming and outgoing flows of water. This is a matter of interest for the student because of the use of software. It is a way to help the student become familiarized with the graphic language used by Vensim.

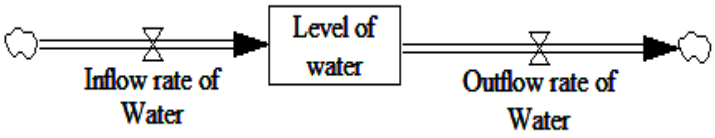


Figure 1. Example 1 of Vensim diagram

LoW(t) = Level of Water

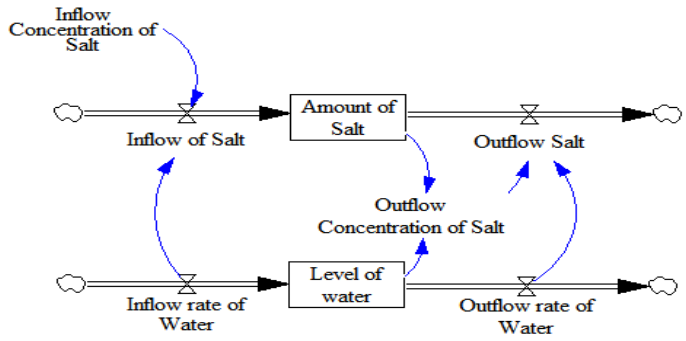
IroW = Inflow rate of Water

OroW = Outflow rate of Water

$$\frac{dV}{dt} = R_i - R_o; V(t=0) = V_0$$

Session 1, Part B. Total Time: 90 minutes; time for this part B: 45 minutes. Week 14/16.

In the second part of the session, the last 45 minutes, the student is asked to observe for a second time the software so that they can adapt it to the studied situation in class. They first studied the system of two tanks, but now salt is added to the incoming flow. Now what we are concerned about is the variable Amount of Salt in the tank S(t) [The figure below is an example of 1 tank with water mixed with salt].



$$\frac{dS}{dt} = IS - OS; S(t=0) = S_0$$

$$\frac{dS}{dt} = (ICS * IroW) - (OCS * OroW)$$

$$\frac{dS}{dt} = (ICS * IroW) - \left(\frac{S(t)}{LoW(t)} * OroW\right)$$

$$\frac{dS}{dt} = (ICS * IroW) - \left(\frac{S(t)}{V_0 + (IroW - OroW) * t} * OroW\right)$$

with

$$S(t=0) = S_0$$

Figure 2. Example 2 of a diagram in Vensim for a tank of water mixed with salt; a related math model

Session 2, Part C. Total Time: 90 minutes; time for this part C: 45 minutes. Week 14/16.

The philosophy of Systems Dynamics is presented again in a new problem during the first 45 minutes of the session. During this time, a mathematical model previously developed in class (week 3) is discussed. The model deals with the situation of a certain population infected by a virus.

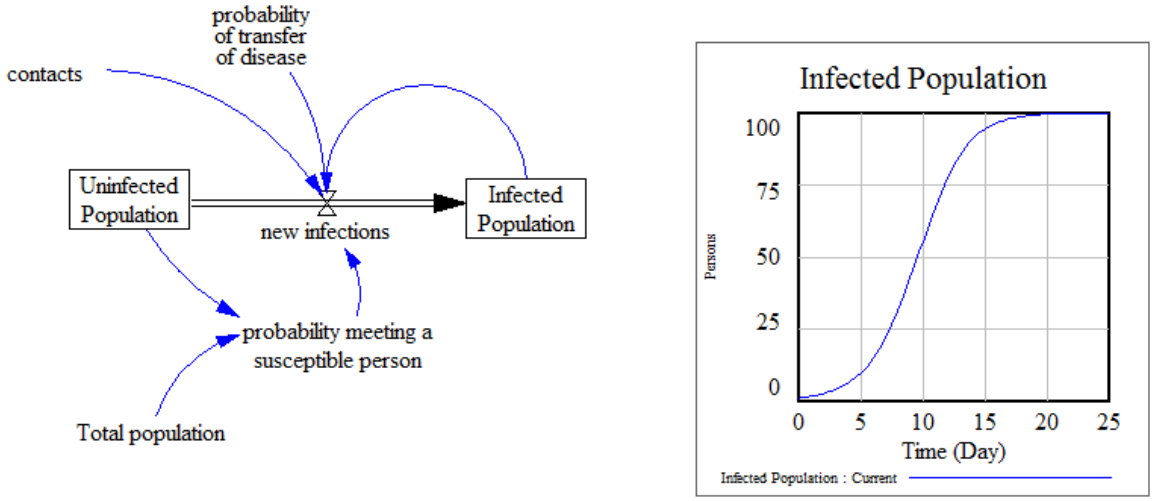


Figure 3. Example 4 of a diagram in Vensim (stock and flow diagram) and graphical representation

Infected Population = I(t); initial value = 1 {person}

Uninfected Population = U(t) ; initial value = 99 {persons}

Contact factor = cf = 10

Probability = p = 0.05

Total Population = N = 100 {persons}

Probability meeting a susceptible person = Uninfected person / Total Population = pmsp/N

*New infections = contact factor*probability*probability meeting a susceptible person*Infected Population*

$$\frac{dI}{dt} = cf * p * pmsp * I(t)$$

$$\frac{dI}{dt} = cf * p * \left(1 - \frac{I(t)}{N}\right) * I(t); I(t=0) = 1$$

This problem has been dealt during week 3 of the course and has been completely solved in an analytical way. This time and based on Blanchard's [11] development and presentation, it is proposed to return to the problem and consider it from a different view.

Session 2, Part D. Total Time: 90 minutes; time for this part D: 45 minutes. Week 14/16.

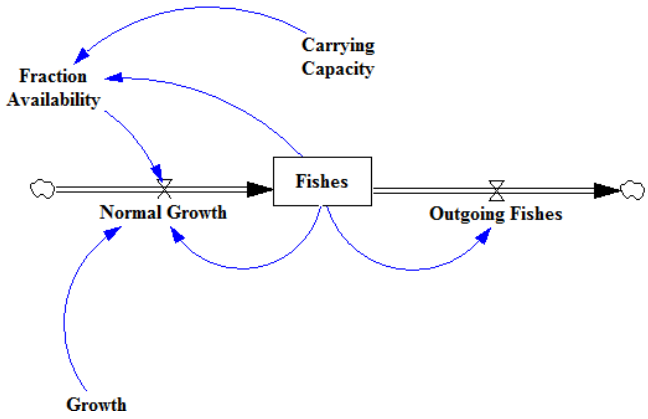
Later on, the students are asked to solve another problem that has been designed based on Blanchard's but rewritten in the case format.

Activity: exercise #15 and 16 from [11, p. 18]

15. Suppose a species of fish in a particular lake has a population that is modeled by the logistic population model with growth rate k, carrying capacity N, and time t measured in years. Adjust the model to account for each of the following situations.
a) 100 fish are harvested each year.
b) One third of the fish population is harvested annually.
c) The number of fish harvested each year is proportional to the square root of the number of fish in the lake.
(From excercise16). Suppose that the growth rate parameter k = 0.3 and the carrying capacity N = 2500 in the logistic population model of the Exercise 15. Suppose P(0) = 2500.
d) Which situation, a, b or c, is the most threatening to the environment? Support your answer.

Question d) has been included in the problem and it is the professors' decision to address it since it was not originally posed in the problem. During the 2014 fall term, the statement of this exercise was modified to introduce it as a case study. We called it the "Fish and Money" (see Appendix 2). The way it is written changes it considerably, giving the student the freedom to make decisions according to the situation described. This time, the problem leads the students to think of a scenario in which they have to make a decision based on three possible ways of fishing. They also have to support their answers based on the simulator according to the statement of the problem in the book.

One example of a student production is:



$$\frac{dF}{dt} = NG - OF$$

$$\frac{dF}{dt} = (G * FA) - OF$$

$$\frac{dF}{dt} = \left(G * F(t) * \left(1 - \frac{F(t)}{CC}\right)\right) - OF$$

$$with$$

$$F(t=0) = 2500$$

Figure 4. Example of the student representation for the case proposed and the related math model

7 OUTCOMES OF THE STUDY

7.1 Phase 2 Qualitative Analysis (2 semesters: spring and fall 2014)

In this last phase, we analyzed some results from an institutional survey since a descriptive statistics point of view. We did focus in the identification of some interesting behavior of the students from our activities that were design-from a systems perspective.

About the student opinion survey

We focused our attention on 3 questions. Since the Likert scale is used institutionally, it was decided that this survey would use the same scale, where number 1 means the best, and number 5 means the worst option. We considered that these 4 questions could help us have a general idea, of how the new exercises in a math class using simulation with a system thinking viewpoint, would allow the students to have better understanding of how mathematics is relevant in real and work life. This proposal, including system thinking, is oriented to show the usefulness and need of this science, from a math education approach, into a framework of a global dimension of a new engineer for the XXI century.

*Question: Theory and Reality (**TR**). "The professor implemented learning activities that allowed students to understand the relationship of the content of the course with reality"*

It is important to emphasize that the indicator Theory and Reality is usually lower for math classes, especially because of the traditional and near-sighted perspective that stops students from realizing the importance of math and its applications in their everyday life. It is also worthwhile noticing that this question is related to generic skill 2 of active learning and practical application.

*Question: Comprehension of Concepts (**CC**). "The professor facilitated the understanding of the content through clear explanations"*

It is mentioned in the mathematical section of this paper, that the introduction of the systems perspective, and the use of a simulator such as Vensim, is to allow the student to have another representation (the fifth representation; the stock-and-flow diagrams) of the DE for them to have better understanding of this math concept.

*Question: Learning Environment (**LE**). "The professor created a suitable learning environment."*

This question is about the active learning environment (generic skill 2) in the classroom where the communication and listening skills (generic skills 4) and the empathy skill (3) are very important in addition to the mathematical ones.

We also included space for the student to express a general opinion about the professor's overall performance during the semester. We called this indicator General Opinion about the Professor

(**GOP**). We considered it was a signal of the complete design of the Differential Equations Course. It is very important to remember that this course is based in active learning, modeling, and simulation practices.

Some charts of the 2013 compared to those of the 2014, both from spring semester, are shown. A total of 95 out of 123 students (77%) answered the survey at the end of the course. Of the four analyzed groups, two were **Honors courses**. An Honors course is a group with maximum 25 students. These students have a minimum average of 90/100 points and they speak at least one foreign language (usually English). We can say that they are special students in contrast with an average group. A more detailed study of these two groups is presented in this paper.

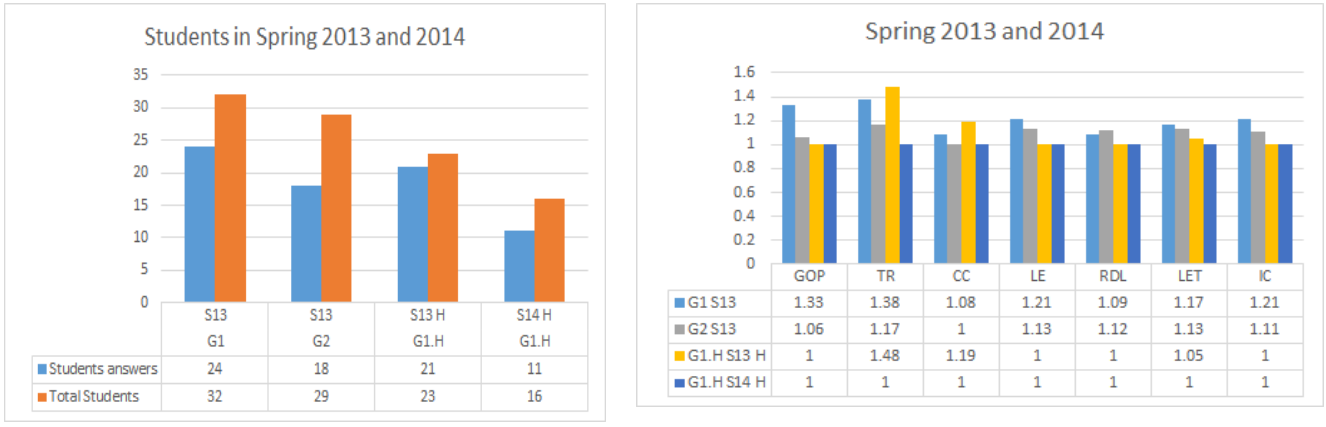


Figure 5. Student Sample and Comparison between Spring 2013-2014

On average, the three groups show an important difference in the TR indicator (1.3 to 1.0); the CC (1.25 to 1.0) and the LET (1.08 to 1.0). In detail, it is clear that the 2013 Honors group had a similar behavior to that of the 2014 Honors group (see detail) and Group G2 behaved closely to these two. It is important to mention that this particular course met three times a week for an hour each time. It makes us assume that this could have a positive influence in the results observed. We first compare two groups, an Honors course and an average course (No Honors, No H) in the same year.

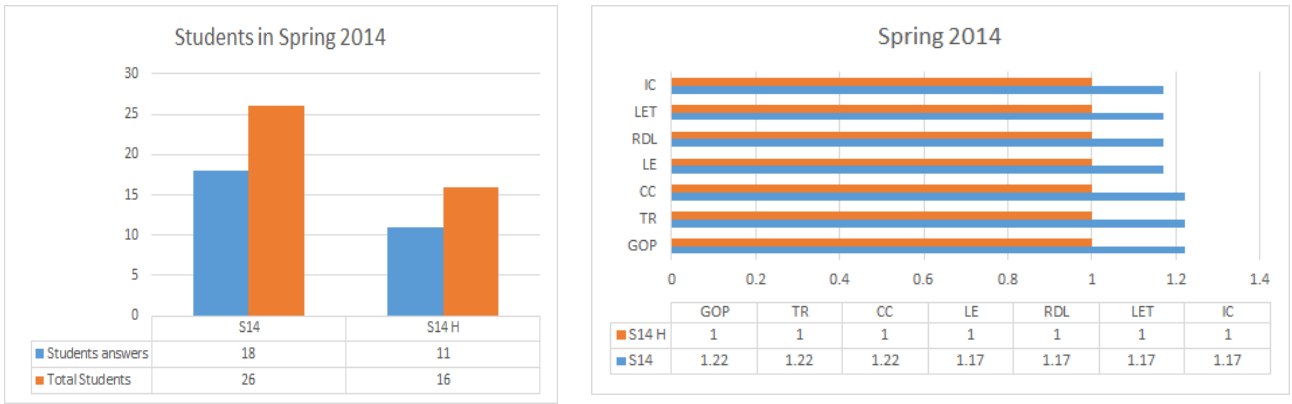


Figure 6. Student Sample and Results in Spring 2014 (honors vs non honors)

It can be seen that in the same semester there is an important difference in the TR and the CC indicators. Additionally, the change in the other four indicators is also relevant. A rise of +0.22 points in the GOP can be seen.

Finally, we analyzed the same semester compared to the same semester but the year before to see the difference in a year without ST. Now, the 2013 and 2014 spring semesters of the Honors courses are compared.

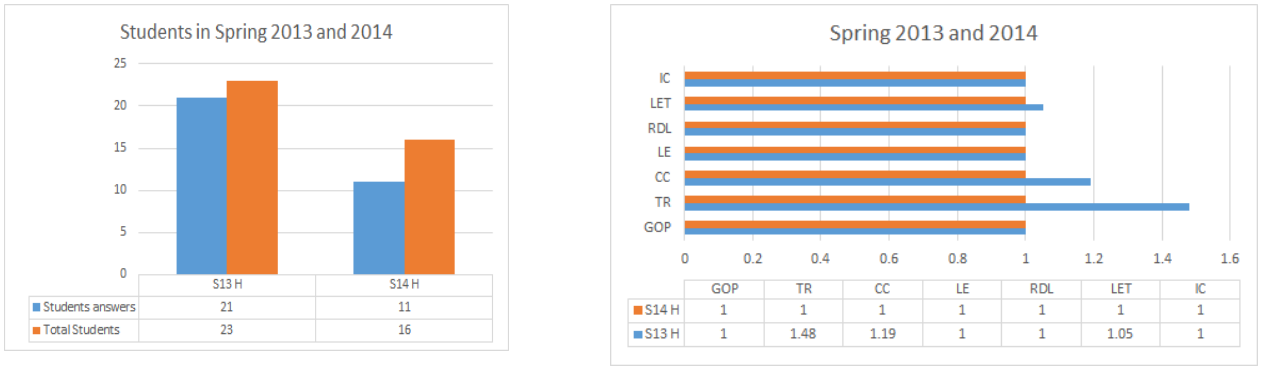


Figure 7. Student Sample and Results in Spring 2013 and 2014 (honors)

It is important to highlight the participation of 91% of the Honors group 1 in 2013. Two important gains can be seen in the TR and CC indicators for the GOP. This indicator is the same for both groups. There is a slight difference in LET.

7.2 SUMMARY OF THE OUTCOMES OF THE STUDY FIRST PART

As a result of the changes implemented in the course, we can conclude that students perceived that the professor facilitated the understanding of the content through clear explanations. We could infer, that the design and incorporation of these activities in the DE course, helped to better understand what a DE is by studying the different representations of this object (analytical, numerical, graphical, stocks and flows diagrams, real situation).

We could also conclude that the use of this technological tool was very important in the course, because it helped the students to have another perception of the math course. The students acknowledged the importance of the DE object since it is helpful to model other real situations in addition to those related to physical phenomena (traditionally studied in a math course).

Finally, about the theory and practice, this was a real challenge from a math perspective because in general, a math professor does not have enough time to introduce real phenomena for studying in a class session. In our case, we had the need to always introduce a real context and skills. This turned out to help the students to a better understanding of the application of Mathematics in their work life.

7.3 Phase 2: Second Part

Qualitative Analysis (2014 Fall semester)

This is concerned with the analysis of the students' opinions about the use of software (19 opinions). We show the results in two particular subjects:

1. **Modelling of phenomena of diverse nature (social sciences and physics)**

Six of the 19 students (31.6%) mentioned the richness of the use of the Vensim software and the System Dynamic technique to deal with real life problems, which should be more complex than the ones usually shown in a traditional DE course. They specifically comment on the Chemistry or Chemical Engineering and Business (logistics: resources and personnel) settings. These settings are seldom dealt with in a DE course.

I liked to use the software as a tool to see the potential of the use of differential equations in different professional scopes, not only the typical class topics.

2. **Use of Software for Specific Modeling**

In this category, the use of the specific Vensim tool for the DE course was highlighted. Especially, 12 out of 19 students (63.1%) emphasized that the software was very helpful for visualizations; other environments (such as the pencil-paper one) do not provide this possibility. They also mentioned its capacity to allow understanding the relations between variables (9 out of 19, 47.3%), explain and solve the model, describe behavior and compute predictions. Besides, its friendly-user feature, its effectiveness and its capacity to communicate mathematical ideas to non-mathematicians (ease in understanding its results) were highlighted.

This useful program ensures the user the convergence of their results and the exploration of the world of mathematics through cognitive organization and visual schematizing.

3. **Doing Qualitative Analysis as a Means of Understanding the Phenomenon**

Regardless of the qualitative analysis, the software allows the students to highlight the correlation of their previous findings, with the capacity and richness to produce graphs (12 of 19 students; 63.1%). They also had positive remarks for the graphic language used by Vensim, which allowed them to understand the relationships between variables (9 of 19; 47.3%)

Some ways of expressing the above were as follows:

I liked the Vensim program because it allowed us to observe the connections in the mixing processes and to visualize the problem in a more general way. I think it is also very useful to visualize the graphs in each tank and how the reasons of change within the system are related.

4. **Solving the Mathematical Model in Question**

Concerning the analytical part, only four out of 19 students highlighted the ease to solve a DE with Vensim because of the graphs it produces thanks to its numerical methods. A student even commented on the advantages of this software since it allows him the understanding of how to solve a DE in a more didactic way.

The problem at hand gets complicated, but with this program it is simple to understand what is related and the effects on time that the different variables will have.

5. Using Numerical Methods to Solve the DE That Represent the Phenomenon

In the section of comments, only four students expressed the capacity to produce tables, and two of them highlighted the numerical methods to find the solution. Although this is the basis of the software, it calls our attention that it is mentioned few times. One student emphasized the little error in calculations, and the accuracy of the use of the software regardless of the numerical methods.

Likewise, I think it is very convenient that accurate tables can be created, and thus, making predictions is easy.

Some comments are related to the software, how it helps to have different representations of a DE (graphic, numeric, and visual) and the stock-and-flow diagrams. It is important to notice how the creator of the model needs to "reach a convergence of results" that help him make some foresights or decisions around the phenomena. Besides, it also let the students explore Mathematics from a visual perspective. A student pointed out how the software does not let one forget or ignore the variables in an equation since they are related (linked with arrows with a stock or a flow) in the software.

The software is very friendly since it does not allow you to ignore variables once you have linked them even if you forget the formula. In the square below you can see the variables that you have to use and with that you remember better. Besides, the arrows and the squares adapted in a good way for the mix problems, since you could see them with the tanks and the flows.

The problem presented is not an easy task to solve ~~it~~ in class, since it is presented as a case. This makes that the different teams rethink their answers and although the problem seems easy, it is not possible to give a unique answer, and also the students have difficulties to establish a conclusion without the help of the software. We assume that incorporating a systemic viewpoint in order to understand the complete setting of the problem is helpful. Some matters of social, ethical and sustainable development interest appear within the arguments the students give to answer the problem. This kind of modeling practice observed in most of the cases of engineering fields is seldom addressed in a traditional math class.

8 CONCLUSIONS

The intention of the present article arises from the idea of improving the teaching and learning of mathematics at all educational levels. The part that interests us is improving the understanding of the future engineers that they have of a basic concept: the differential equation tool is of great value for the professionals at all levels in many disciplines. At this study, we identified, which is the use of Differential Equations that Industrial Engineers with a minor in Systems Engineering do. This was done in a specific course called Dynamic Systems. We also observed that the math professors could retake some important ideas of the Dynamics System Approach to promote the development of generic skills, such as: holistic thinking/systems thinking, active learning, ~~and~~ practical application, empathy, and also communication and listening skills. These skills are

additional to the mathematical ones. These skills are important to educate the new global engineer of the 21st century.

To understand how the introduction of the Dynamics Systems based on activities in a Differential Equation course affect our students, we decided to use 3 different analyses:

* From an institutional survey, we had elements to observe that the students had a better and a clearer understanding of the DE object. The use of technology was very helpful to have another representation of a DE, and to see that the relationship between theory and reality (practical application) showed a positive influence when we implemented this design.

* From an open-ended questionnaire, we observed that the students had better understanding of complex situations. They observed the generic structures in the mathematical models. They had a better idea of the relations in the DEs, the "big picture" of the different components of the problem, and they also had to further work in developing the skill to understand the dynamic of the phenomena. These skills are related to the 8-system skills proposed by Richmond to develop critical thinking; another important skill to be developed by the future engineers of the 21st century.

* The answers for the case presented at the end of this paper show several responses of the students to this problem: how helpful the software is to understand the different situations and scenarios related to different hypotheses on the behavior of social phenomena.

To conclude this paper, we found it very helpful and enriching to introduce a systems perspective into a math course. In future studies, we would like to explore the possibility to better understand, in more detail, the benefits of this approach in a Math Course. It is important to remember that the holistic thinking is an important global skill for the future engineer of the 21st century. As researchers of math education, we are interested in working in this direction.

REFERENCES

[1] Organization for Economic Cooperation and Development [OCDE]. (2009). *PISA 2009 Results. What Students Know and Can Do: Students Perfomance in Reading, Mathematics and Science*. Retrieved for http://www.oecd.org/pisa/keyfindings/pisa2009keyfindings.htm

[2] Bourn, D. & Neal, I. (2008). *The Global Engineer. Incorporating global skills within UK Higher Education of Engineers*. Engineers against Poverty. Leading Education and Social Research. Institute of Education. University of London.

[3] Jhori, A. (2009). Preparing Engineers for a Global World: identifying and Teaching Strategies for Sensemaking and Creating New Practices. Proceedings of the 39th ASEE/IEEE Frontiers in Education Conferencia.

[4] Rodríguez, R. (2013). Innovation in the Teaching of Mathematics for Engineers through Modeling and Technology: a Mexican Experience. ASEE International Forum 2013.

[5] Senge, Peter M. (2006, 1990). The 5th Discipline: the art and practice of the learning organization. New York: Doubleday/Currency.

[6] Meadows, D. (2008). Thinking in Systems. A primer. Chelsea Green.

[7] Richmond, B. (1990). Sytems Thinking. A critical set of Critical Thinking Skills for the 90´s and beyond. Systems Dynamics. http://www.systemdynamics.org/conferences/1990/proceed/pdfs/richm934.pdf

[8] Richmond, B. (2010). Tracing Connections. Voices of Systems Thinkers. ISEE.

[9] Arslan, S., Chaachoua, H. y Laborde, C. (2004). Reflections on the teaching of differential equations. What effects of the teaching of algebraic dominance? Memorias del X Congreso Internacional de Matemática Educativa (ICME XI). Dinamarca.

[10] Artigue, M. (1992). Functions from an Algebraic and Graphic Point of View: Cognitive Difficulties and Teaching Practices, en Dubinsky, E. & Harel, G (eds), The concept of function: Aspects of epistemology and pedagogy, MAA notes 25. Washington, DC: MAA.

[11] Blanchard, P. (1994) Teaching differential equations with a dynamical systems viewpoint, *en The College Mathematics Journal*, 25, 385-393.

[12] Blanchard, P., Devaney, R. y Hall, G. (2006). *Differential Equations.* (3a edición). Belmont: Cengage.

[13] Niss, M., Blum, W. y Galbraith P. (2007). Introduction. ICMI Study 14: Applications and Modelling in Mathematics Education. New York: Springer, 3-32

[14] Fisher, D. (1997). *Seamless Integration of System Dynamics into High School Mathematics: Algebra, Calculus, Modeling Courses*. Recuperado en: ftp://www.clexchange.org/documents/implementation/IM1997-07IntegrationSDMath.pdf

[15] Bourguet, R. E. y Pérez, G. (2003). On Mathematical Structures of Systems Archetypes. En *Proceedings of the 21st International System Dynamics Conference*. Nueva York, E.U.A.: System Dynamics Society.

[16] Bourguet, R. E. (2005). Desarrollo de Pensamiento Sistémico usando ecuaciones diferenciales y dinámica de sistemas. En Reunión de Intercambio de Experiencias en Estudios sobre Educación del Tecnológico de Monterrey (RIE). Monterrey. Recuperado en: http://www.mty.itesm.mx/rectoria/dda/rieee/

[17] Fisher, D. (2001). Lessons in Mathematics: a Dynamic Approach.

[18] Fisher, D. (2011). *Modeling Dynamics Systems: Lessons for a First Course*. 3rd Edition.

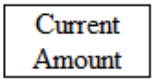
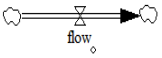
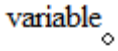
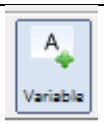
[19] Rodríguez, R. y Bourguet, R. (2014). Diseño interdisciplinario de Modelación Dinámica usando Ecuaciones Diferenciales y Simulación. Latin American and Caribbean Consortium of Engineering Education (LACCEI 2014). Guayaquil, Ecuador. http://www.laccei.org/LACCEI2014-Guayaquil/index.htm

[20] Sterman, J. D. (2000). Learning in and about complex systems. En *Business dynamics: system thinking and modeling for a complex world*. (Cap. 1, 5-10). Estados Unidos de América: Irwin/McGraw-Hill.

[21] Vensim www.vensim.com

Appendix 1

A language of system thinking in mathematics courses (based in Fisher works)

Concept	Mathematical form	Vensim form	Vensim button
Stock It represents the main quantity that is to be accumulated. The values increase or decrease over time. How things are.	Current Amount = ***A(t)*** (it is an unknown or dependent variable; time or t is usually the independent variable)		
Flow It represents actions or activities that cause the stock value to increase or decline over time.	Flow = $\frac{dA}{dt}$ (it is a rate of change or derivative)		
Arrow/connector It serves as an information wire or as an action wire. It shows the relations between the unknown (variable A(t)) and its derivative(s).	(Assumptions or hypotheses that we made, physical laws that govern the phenomenon)		
Variable/Converter It is used to represent additional logic important to the model (it is often a modifier for the flow)	(it is a parameter of the equation, usually called ***k***)		

Appendix 2

More fish, more money. What strategy to follow?

Description of the case

On November 19, 2014, as head of the Department of Environmental care of your own company, you should decide on the policy to be followed for the next 8 years. Tomorrow you will report the decision to the board of investors.

His company has been diversifying into canned fish market. You have found an extraordinary wealth of this in the lake, which is located two hours from your headquarters. The departments of planning, engineering and community outreach gave their recommendations last week. You are about to evaluate the recommendations and then commit to one of them today.

The lake and its fish

The lake has been evaluated for its fish population dynamics. It estimates that its recovery rate is about 30% per year with a carrying capacity of 2500. Currently, this is the number of fish that has the total population. The ecological system of the lake has remained in balance for several years. However, the quality of life of people living around can be significantly improved with schools, paving roads, a health center. Undertake an initiative to generate wealth through a fish cannery has been welcomed by the community. You have worked on the project and its investors will listen after tomorrow.

The recommendations of the Departments

Recommendations for extraction of fish came from the corporate, specifically from the departments of planning, engineering and community outreach. The first one recommended to fish a third of the total population each year. This is based on your maximum recovery rate; the second one, has proposed based on their mass balance calculations that removal of 30% of the square root of the current fish population is appropriate for obtaining maximum profits and environmental stewardship. And finally, the third one has proposed only take 100 fish each year, as is the conventional way that fishermen have worked the surrounding community for many years.

The decision and the strategy

Your department has made environmental stewardship in computer modeling of the population dynamics of the fish in the lake. The logistic curve has managed to capture the dominant dynamics of this population. Now it's time to include the recommendations you have into the model and find the new equilibrium point, if any. The question to answer is: What strategy causes less damage to the fish population in order to have a sustainable business for many years?