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Case Study: Numerical Convergence Study on Simulated Spaceborne Microwave Radiometer Measurements of Earth

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Abstract

This paper describes a NASA internship case study in which the outcome can be implemented in a classroom setting. Through this case study, the students can learn numerical interpolation and integration of functions in a real world project as well as the error analysis. A tradeoff expected to be observed is between the speed of obtaining results and getting accurate results. As a result, recommendations are expected for various data sets to ensure fast and accurate results. The simulation can also be carried out using Matlab.

The case study discusses numerical convergence of simulated space-borne microwave radiometer measurements of earth brightness temperatures so as to get fast results. The results are obtained by numerically evaluating a double integral. The integral relies on antenna pattern measurements and observed brightness temperature distribution over the earth's surface. Accurate antenna temperatures are obtained by modifying the step sizes while getting faster results. The accuracy of the numerical methods is analyzed and recommendations are given to improve the process. Such recommendations will be seen to vary for different data patterns. The study will also include antenna theory in order to understand its parameters, and antenna equations that affect the accuracy of the results, as well as antenna equipment, radiation patterns and radiation propagation.

I. Introduction

The main goal of this project is to enable students to realize the applications of mathematic and numerical techniques in antenna theory. Students that have a background in mathematical topics such as log scale, frequency, wavelength, spherical coordinates, integrals of several variables, and numerical integration can be introduced to antenna theory and measurements.

Antennas are devices used to efficiently transmit and/or receive electromagnetic waves. They serve as interface between wireless channels and circuits. Most antennas reversibly link radiation fields to currents flowing in wires at frequencies ranging from sub-audio through the far-infrared region[1]. Each antenna is designed for a certain frequency band. Beyond the operating band, the antenna rejects the signal. The antenna is an essential part in a communication system that is installed in satellites to receive the signals. Special instruments inside the satellite use these signals to measure air temperatures, soil moisture, wind, water currents, and much more. One of the instruments installed in a specific satellite is the Soil Moisture Active Passive (SMAP) mission that observes soil moisture and freeze/thaw state from space to improve estimates of water, energy and carbon transfers between the land and the atmosphere[2]. This will reduce uncertainties in quantifying the global energy and carbon balance. Areas directly addressed by SMAP are weather and climate forecasting, droughts, floods, agricultural productivity, human health, and national security. These areas, as well as others, help to improve the understanding of linkages between water, energy and carbon cycles. Data for measurements in SMAP are carried out by antennas. Specifically the SMAP concept utilizes L-band radar and radiometer instruments sharing a rotating 6-m mesh reflector antenna to provide high-resolution and high-accuracy global maps of soil moisture and freeze/thaw state every two to three days.

A SMAP mission is depicted in Figure 1 as below.

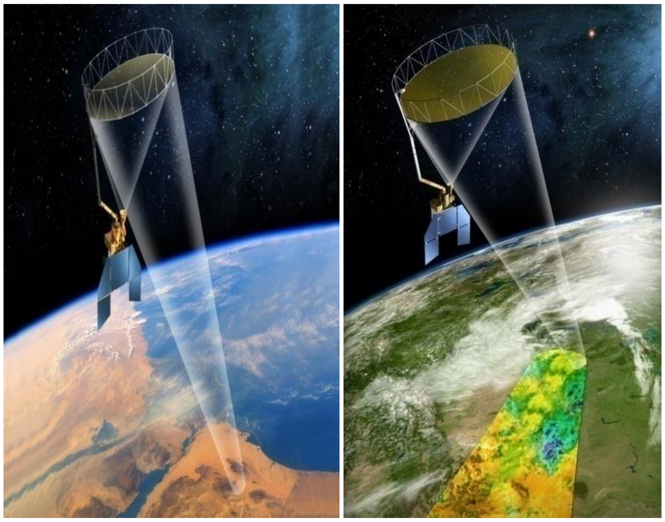

Figure 1: SMAP Mission

Some basic parameters such as power, radiation pattern and efficiency, directivity, beam solid angle, polarization, and bandwidth affect antenna performance and are of interest to scientists.

The antenna radiation efficiency e_{cd} is given by

$$e_{cd} = \frac{P_{rad}}{P_{in}}$$

where P_{in} is the total power delivered to the antenna terminals, and P_{rad} is the total power radiated by the antenna. The efficiency is close to unity for most antennas. Ideally, this power would be radiated exclusively in the intended direction, but in practice some fraction is radiated into sidelobes instead of the side, or to the rearward 2π steradians in the form of backlobes.

Although power is usually the parameter of interest in communications or radar systems, antenna temperature is often preferred when passive sensors for radio astronomy or remote sensing are of interest. A full Stokes vector formulation of an antenna temperature accounting for the entire antenna pattern, including polarization mixing in the main-beam and side lobes, is presented in reference [3].

The scalar formulation for the antenna temperature of interest is[4]

$$T_A = \frac{A_e}{\lambda^2} \int_{4\pi} F_n T_B \, d\Omega$$

where T_B is the brightness temperature distribution, F_n is the normalized antenna pattern, A_e is the effective aperture area of the antenna, and λ is the wavelength of the mean frequency of interest. Both T_B and F_n are functions of direction. Accurate antenna temperatures are obtained by modifying the step sizes while getting faster results. The rate of convergence in numerical integration can be slowed down or even reversed due to a singularity at the boundary of the region of integration in the integrand function or data.

II. Methodology

The recent development in computational capabilities, along with increased software reliability, made the numerical method and simulation approach more favorable. Examples of radiation patterns can be used to evaluate the integrals that reflect different kinds of antennas, such as traditional versus a focused antenna. The study includes the numerical evaluations of antenna temperature, directivity, and beam solid angle.

Study 1: In this section antenna temperature is calculated numerically. The numerical integral in cylindrical coordinate form for the antenna temperature corresponding to scalar formulation is given by

$$T_A = \int_0^{2\pi}\int_0^{\theta_h} X(\theta,\phi)Y(\theta,\phi)\sin\theta d\theta d\phi$$

where θ_h is the visibility range of the antenna to the earth from the vertical axis with respect to the antenna; $\boldsymbol{X}$ being the earth brightness and $\boldsymbol{Y}$ being the antenna temperature. Three sets of data are collected for the earth brightness. The first set of data is collected across the sea which is smooth. The second set of data is collected across the earth's soil surface which is relatively smooth, and the third set of data is collected across the earth coastline. In order to study the convergence, the numerical method approach is utilized by first interpolating the numerical data and then evaluating the interpolating function.

As an example, setting $\Delta\theta$ changes from 0.1 to 1, the integral for T_A was calculated numerically in Matlab and the mean temperatures obtained. In the numerical calculations, data is interpolated linearly and the interpolating function is used in the double integral. Each calculation was timed.

Figure 2 shows computational time will decrease while $\Delta\theta$ is increased. The computation time decreases as the step size of the computed integral increases, which is a desired property. However, Figure 3 shows that the computation of the earth temperature may be inaccurate with large step sizes.

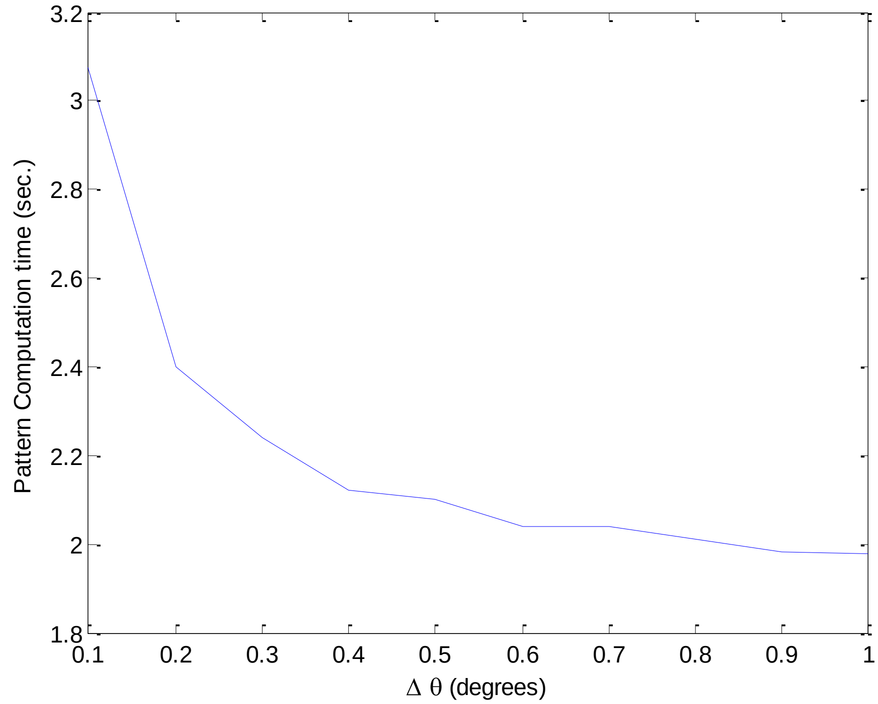

Figure 2: Antenna Pattern Computation Time ($\Delta\phi = 0.1$)

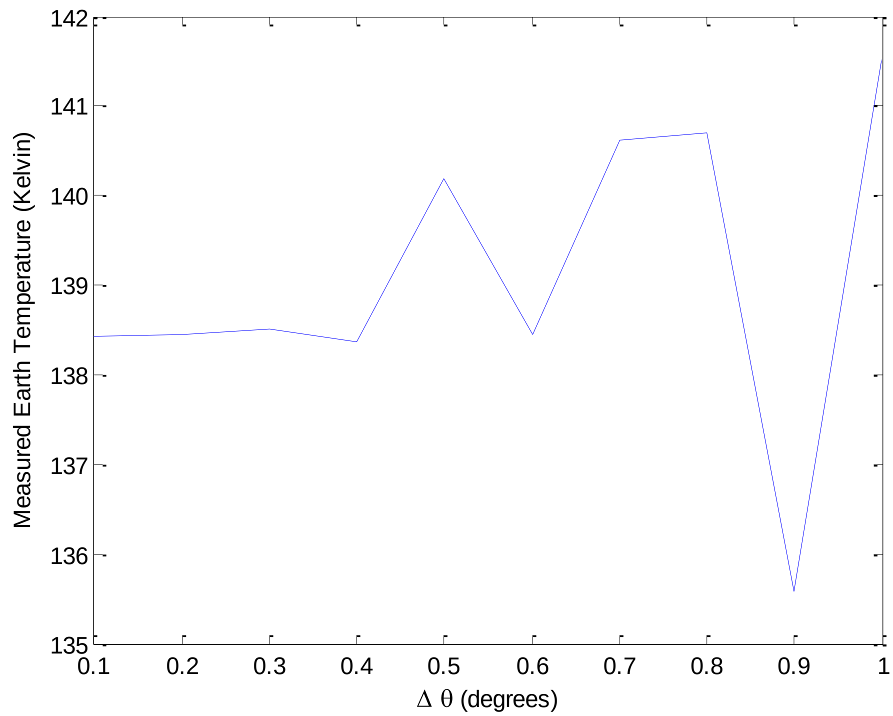

Figure 3: Antenna Pattern Earth Land Temperature ($\Delta\phi = 0.1$)

Figure 3 indicates that temperature result remains accurate till $\Delta\theta = 0.4$. This is expected because fewer points are used in the integral with coarser resolution when using Linear Interpolation. The same accuracy is obtained when using cubic Spline Interpolation for the antenna pattern values. They take longer to compute and are not worth the effort. It's recommended that when $\Delta\phi$ is stationed at 0.1, $\Delta\theta$ should be no greater than 0.4 because fluctuations on the temperature calculations begin to appear in the graph when $\Delta\theta$ is greater than 0.4.

The fluctuations in the measured earth temperature are due to singularities inherent in the data (since it is observed from various earth's topography such as rough places, rocky areas, or even across coastline from land to sea). They can be avoided by taking a coarse enough grid of numerical integration that hopefully skips the discontinuities in the data. This can be done as long as the errors in results are still numerically acceptable. This is the case in the graph in Figure 3. If fluctuations persist for fine grids due to singularity at the boundary condition, a numerical transformation in the data can be first be done before carrying out the integral, to avoid its divergence.

Study 2: In this section computational timing for linear and spline interpolation methods to calculate the temperature are studied**.** Table 1 shows that linear interpolation takes the shortest time to run, which is expected. It's recommended to leave the earth's antenna integral with linear interpolation over the earth brightness temperature distribution data.

Table 1**:** Computation Times and Temperatures with Spline and Linear Interpolation

Interpolation	Mean time over Coastline at $\Delta\theta$ and $\Delta\phi = 0.1^\circ$	Mean time over Coastline at $\Delta\theta$ and $\Delta\phi = 0.4^\circ$	Mean temp. over Coastline at $\Delta\theta$ and $\Delta\phi = 0.1^\circ$	Mean temp. over Coastline at $\Delta\theta$ and $\Delta\phi = 0.4^\circ$
Linear	5.9 sec.	5.6sec.	210.3 K	210.8 K
Spline	226.6 sec.	625.4 sec.	209.2 K	209.7 K

Study 3: Directivity and beam solid angle are other parameters that affect antenna performance. An antenna radiation pattern is a graphical representation of the antenna radiation properties as a function of position in spherical coordinates. An antenna's radiation intensity pattern function can be written as a two dimensional function $F(\theta,\phi)$ in spherical coordinates. Different antennas can be introduced specifying their radiation intensity pattern function $F(\theta,\phi)$. Directivity is defined as the radiation intensity of an antenna created in a particular direction per average radiated power density. The directivity is given by

$$D(\theta,\phi) = 4\pi \frac{F(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta d\theta d\phi}$$

An antenna that radiates equally in all directions (isotropic radiator) would have directivity value of 1 (or 0 dB). The beam solid angle represented as Ω_A is defined as the solid angle which all power would flow through if the antenna intensity is constant and at maximum value.

$$\Omega_A = \frac{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta d\theta d\phi}{\left[F(\theta,\phi)\right]_{\max}}$$

And the maximum directivity D_0 is given by

$$D_0 = [D(\theta,\phi)]_{\max} = \frac{4\pi}{\Omega_A}$$

As part of the case study various antennas can be used in which the parameters are computed analytically and numerically. Analytically, $\left[F(\theta,\phi)\right]_{\max}$ occurs when $dF(\theta,\phi)/d\theta = 0$.

III. Module Implementation

This paper can be used as a three-week module to apply application of mathematics in antenna theory. The module can be presented to students with a background in Calculus III, Numerical Methods, Complex Variables, and Physics I. It can be presented on a weekly basis as follows:

Week 1: Special emphasis can be paid to define antenna and how an antenna transfers power to/from a receiver/transmitter (energy is contained in voltages and currents) into electromagnetic radiation (where the energy is contained in the E- and H-fields) travelling away from the antenna. Types of antennas can be introduced such as wire antennas, aperture antennas, reflector antennas, lens antennas, etc. Finally, the question “Why Antennas Radiate?” is to be addressed.

Week 2: Topics in surface spherical coordinates integration, numerical interpolation, and numerical integration such as Simpsons and midpoint rules need to be reviewed. Terminology is introduced such as frequency and frequency bands, radiation patterns, fields, gain, directivity, antenna temperature, and antenna efficiency.

Week 3: Antenna measurement formulas finally are introduced and their calculations are done. The solution to some formulas can be obtained exactly or numerically. Numerical integration schemes will be based on both one-dimensional and two-dimensional independent variable data. Graphical representation of measured quantities, such as radiation pattern and gain, are presented in software such as Mathematica or Matlab. Feedback from the students can be obtained via exercises that test ability to carry out calculations to get directivity, beam solid angle approximations, and radiation efficiency.

In this section, an example is used to illustrate our approach.

Example: Determine the directivity $D(\theta,\phi)$, the beam solid angle Ω_A and the maximum directivity D_0 of an antenna defined by radiation intensity pattern $F(\theta,\phi)=\sin^2\theta\cos^2\theta$. Graphing the pattern using the Mathematica command

$$\texttt{SphericalPlot3D}[Cos[\theta]^2 * Sin[\theta]^2, \{\theta, 0, \pi\}, \{\emptyset, 0, 2\pi\}]$$

gives the graphical representation:

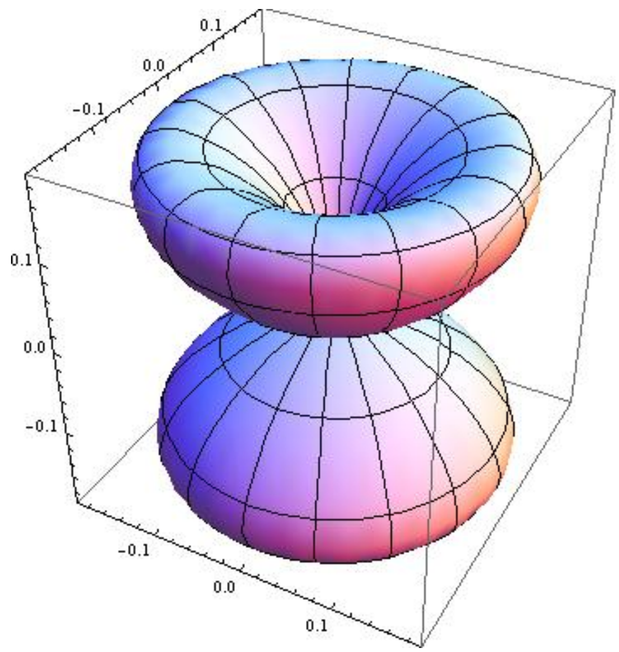

Figure 4: Intensity Pattern $F(\theta,\phi)=\sin^2\theta\cos^2\theta$

Exact (analytic) calculations give the results:

$D(\theta,\phi)=\dfrac{15}{2}\sin^2\theta\cos^2\theta$

$\Omega_A = 6.7\,\text{rad}^2$

$D_0 = 2.73\,\text{dB}$

More examples can be used such as $F(\theta,\phi)=\sin\theta\cos\theta, F(\theta,\phi)=\sin\theta\cos\theta\sin\phi$. Then, data can be generated based on these patterns to test interpolation and integration numerical methods and compare the results with those obtained analytically.

For antennas with narrow major lobes and negligible radiation in their minor lobes, $\Omega_A=\theta_1\theta_2$, where θ_1 and θ_2 are the half-power beam widths (in radians) which are perpendicular to each other. The evaluation of directivity can be carried out numerically if radiation intensity pattern function $F(\theta,\phi)$ is separable in form of $F(\theta,\phi)=f(\theta)g(\phi)$.

IV. Conclusion

This paper presents a novel method to introduce antenna theory and antenna parameter calculations to Engineering, Technology, Science, and Mathematics students. The topics in spherical coordinate systems, double integrals, numerical interpolation, and numerical integration have been applied to yield the results. In particular, antenna patterns, antenna temperature, directivity, and beam solid angle are introduced for various antennas. Antenna pattern data that is collected from various earth topologies are used to measure the resulting earth heat pattern.

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