

## **Challenges in Teaching Ideal Flows to ME Students Concurrently with Senior Design**

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## **Abstract**

Students in mechanical engineering need to learn important analytical and mathematical concepts of computational fluid dynamics (CFD) if they wish to choose a career in fluid mechanics. However, these tools are challenging to learn and are not always interesting to most students attending group activities in a multidisciplinary senior design class. This paper presents implementation details of motivational strategies presented in three earlier papers together with a suggested approach to deliver them. The student performance data is from a well-acclaimed, ABET accredited, career oriented mechanical engineering curriculum. The paper clearly demonstrates both horizontal and vertical integration of engineering mechanics concepts in the curriculum beginning with freshman level and ending with the upper level elective classes. Mathematics is delivered in a meaningful way enhancing reinforcement and understanding. In addition, assessment adjustments are made to encourage increased mathematical rigor and practice of logical arguments. Overall the approach improves retention and recall of mathematical and physical concepts appropriate for analysis. Discussion of specific examples and performance data are presented from course topics of ideal flows in a class of advanced fluid mechanics. The paper lists relevant focal concepts and how conceptual links are further enhanced using follow-on applications.

## **Introduction**

The author conceptualized and managed Engineering Sciences Core Curriculum (ESCC) for several years as a part of ABET assessment for continuous learning improvement in Mechanical Engineering (ME) at Rochester Institute of Technology (RIT). The faculty participating in ESCC collectively designed a seamless learning environment which may be emulated by others. ESCC assessment guidelines have been followed for the past 12 years. Details of the assessment may be found in references [1 – 3]. An important difference of ESCC from traditional curricula elsewhere is our program is student-centered. All difficulties in concepts have been researched and presented below from a student's learning point of view.

Modern computational focus requires mastery of analytical thoughts to properly understand and improve computational models. There are some mathematical bottlenecks in achieving this feat which are discussed separately in another paper [4]. The approach requires reinforcing mathematical understanding in parallel with engineering applications. Many examples and attractive demonstrations are necessary before and during active learning of mathematics. The recommended mathematical concepts are reinforceable similar to engineering concepts discussed herein. Only when the breadth of engineering concepts is connected well with the depth of the mathematical understanding, a true T-shaped curriculum [4] may be claimed.

We begin with the connectivity of ideal flow topics which were presented for four weeks as a part of an upper undergraduate level elective course (Fluids II). Dynamics, which reinforces and extends statics thoughts and introduces fluid mechanics (which later leads to ideal flows), serves as the focal data source for ESCC. If mastery of concepts is not established in Dynamics,

thoughts become sketchy and disappear by the time students reach ideal flows. For average students to retain these thoughts, gaged reinforcement and incentives are necessary. These are discussed below beginning with a suggested focus on force-couple equivalency. We prescribe clear and coherent connectivity of dynamics and fluid mechanics thoughts first, followed by some performance data to establish successful connectivity. Finally, we present some current concerns and recommendations to conclude this paper.

### **Pedagogical Notes**

In the first lecture of Dynamics an instructor asks “Can anyone summarize in one sentence what you learnt in Statics?” The discussion would open immediately a quick recall of concepts and take students back to physics review of Newton’s laws of motion. Students are led to recall many concepts in system of units, equilibrium, particles, rigid bodies, deformable media, Hooke’s law (if seen before), algebra, calculus and differentiation in a single discussion. The clarity of this discussion is very important so that students would start taking notes for the first time. The instructor’s presentation skill is also very important. ESCC recommends a discussion based upon common understanding so that all instructors in core ME classes (Statics, Mechanics of materials, Dynamics, Thermodynamics Fluid Mechanics and Heat Transfer) are able to deliver the same material uniformly. If students miss an 8 am class they can attend the 3:30 pm section of the same class and learn almost the same thought process, complete with examples. A suggested way to present fluid mechanics connected with dynamics and mathematics is presented below.

### **The equilibrium connection**

In the context of a statics course, equilibrium represents absence of translational and rotational motion for rigid bodies. In dynamics, this static equilibrium is violated since rigid bodies will undergo both translational and rotational motions in general. Furthermore, how equilibrium is perceived is simplified in the superposition of two motions. In Statics students first learn the concept of a Free Body Diagram (FBD), which sketches only external forces and/or couples on the system of interest. When FBD is constructed in fluid flow systems, not only is the motion more generalized to include shear and volume deformations of the system, it introduces two distinct viewpoints (e.g., Lagrangian and Eulerian) to analyze those motions. This paper will focus only on rigid bodies, except it will indicate generalization of ideas into calculus using the continuum and discrete concepts.

Alternate methods such as the D’Alembert’s technique [5], wherein all equilibrium questions are changed to the sum of forces equal to zero, is a popular idea practiced by many professors to deliver upper level ME courses. Such equilibrium is called a dynamic equilibrium in principle of virtual work. But a struggling novice may find the concept difficult to understand why and how to use the *reversal of inertia forces* in equilibrium equations. Instead, we propose that both static and dynamic equilibrium should be taught clearly by **a cause and effect logic** presented in the Newton’s first and second laws of motion. If there is an unbalanced external force or couple resulting from the sum of forces and moments, it will initiate a translational and/or rotational acceleration on a rigid body about a certain point.

The action-reaction principle (Newton's 3<sup>rd</sup> law) is not necessary at early stages of delivery in Statics. It will be introduced when the course reaches trusses, followed by frames and machines. Our experience shows that D'Alembert's principle is the single most stumbling block in the correct learning of FBD's. *Students who are unable to successfully isolate the selected system from its surroundings suffer the most.* If a simple truss member is in equilibrium, are there two or, one external force on it? Overcoming doubts such as this, and understanding support reactions must be top priority in learning Statics. Proper system isolation must be emphasized when students sketch complex FBD's of interconnected systems such as trusses, or, frames and machines. A novice must learn to verify cancellation of all internal forces to create an overall (or, external) FBD from all component FBD's. This is similar to internal flux cancellations in CFD which would later be learnt in Fundamentals of Computational Fluid Mechanics.

In Statics, average students have enough trouble learning the difference between external and internal forces, and how to choose and analyze a system using algebraic equations. Body and surface forces are taught for the first time. More discussion is necessary to understand why the body force caused by gravity is an external force and why inertia forces are not external forces. If possible, an instructor should design suitable experiments (in our program this task is managed by three experiential learning courses) to present the distinction between surface and body forces. Although this helps students develop a feel about surface forces, body forces are not clearly understood as external forces by majority of students without their first taste of conceptual idealizations. More abstract ideas will be offered later to students during coverage of center of gravity (CG), center of mass, centroid and moments of inertia in Statics.

In recent years, widely followed dynamics textbooks [6, 7] have introduced the concept of a Kinetic Diagram (KD) which distinguishes the external from inertia forces. It reminds a student how to identify external inertia forces and couples and place them at the center of gravity in the same way a body force is presented on the FBD. Instructors may also reinforce force-couple equivalency concepts to calculate forces and moments on submerged objects and buoyancy in fluid statics. Like CG, the net buoyancy force is represented at the metacenter (the center of buoyancy). A preview of these later topics in Statics may excite a keen novice to seek more confirmations elsewhere on the Internet [8, 9].

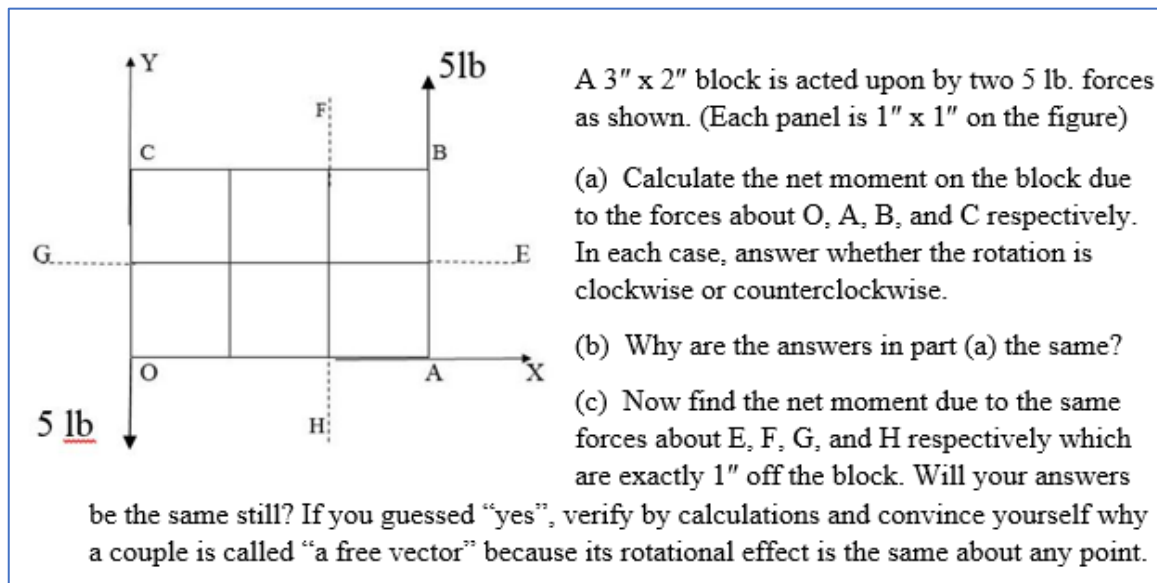
Another connectivity example could be made by demonstrating experiments of hydrodynamic and aerodynamic stability. An instructor may ask students if they knew why the engine compartment of a ship is located at the bottom of the ship, then demonstrate how the location of the CG below the metacenter creates a corrective couple when the model ship's equilibrium is perturbed. Universities where aerodynamics is taught, an instructor may also demonstrate that negative slope of the pitching moment vs angle of attack curve creates the static stability. These are some suitable examples of reinforcing the concept of force-couple equivalency.

### **Rotational Equilibrium**

Since particles do not have measurable size, they can possess only a translational motion. A curvilinear motion of a particle is quite different from the curvilinear motion of a rigid body when it rotates during its motion. Chasle's theorem [10] (which is typically introduced in

Dynamics) may be previewed during Statics by decomposing a general motion of a rigid body into a translation, plus a fixed axis rotation about the CG. Furthermore, contrasting examples of placing objects on turntables vs. roller-coasters and Ferris wheel rides, plus previews of fluid rotation experiments [11] would excite more independent learning of such topics. More hands-on experimentation will induce easier conceptual learning, and (if steps are carefully executed) may excite students to learn analytical formulations on their own.

Since a couple cannot cause a net force but a net force can cause a moment about a point, it is hard to separate the translational and rotational effects due to a force alone without the introduction of couples. Careful experiments must be devised to present various ways couples may work. Some field trips to the science museums may be necessary. Simultaneously students need to master both the principle of transmissibility and the force-couple equivalency. By adding and subtracting the same force at a point not lying along the axis of the original force equivalently represents the same force at a new point of application, plus a new couple. In addition to observing experiments in a laboratory setting, students must solve some illustrative examples (e.g., Figure 1) to realize why couples may be called free vectors. It is advisable to assign homework and quizzes for them to practice moving forces on the same rectangular block ABCO as illustrated below. Finally, similar technique must be presented for three dimensional systems on the same block of solid by moving across the diagonals. Vectors will be used for such couples.

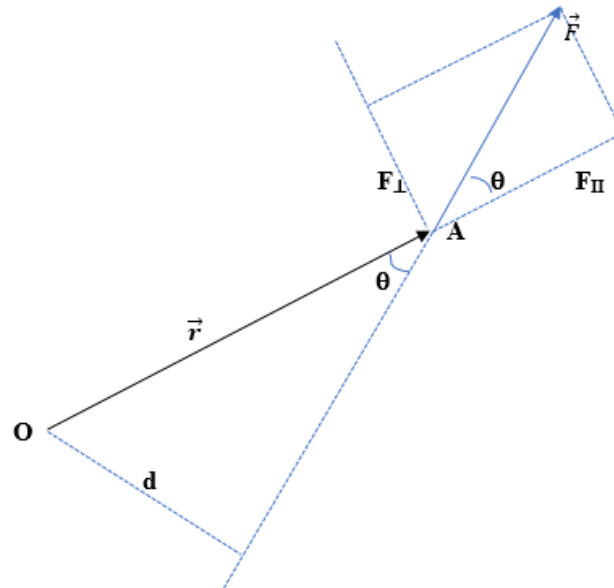


**Figure 1. Why a couple is considered a "free vector"**

### Force-Couple Equivalency

So far, arguments presented above established the need for emphasizing force-couple equivalency. Now we illustrate the concept of moments and give reasons why the association to force-couple equivalency is lost sooner than anticipated during the final examination time in Statics. Rotational effect caused by couples are limited only by their rotational direction and sense. We shall first discuss the moment calculations by vector and scalar methods, and to what extent these topics need to be presented for retention of concepts.

The moment of a force applied at A about the point O is given by  $\vec{M}_O = \vec{r} \times \vec{F}$ , where,  $\vec{r}$  is the position vector from O to A. The non-zero cross product implies that  $\vec{r}$  and  $\vec{F}$  are not parallel to each other. When they are parallel, i.e.,  $\theta$  is  $0^\circ$ ,  $\vec{F}$  may be extended to pass through O causing a zero moment. Now decompose  $\vec{F} = \vec{F}_\perp + \vec{F}_\parallel$ , where,  $\vec{F}_\perp$  and  $\vec{F}_\parallel$  are mutually orthogonal to each other, forming the basis vectors mathematically (see Figure 2).

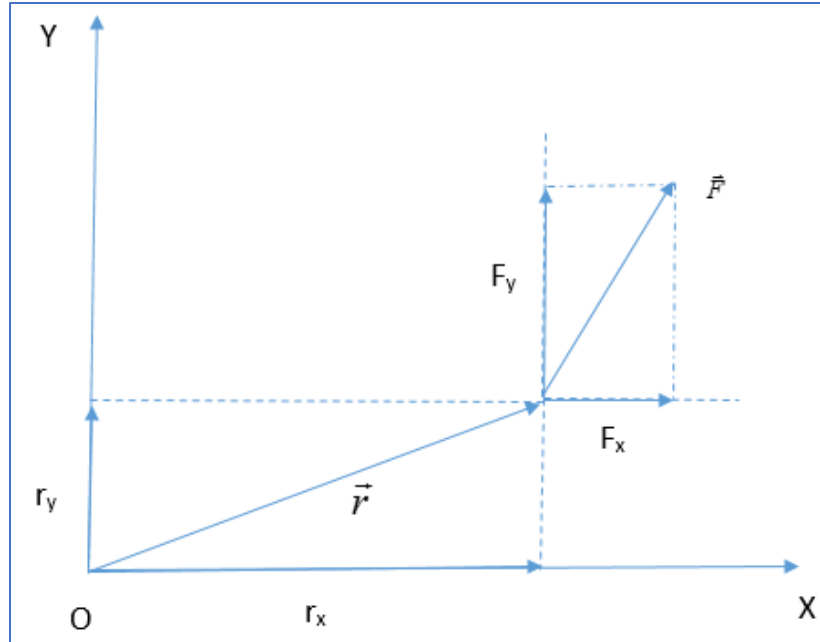


**Figure 2. Two alternate ways to express a moment vector's magnitude**

Since  $d = |\vec{r}|\sin\theta$ , and,  $F_\perp = |\vec{F}|\sin\theta$ ,  $\therefore |\vec{M}_O| = |\vec{F}||\vec{r}|\sin\theta = |\vec{F}|d = F_\perp|\vec{r}|$  represents two alternate ways to express the same moment by the construction process shown above.

Another way to view this result is as follows. Since  $\vec{F}$  has two possible effects on a rigid object – viz. a translation and a rotation, one extreme case is when the force loses its power to cause a rotation about the point O. The other extreme case is when the angle between  $\vec{r}$  and  $\vec{F}$  is  $90^\circ$ . Then the distance between  $\vec{r}$  and  $\vec{F}$  becomes a perpendicular distance making the moment the largest possible magnitude of  $\vec{r} \times \vec{F}$ . Geometrically speaking, the area of the parallelogram formed by placing  $\vec{r}$  and  $\vec{F}$  along two adjacent sides turns into the area of a rectangle when the bearing angle becomes  $90^\circ$ . Mathematically, these two cases form two different scalar forms of calculating moments which was generalized by Varignon in the form of his famous theorem [12]. The magnitude of the perpendicular distance  $d$  between the point O and the force  $\vec{F}$  forms the most popular scalar form of moment  $|\vec{M}_O| [= F \cdot d]$ , that most Statics students memorize to use.

To complete the discussion of the general case of the moment calculation, let us now decompose both the position vector and the force into their respective Cartesian components (see Figure 3).



**Figure 3: Using orthogonal components for moment calculation**

$$\begin{aligned}\vec{r} \times \vec{F} &= (r_x \hat{i} + r_y \hat{j}) \times (F_x \hat{i} + F_y \hat{j}) = r_x F_x \hat{i} \times \hat{i} + r_x F_y \hat{i} \times \hat{j} + r_y F_x \hat{j} \times \hat{i} + r_y F_y \hat{j} \times \hat{j} \\ &= (r_x F_y - r_y F_x) \hat{k}\end{aligned}$$

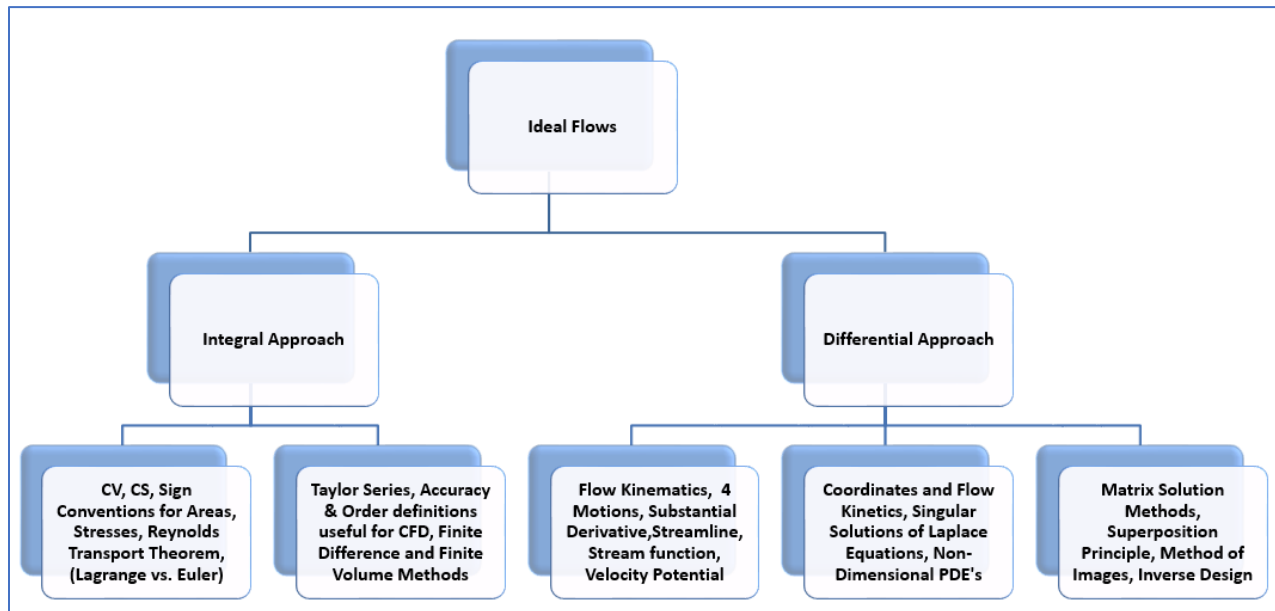
Due to the cross-product rules of unit vectors, the above expression simplifies and confirms the clockwise and the counterclockwise rotations of each of the components about O – a difficulty many students fail to realize while learning only scalar approaches. Therefore, *it is advisable to present up to this mathematical level suitable for all students* (having learned vector calculus). Moreover, it is important to note the direction of rotation offered by the cross products is by the **right-hand rule (RHR)**. *Why clockwise rotation is produced by a vector in the negative  $\hat{k}$  direction is often not clear to many struggling students.* An effective illustration would be to ask a struggling student to torque a sheet of paper placed on a flat table with one finger.

The last equation discussed above also provides a necessary introduction and reinforcement of the famous Varignon's theorem:  $\vec{r} \times \vec{F} = \vec{r} \times (\vec{F}_1 + \vec{F}_2) = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2$ . In other words, if the force is conveniently resolved into two components, the sum of moments due to each component about any point is the same as the moment due to the force itself about the same point. The most useful form of this theorem comes during problem solving when each Cartesian component of the position vector as well as the force vector is computed by selecting preferred axial directions. After the disappearance of about 30 years, the *name* Varignon's theorem has reappeared in most text books because of the emphasis on this area in recent years [12, 13]. *ESCC recommends use of this name* instead of Law of Moments. Many professors use the term moment and couple synonymously, which should be avoided. Instead remind students that moment is a measure but couples are free vectors which can cause same rotational moment about any arbitrary point.

For FBD's in Statics with many forces and couples, force-couple equivalency is used to move each of the applied forces plus the external couples to a single point at O (say) where a support introduces reactions. All the reactions necessary to create an equilibrium of the rigid body must be equal and opposite of the net unbalanced force and couple at O. Thus, the equilibrium equations may be written as  $\sum \vec{F} = 0$ , and,  $\sum \vec{M}_O = 0$ .

Once the students reach Dynamics, they begin processing the dynamic equilibrium as a cause and effect process mentioned before. Complexities in dynamical motion begin with reinforcing Chasle's theorem. Conceptually contrast n-t and cylindrical coordinates. Derive *the rate of change of unit vectors* for n-t coordinates but conduct student group discussions in cylindrical coordinates. *This derivation should not be skipped as it reinforces concepts of secants and tangents to a curve also*, and extends readily into derivation of differential equations in ideal flows. Adopting the connected approach students should be able to learn the integrated concepts introduced in momentum and energy methods for particles and rigid bodies. Once the students understand the trigonometric, vector and the scalar approaches well, the remaining tasks concentrate on analyzing relative motion and employing the use of KD's.

What was learnt in the form of forces and moments may equally be extended to motional concepts. Students should be exposed to concepts of continuum mechanics from here on. Summary of fluid flow tasks in ideal flows are shown in the figure below.



**Figure 4. Organization of Ideal Flow Mathematical Topics [4]**

The above flowchart (Figure 4) shows connectivity of relevant mathematical concepts which must be strengthened as students proceed from Statics to Ideal Flows. By the time students reach Fluids II all mechanics concepts necessary to understand formulations and solutions have already been presented and reinforced. The breadth of applications in our program begin rapidly from this point onward. Fluid Mechanics concepts similar to Statics require recall of many of the



above ideas. So frequent reinforcements are necessary. Macro fluid mechanics begins with the continuum hypothesis which allows better connectivity with calculus as a tool. After initial coverage of fluid properties Reynolds Transport Theorem is introduced as the sole synthesizer in developing a unified theory of fluid motion and forces in both control volume and differential approaches. Later it implants conceptual foundations for MECE725- Fundamentals of Computational Fluid Dynamics. The first fluids course ends with Engineering Bernoulli equation and applications. Some important non-fluid concepts such as Buckingham pi theorem and similarity principles are introduced. It is important to take students into nondimensionalization and its benefits first with algebraic equations and then with differential equations in the elective course. This completes connectivity of lower level core classes and also heat transfer with advanced fluid mechanics. Scaling of fluid flows which is never well-understood by students finds support from experiments and digital media. The classes then advance to electives such as ideal flows, convective phenomena and CFD. At these levels continuum mechanics ideas should be introduced but with a standpoint of vector calculus, matrices and boundary value problems. By the end of the 5<sup>th</sup> column applications in figure 4 students see examples from tornadoes, wind tunnels, superposition in design methodology, wall interference correction schemes, in addition to some wave equation demonstration and interesting shock tube experiments. The learning experience is made lucrative by a unique re-grading motivator (see Appendix C for some student comments) which attracts student engagement/learning. Motivated students implement codes and likewise may design them [14] knowing their fundamentals, or take charge of contributing to better education for future students through projects with paper [15].

## Results

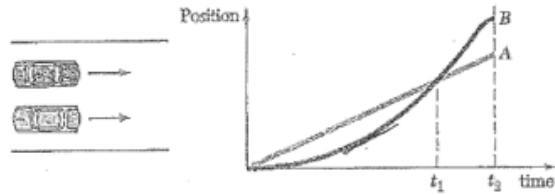
Our unified performance-based direct assessment process has several benefits at a modest increase of workload for faculty. First it increases the assessment pool to provide more statistical reliability. The target assessments are easily trackable due to a well-maintained archive of examinations for comparisons of different courses laterally, and/or testing retention of the same concept longitudinally over time. The examples below show some reinforced results from ESCC archives. The examples demonstrate tracking of vectors and coordinates with relevance to mechanics courses which will lead to ideal flows later. Another longitudinal tracking method for vectors may be noted on impact problems in Dynamics. The reasons impact questions suffer in performance are well understood now [16, 17]. These involve misunderstanding FBD's, which conservation law applies for inelastic impacts, whether to conserve momentum singly or the two particles together as a system, and oblique impact solutions. This area is not discussed here due to its lack of connectivity with fluid mechanics.

The first two questions on figure 5 are borrowed from 2013Spring and 2015Spring Dynamics final examinations showing connectivity with velocity components and physical interpretation of slopes, and the third question is taken from a 2017Spring final examination of Fluid Mechanics reinforcing concepts of streamlines. The first question was correctly answered by 51% students, while the second and third questions were correctly answered by 66%, and 80% of the classes respectively.

- 1.1 The path of a particle is defined by  $y = 0.5x^2$ .  
 If the component of its velocity along the x-axis at  $x = 2$  m is  $v_x = 1$  m/s, its velocity component along the y-axis at this position is:
- a) 0.25 m/s
  - b) 0.5 m/s
  - c) 1 m/s
  - d) 2 m/s**

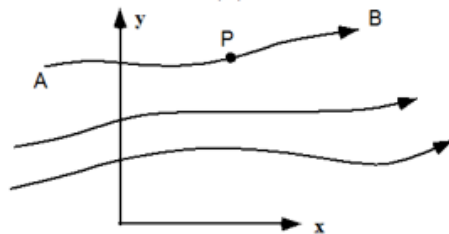
1. Two cars which are next to each other race down a straight road. The positions of each car from the current location are shown over time. Select the correct statement below

- a. At time  $t_2$  both cars have traveled the same distance
- b. At time  $t_1$  both cars have the same speed
- c. Both cars have the same speed some time  $t < t_1$**
- d. Both cars have the same acceleration some time  $t < t_1$
- e. None of the above



1. The point P is on the streamline passing through points A and B. The velocity at the point P is determined to be:

$$\vec{V}(P) = 3\hat{i} + 4\hat{j} \text{ m/s}$$



The slope of the streamline at point P is:

- A) Zero
- B) Infinity
- C) 1.33**
- D) 0.75

Figure 5. Dynamics and Fluid Mechanics Connectivity

Also, we tracked the same area of streamlines as we move from Fluid Mechanics to the elective Fluids II as shown on the figure 6 below. This time the question is related to the topic of streamline evaluations as shown in the two quiz questions, the first as a part of a set of MC questions, and then as a part of a stand-alone quiz of 10-minute duration on two different tests. On each the class performance was better than 80% correct for the streamline evaluation, plus on the part (c) related to the ideal flows in the stand-alone quiz.

9. A velocity field is given by  $\vec{V} = y\hat{i} - x\hat{j}$  (m/s), where, x and y are in meters. A streamline plotted at (2, 1) in this flow is given correctly by (You must show your work to get full credit)

- (a)  $\ln x = -\ln y + \ln 5$ , **(b)  $x^2 + y^2 = 5$** , (c)  $xy = 5$ , (d)  $y = 5x$ , (e) None of the above. (2 points)

**Quiz No. 3**

**(Maximum Time: 10 min)**

A water flow in the horizontal plane ( $x, y$ ) is described by the velocity vector:  $\mathbf{V} = y \mathbf{i} - x \mathbf{j}$  (m/s), where  $x$  and  $y$  are in meters.

- (a) Find the equation of the streamline passing through the points (1, 1).
- (b) Show the flow direction on the streamline in part (a).
- (c) Is this flow irrotational? Verify your answer by a suitable check.

**Figure 6. Streamline concepts carried over from fluid mechanics to ideal flows**

Note that of the above two questions, the MC question asks for the *details* to get the full credit.

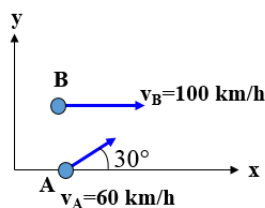
The work load is not much beginning with the equation (available on the formula sheet)  $\frac{dx}{u} = \frac{dy}{v}$ .

The important task is to check if students carry out the integration and the constant evaluation correctly by fitting the point (2, 1) on the streamline. If they do not show the work, they get only one point usually allocated for MC questions on a class quiz. They may also incorrectly perform the integration, simplify the ratio incorrectly, or, evaluate the constant incorrectly, which would be revealed from the given choices. If the class performance was not up to expectations, students would still get the opportunity to improve their score on the question by resubmitting the quiz [18]. If the class size is small enough, alternate and accurate assessment may be performed using a stand-alone quiz as the Quiz No. 3 shows. As always, a prompt return of the original quiz is necessary so that students can relearn the missed topic over the approaching weekend (see Appendix B for a note sent to students).






The relevant flow viscosity questions are preceded by understanding of dry friction first in dynamics and then (with the help of strength discussions) contrasting each with fluid friction behavior. Much of the fundamental review of viscous boundary layer comes after the ideal flows coverage. Let us therefore focus solely on mass and momentum conservation laws as shown on figure 7. Here is an example of tracking three questions on relative motions.

Determine the relative velocity of particle B with respect to particle A.

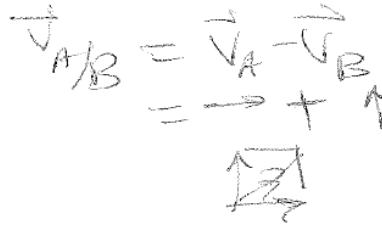
- A)  $(48\mathbf{i} + 30\mathbf{j})$  km/h
- B)  $(-48\mathbf{i} + 30\mathbf{j})$  km/h
- C)  $(48\mathbf{i} - 30\mathbf{j})$  km/h
- D)  $(-48\mathbf{i} - 30\mathbf{j})$  km/h

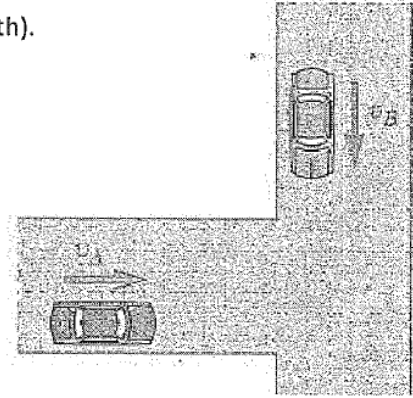


- 4) Car A (going east) is being tracked by a passenger in car B (going south).  
Which direction will the velocity of car A appear to him?

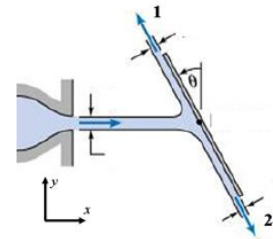
- (A)   
 (B)   
 (C)   
 (D)   
 (E) 

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

$$= \vec{v}_A + (-\vec{v}_B)$$




8. In the following figure, if a control volume is placed around the plate, which of the following is true about the momentum flux out of the control volume at points 1 and 2?



- A) Negative x-momentum flux at 1, negative x-momentum flux at 2  
 B) Negative x-momentum flux at 1, positive x-momentum flux at 2  
 C) Positive x-momentum flux at 1, negative x-momentum flux at 2  
 D) Positive x-momentum flux at 1, positive x-momentum flux at 2

**Figure 7. Use of vectors tested by Component method and parallelogram law in Dynamics (2015 & 2016 Fall final exams), and using dot products in Fluid Mechanics (2016 Spring Final)**

For the first two questions taken from Dynamics, the focus is component addition/subtraction to obtain relative velocity - but the second one uses geometrical construction of a negative vector and parallelogram/triangle of vectors. The third example on the figure is from fluid mechanics involving dot product and its interpretation in finding fluxes of fluid mass and momentum. Once again, the performance was better than 65% correct on each examination demonstrating that such instructional reinforcement is very effective.

One of the areas of prime importance extending from dynamics to fluids is the understanding of rigid body rotation and contrast it with fluid rotation. Ideal flow behavior is simplified with potential flow fields but singular flows are abundant to check on the rotation feature. The instructor screens the movie Vorticity [11] to clarify many conceptual ideas such as vorticity, circulation, starting and bound vortices, downwash, induced drag, etc., plus Crocco's, Kelvin's and Helmholtz's theorems. The performance in ideal flows is assessed using some MC questions on quizzes 1- 4, a 15-minute long class quiz 5, the first midterm examination and the final examination (see four sample questions from quiz 5 in Appendix A) where students demonstrate the concepts and math skills in ideal flows after the 5<sup>th</sup> week of the class. The emphasis is on conceptual recall and mathematical skill reinforcement, and not just the engineering modeling.

Current trend in flow related industrial work would most certainly place our students on software and flow modeling for which understanding numerical accuracy in modeling, and mathematical forms of equations would be necessary. Through enticement of various types (see Appendix B for an unusually attractive incentive, and positive student comments in Appendix C), instructors must excite students to take ownership of the questions and work independently on model development. One way to do this is to introduce applications right from the dynamics or, fluids course which students regularly use or, have seen before, and *prepare a mathematical model around it* [18]. Note that the first two questions in Appendix A are attempting just that. Since students would not recall the speed of sound and Mach number (M) check for incompressibility, the second question provides the value of the limiting airspeed of 102 m/s. It does give the instructor opportunity to address the  $M < 0.3$  limit while discussing the solutions with students. The location of stagnation points, the concepts of stagnation streamlines and their relation to control the design of the body shape, the sketching of streamlines in  $r-\theta$  coordinates, etc. are all focal points of the quiz 5 samples. Figure 8 shows a summary of the performance on quiz 5 from 2014Spring semester through 2018Fall offering of the Fluids II. On each term the maximum, minimum and the average score in the class are displayed. The re-grading offer (if students relearn the material) is a very strong incentive which the author discovered over the years. Initially a 10-point quiz is graded with 5 points for attending the quiz, plus 5 points for the earned grade. But students must demonstrate they correctly know the missed answers to earn a better score. Notice that on the 2017Spring semester minimum score is zero which means that at least one student in the class never appeared on the quiz and never made up the score by retaking later. Also, for the most recent 3 semesters there has not been a single perfect score in the class. Only time would tell if this is a developing trend or not.



**Figure 8. Class performance on ideal flow quiz 5 for the last four years**

## Challenges

Remembering definitions and recalling procedures are difficult for ME students. Often concepts get confused because of mixed symbols. In ESCC guidelines these are resolved by several steps: 1) If possible, all instructors must use common symbols introduced uniformly in all sections of the course. 2) Notify students/faculty of confusing symbols from earlier courses while reviewing conceptual steps from previous courses. 3) Communicate with instructors of later elective courses so that the same symbols may be carried over in follow-up courses. 4) Common symbols that are conflicting in textbooks should be pointed out clearly by the course instructors of any ESCC course. For example, the symbol  $h$  is used as height of a triangle in geometry, or, in Statics; altitude or, distance from ground up in Dynamics; the depth of a submerged object from the free surface in Fluid Mechanics; specific enthalpy of a fluid in Thermodynamics; and finally, coefficient of convection in Heat Transfer. Typically, units for the last two uses mentioned here are forgotten during tests. To compound the complexity, average students do not remember to recall definitions so that correct units can be quickly verified. Nowadays with accelerated learning efforts, we urge faculty members of different core courses work together with upper level course faculty to implement and reinforce conceptual recalls in all engineering classes. As a result of our unifying approaches for several years we seem to have achieved moderate success in examining conceptual recalls demonstrated in the ideal flows topics. Figure 9 displays performance on first midterm examination for AY 2014 – 2018 summarizing effects of conceptual connectivity.



**Figure 9. Class performance on ideal flow Test 1 for the last four years**

In future we will revive a focus on engineering formulations. Many engineering professors introduce engineering formulations without any conceptual relevance to mathematics. Instead formulations must be understood *a priori* by solvability and uniqueness of solution to

mathematical equations. Disallow all random assumptions. Instead ask students to make assumptions to better simplify a formulation and only those which assist solvability of equations. Explain earlier used examples from Dynamics and Fluid Mechanics to reinforce these ideas.

A flipped course structure is a noticeable flaw in many core courses. Not all engineering topics are suitable for a flipped course structure. Topics which require breadth only are perfect for senior design groups to search on the Internet. But students often pick incorrect formulae for analytical models because they lack the depth of understanding. Once shortcuts develop in a mathematical thinking process, it becomes very hard for those students to unlearn them. Negative feedback was experienced in the early stages of data collection when unsupervised student groups proposed mathematical models. This may be due to the number crunching solution style adopted by many ME faculty. ESCC collects and preserves the actual final examinations of both MC and analysis questions to resolve these issues. We have just begun a new symbolic and mathematical emphasis [18] in ESCC. We hope that this trend continues in future to broaden the scope of connectivity steps proposed here.

Similar work needs to be performed to implement specific strategies in other elective courses. Once students develop the pattern of attaching mathematics with its conceptual understanding, they become self-guided operators. But faculty guidance and mentor scaffolding are essential.

### **Concluding remarks**

A subject like ideal flows has far reaching concepts from the standpoint of physics and equally challenging mathematics. A simplified treatment is presented here (suitable for our mechanical engineers) by focusing only on the classical solution methods. If complex analysis was available as a tool, it could expand the understanding even more for aerodynamicists.

Strengthening and linking dynamics and mathematics are important for ME students because all formulations in fluid mechanics and ideal flows depend on extension of dynamics concepts.

More experiential learning must be developed from ESCC leads. Other upper level elective courses should also be brought into this effort and connected as shown in this paper to provide sufficient breadth in the whole curriculum. If faculty groups prepare tutorials with completely worked out examples and constantly monitor student group performance with complete rigor in analysis, engineering learning of mathematically challenging courses may bloom even more. This is our goal to meet in future.

### **Acknowledgments**

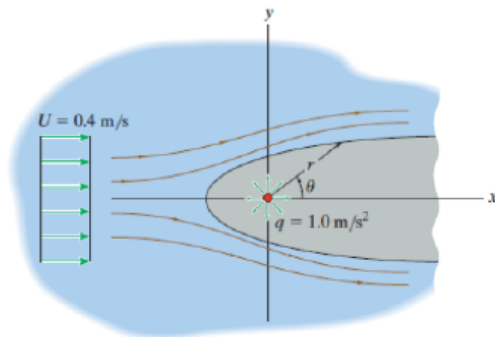
This study would not have been possible without active participation of all members of various ESCC instructional teams for the past twelve years. All participating faculty members and trained teaching assistants helped in designing, discussing, and evaluating examination archives from which examples are presented here. The author, who was also the ESCC coordinator in the past, gratefully acknowledges all their contributions. The author would also like to thank Drs. E. C. Hensel and R. Robinson for helpful discussions and support.

## **Appendix A**

This appendix contains four examples of quiz 5 questions. Each question involves a fair amount of mathematical use, thoughtfulness for proper modeling and accuracy in execution, all of which are required for successful development of CFD understanding. Moreover, these questions present conceptual links in understanding applications mentioned in this paper. Streamlines which are typically used as flow visualizers may act as concept builders for wall models in ideal flows. The boundary conditions for slip flows must be reviewed. Streamlines and equipotential lines lead to development of curvilinear grid systems which are commonly used later in CFD software. At the undergraduate level, creating some models of wind tunnel applications on the computer is very interesting for students. Our current laboratory exposure also offers application-oriented demonstrations for wave equations, shock-tubes, etc. Overall clarity in understanding mathematical depth acts as a big motivator for students at upper undergraduate levels (see student comments in Appendix C).



The profile of a torpedo when viewed from above the sea is approximated by a half-body, for which  $U = 0.4 \text{ m/s}$  and  $q = 1.0 \text{ m}^2/\text{s}$ . As the water flows over it, (a) determine the magnitudes of velocity and pressure at the point  $r = 0.8 \text{ m}$  and  $\theta = 90^\circ$ . The pressure within the uniform flow may be taken as  $300 \text{ kPa}$  and the density of seawater is  $1000 \text{ kg/m}^3$ . (b) Is this point on the surface of the torpedo? (c) Plot the nose section of the torpedo from  $\theta = 45^\circ - 180^\circ$ . Justify by calculations and state any assumptions.



**For 2-dimensional Ideal Flows:**

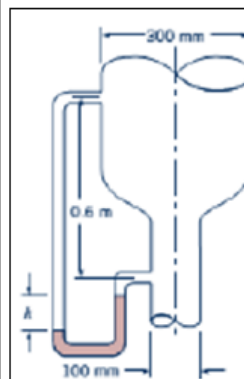
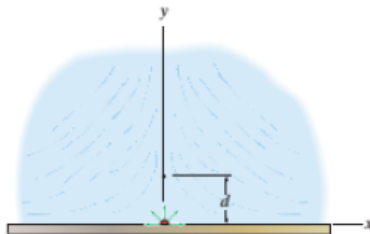
$$\psi_{U\infty} = Uy = U\tilde{r} \sin \theta, \quad \psi_{Source} = \frac{q\theta}{2\pi}, \quad \psi_{Vort} = -\frac{\Gamma}{2\pi} \ln r, \quad \psi_{Doublet} = -\frac{K \sin \theta}{r}; \quad V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad V_\theta = -\frac{\partial \psi}{\partial r}$$

$$\text{Bernoulli Eq: } \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

A crude model for a tornado flow field (as seen from a space satellite) is formed by the superposition of a sink of strength  $2800 \text{ m}^2/\text{s}$  and a vortex of strength  $5600 \text{ m}^2/\text{s}$ . In standard atmospheric conditions ( $p_{atm} = 101.3 \text{ kPa}$ ,  $\rho = 1.225 \text{ kg/m}^3$ ), an air flow may be treated as incompressible if the airspeed is lower than  $102 \text{ m/s}$ . By placing the two singularities at the origin of coordinates, first determine the stream function, velocity components  $V_r$  and  $V_\theta$  for the tornado at any field point  $(r, \theta)$ . Then determine at which distance,  $r$  from the origin the flow field may be treated as incompressible. Also, what is the gage pressure at that radial location?

$$[\text{Note: } \psi_{Source} = \frac{q\theta}{2\pi}, \quad \psi_{Vortex} = -\frac{\Gamma}{2\pi} \ln r, \quad V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}, \quad V_\theta = -\frac{\partial \psi}{\partial r}, \quad \frac{p}{\rho} + \frac{V^2}{2} = \text{Const}]$$

**7-66.** A source  $q$  is emitted from the wall while a flow occurs toward the wall. If the stream function is described as  $\psi = (4xy + 8\theta) \text{ m}^2/\text{s}$ , where  $x$  and  $y$  are in meters, determine the distance  $d$  from the wall where the stagnation point occurs along the  $y$  axis. Plot the streamline that passes through this point.



Oil ( $\rho = 900 \text{ kg/m}^3$ ) is flowing downward between station 1 and 2 (see fig). If the mercury ( $\rho = 13600 \text{ kg/m}^3$ ) manometer reads  $h = 100 \text{ mm}$  in steady frictionless flow through the pipe reduction, find the flow speeds at stations 1 and 2. Will the volumetric flowrate increase or decrease if the flow was viscous? Clearly sketch CV and show streamline which you use to apply theory correctly.

Figure 10. Preparing Students for CFD through Ideal Flows

## Appendix B

The opportunity to relearn missed concepts for a better grade is a very powerful motivator. The following instructional style was adopted a few days from the beginning of the course after the first concept review quiz [18]. See the memo to the class below.

As I mentioned in class today, we begin the course with a quick recall of topics which must stay in your active memory when you learn Fluids II. The first quiz that you took in class today is a representative sample of the topics that would be necessary for you to recall. Each of these topics is available (if you search the 550CD over the weekend) from myCourses. This is why I have placed a blank quiz in a folder called "Quiz Blanks" on myCourses. You may have missed to answer some of the questions correctly in today's quiz. But if you relearn these topics over the weekend, print out a copy of the quiz and return your corrected quiz with reasons why you answered now differently.

For example, question 1 is related to the definition of a Newtonian fluid. In this semester all flow questions attempted would involve incompressible flows. So you must know what type of flow it involves. Similarly, we'd solve differential equations with assumed velocity profiles. So you must know what that involves in question 4, etc. If you demonstrate to me that you have learned the topics correctly now, I promise to average your newly earned grade with the original grade received and replace your original score earned on my grade book. Note that grade correction is only possible if your new score is more than your original score. So find out the correct reasons for the answers by the time you re-submit.

## Appendix C

Here are three different student comments that were given on the course review

*I could see CLEARLY how much interest you took to make sure we all understand the information and topics. This is very refreshing to me, especially when most engineering professors don't show nearly the passion and interest that you demonstrated every day. You were always willing to meet with us out of class and actively sought to improve our understanding of the material, not just for the class, but for our futures as engineers. I thank you for that. You are very knowledgeable in the topics of the class and made sure all the time of each class was used effectively. You always asked for our honest feedback throughout.*

*I love the fact that we can do the quiz and test corrections. That's great, because it encourages us to go back and understand what we got wrong, and the reason we missed a question. I think that is a much better way to grade and motivate/encourage learning, than not offering corrections. The lectures are very interesting. I finally started to feel pretty comfortable with a lot of the calculus and math concepts behind a lot of engineering in this class because Dr. Ghosh explains it so well (and spends so much time on it). The 550CD notes are a really powerful reference. I loved the movie that he showed in the beginning of the semester. It really helped, have a physical understanding of fluid mechanics and understand the conventions and math principles. I really appreciate how Dr. Ghosh focuses on making sure you understand the physical reasons behind the math and formulas.*

*I think his grading appropriately reflects his teaching/testing style, which I like. Dr. Ghosh wants people to understand his material on an intimate, conceptual level which requires tests and quizzes be more difficult because you cannot simply rely on knowing formulas. He therefore is quite lenient with grading and awarding points to wrong answers that he believes were conceptually on the right track. He asks a lot from his students but does not punish a more difficult style of teaching/learning with harsh grading, if that makes sense...*

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