AC 2007-2248: COMPARISON OF THE STRENGTH TO WEIGHT RATIO OF VARIABLE SECTION BEAMS WITH PRISMATIC BEAMS

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Comparison of the Strength to Weight Ratio of Variable Section Beams with Prismatic Beams

Abstract

An experiment for examination of the stress distribution in a prismatic cantilever beam and a tapered cantilever beam of equal weight is proposed for national adoption. Strain gauges attached at four identical locations of the two beams will provide data regarding the state of stress for comparison purposes. Obtaining meaningful strain levels, and at the same time avoiding damage due to permanent set of the precious gauged specimens is always a challenge. To control this catch 22 situation, a backward analysis has been conducted to prescribe the allowable range of loads. The students must develop the mathematical model for predicting the levels of strain and stress in the two beams and manually calculate the expected levels of strain and stress. Modeling of the two different beams in ANSYS and comparison of the behavior of the beams may be added as an optional integral part of the project. This experiment vividly illustrates the advantages of the tapered beams over the prismatic ones. Students however, must comment on the extra manufacturing cost of the tapered beams in justification of their final design decisions in different applications.

I - Introduction

Laboratory experimentation is a critical final link for a thorough understanding and appreciation of scientific and engineering theories and principles. Every possible effort should be made not to deprive the future engineers or educators from this vital component of their education[1]. It is therefore necessary to continue development of effective and efficient pedagogical methods and techniques for the engineering laboratory experience[2].

Laboratory apparatus is generally expensive due to low production levels, specialized features and significantly higher *Design Costs* built into the final cost. Such high costs may lead to the lack of vital laboratory apparatus and in turn deprive the engineering students from being sufficiently exposed to important concepts such as verification of the theory through experimentation, interpretation and analysis of data and gaining sufficient background for designing experiments. However, if blueprints of the designs of a (desired) apparatus are available, and on site machining capabilities exists, a major cut may be expected in the final cost. Such designs and blueprints may be generated in-house in collaboration with undergraduate engineering students[3].

II - Objectives of the Experiment

The following major objectives were set at the inception of the project;

1. To develop an experiment for examination of the stress distribution in non-prismatic beams and comparison of their *Strength to Weight Ratio* with the prismatic beams,
2. To create an opportunity for collaborative research and design efforts between undergraduate engineering student(s) and faculty,

3. To design and produce a cost-effective, reproducible apparatus with outstanding features.

4. To incorporate (optional) use of ANSYS for comparison of the measured and calculated results,
5. To make all information necessary for fabrication of the apparatus and conducting the experiment available to engineering programs nationwide.

The authors invited three rising junior engineering students (Daniel Salman, Greg Reese, and John Martin) to collaborate with them in materializing the above goals. The parameters in successful implementation of the processes involved for achieving the above goals were comprehensively discussed, outlined and a preliminary Gantt chart was generated. Through four weekly scheduled meetings, alternative designs and approaches for each of the components and processes were evaluated, chosen, and optimized. It took another two weeks to fabricate, modify, and test the reliability of the apparatus.

III - Design of the Beams and the Associated Apparatus

To facilitate the running of the experiment, the following must be available:

1. The Prismatic Beam,
2. The Tapered Beam,
3. Frame and the Cantilever Beam Support System/Stand(s).

The role and design characteristics of each of these components are briefly discussed.

1. The Prismatic Beam

Starting with a ¼" thick by 1.5" wide *certified* stock of Aluminum 6061-T6, the prismatic beam is machined to have the dimensions shown in figure (1). The beam is designed to have an "effective" length of 18 inches. The two inch extension (on the left) is required to support the beam and the one inch extension on the right allows for placing an eye-bolt to support the externally applied load.

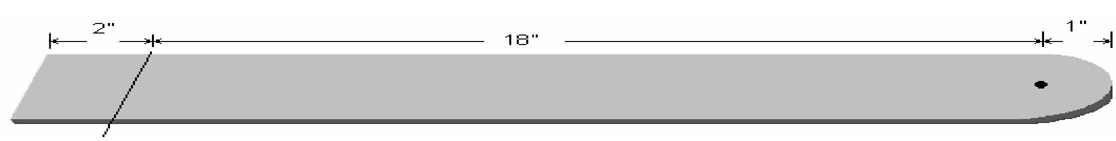

Figure 1. The Prismatic Beam.

2. The Tapered Beam

Figure (2) portrays the selected geometric shape and dimensions of the tapered beam when compared with the prismatic beam. For ease of comparison, it is made of the same material and thickness used for the prismatic beam. It will be shown (in the modeling section) that the effective weights of the two beams are (nearly) identical.

Four (4) temperature compensated strain gauges have been installed on each of the two beams. These are symbolically represented by the black rectangles in the figure. The selection of strain gauge locations is arbitrary, as long as the identical locations are chosen on both beams. Also, the number of strain gauges may be varied according to the experimental need. In the proposed arrangement, the distance of the first gauge (outmost left) from the support is chosen so that potential effects of stress concentrations would be minimal without (much) sacrificing the maximum moment arm. Also, the remaining gages are arranged so that the collected data would lead to generation of tables and graphs for clear comparison of distribution of strain and stress on the two beams.

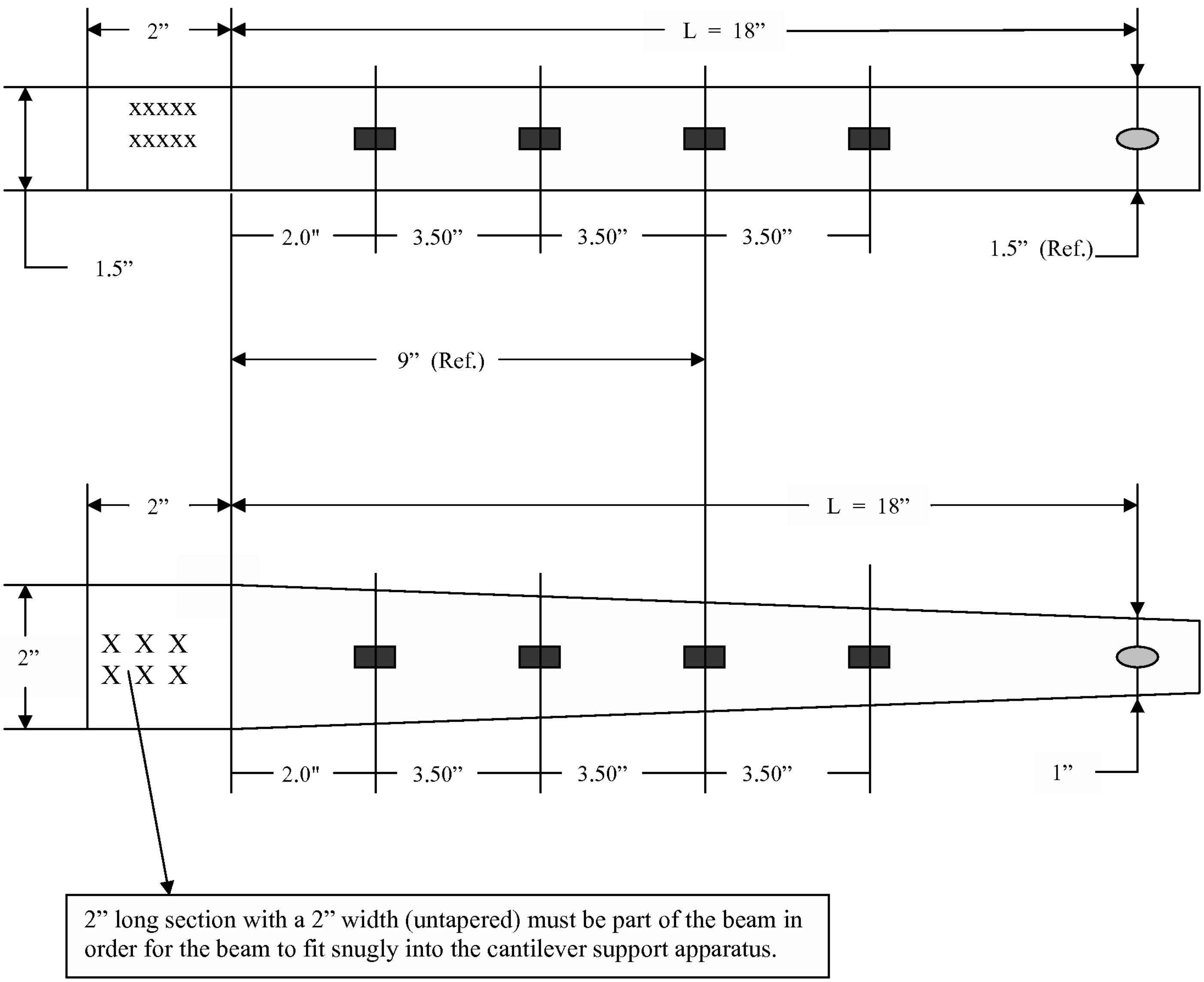

Figure 2. Dimensions for Manufacturing of the Prismatic and Tapered Cantilever Beams and Installation of the Strain Gauges.

3. The Frame and Beam Support Stand

The recommended frame is constructed of MiniTec components due to their durability, cost effectiveness, and aesthetic appeal. MiniTec frames are capable of being rearranged to fit additional demands.

This modular nature allows for upgrading the system in the future by only purchasing a few additional components instead of purchasing a completely new frame. The frame allows for supporting and running two testing devices simultaneously. It is equipped with four locking casters (that also resist rotation). The modularity of its design allows for conducting many other experiments at different periods of a typical laboratory course and easy storage when not in use.

The Beam Support System/Stand is required to support the specimen in a cantilever mode. These are made of aluminum and may be used for other experiments. Figure (3) shows the frame, and the two stands used to run tests on two different specimens at the same time.

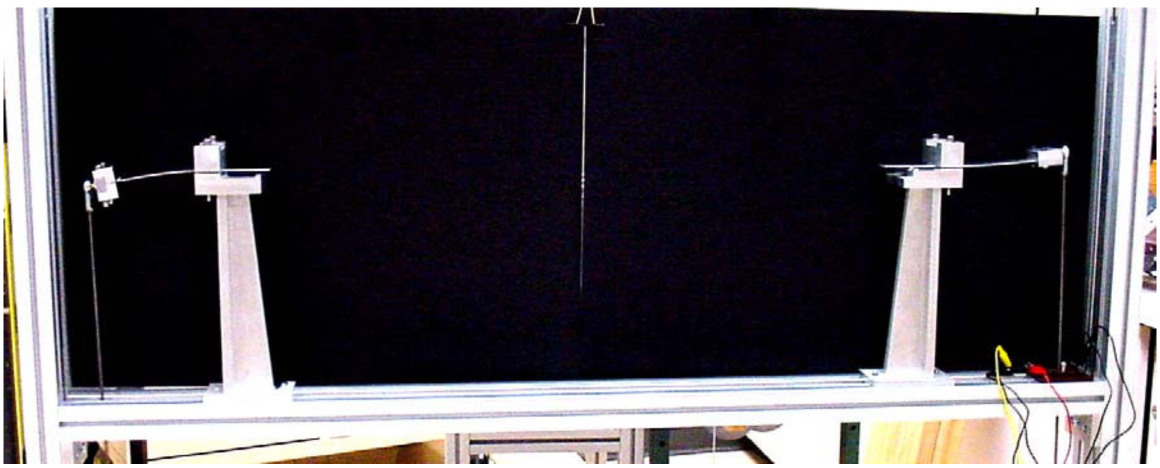

Figure 3. The Frame and the two Beam Support Stands used for running two tests simultaneously. In this photo, the Frame and the two Beam Supports are used for a different experiment (Fatigue Testing).

VI - A Simple Mathematical Model

The surface strains ***(ε)*** at the four sections of interest will be measured by the axial reading of a strain gauge bonded at that point. Alternatively, knowing that the *Modulus of Elasticity* of aluminum is[4]: ***E ≈ 10 E 6 psi***, we may easily obtain the stress value by recalling that: $\sigma = E\varepsilon$. The use of this relationship will generate the *Experimental* values of stress. It is clear that since the modulus of elasticity of the material used is a constant, the higher the value of strain, the higher the stress.

Our task is then reduced to the development of the equations that may express strains as functions of locations on the lengths of the beams. These will generate the *Theoretical* values of stress / stress.

1. Proposed Dimensions for the Two Beams

Figure (4) displays the effective dimensions of the two beams for comparison of their effective weights. We note that:

$$\mathbf{W}_{\text{Prismatic}} = \mathbf{V}_{\text{Prismatic}} \cdot \rho_{\text{Prismatic}} = \mathbf{A}_{\text{Prismatic}} \cdot \mathbf{t}_{\text{Prismatic}} \cdot \rho_{\text{Prismatic}}$$

$$\mathbf{W}_{\text{Tapered}} = \mathbf{V}_{\text{Tapered}} \cdot \rho_{\text{Tapered}} = \mathbf{A}_{\text{Tapered}} \cdot \mathbf{t}_{\text{Tapered}} \cdot \rho_{\text{Tapered}}$$

Since the material of the two specimens *(ρ)* as well as the thicknesses *(t)* are the same;

$$\mathbf{W}_{\text{Prismatic}} / \mathbf{W}_{\text{Tapered}} = \mathbf{A}_{\text{Prismatic}} / \mathbf{A}_{\text{Tapered}} =$$

$$\mathbf{1.5'' \times 18'' / [(2'' + 1'') / 2] \times 18'' = 1 / 1}$$

Therefore, the proposed dimensions for the two beams results in (almost) identical weights. This arrangement will further ease the process of comparison of the "Strength to Weight Ratio" of the two beams.

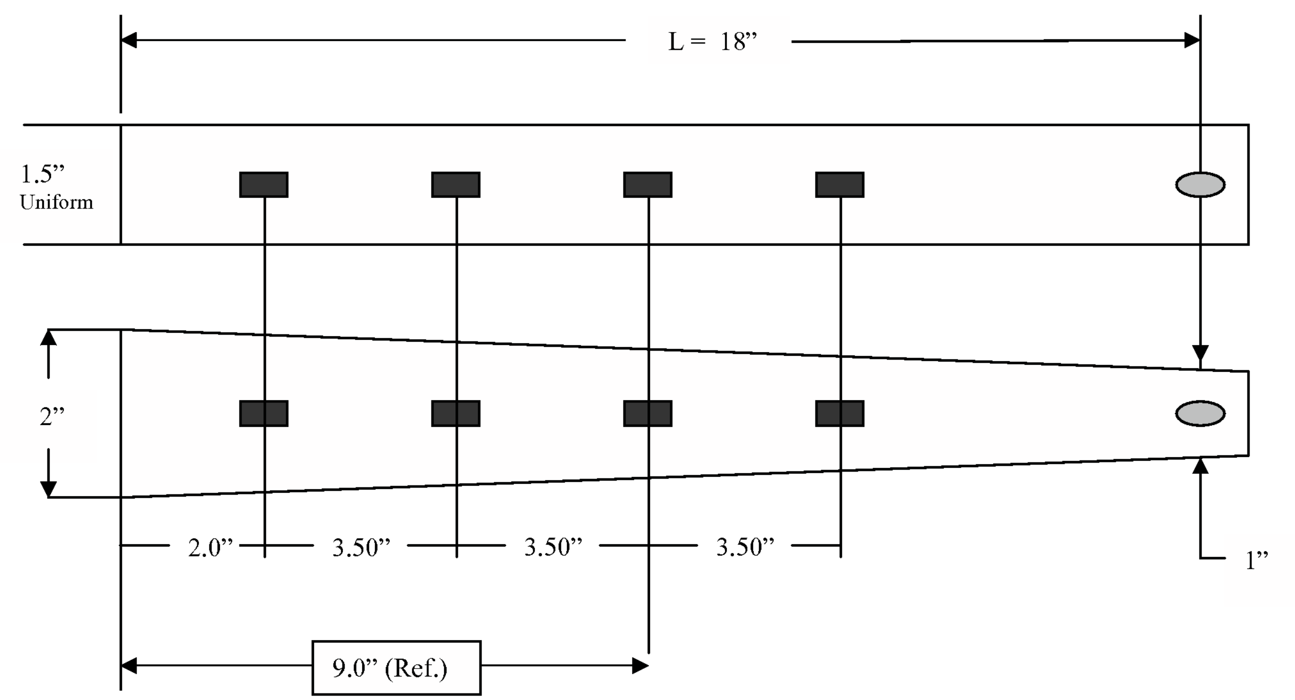

Figure 4. Effective Dimensions of the Prismatic and Tapered Cantilever Beams for Comparison of their Effective Weight.

2. Strain Changes due to Tapering

From basic mechanics of materials[5];

$$\sigma = \frac{Mc}{I} \qquad \text{and} \qquad \sigma = E\varepsilon$$

Rearranging these expressions:

$$\varepsilon = \frac{\sigma}{E} \qquad \text{or} \qquad \varepsilon = \frac{Mc}{IE}$$

After simplification: $$\boxed{\varepsilon = \frac{6FL}{bt^2 E}} \quad (\mathbf{\Omega})$$

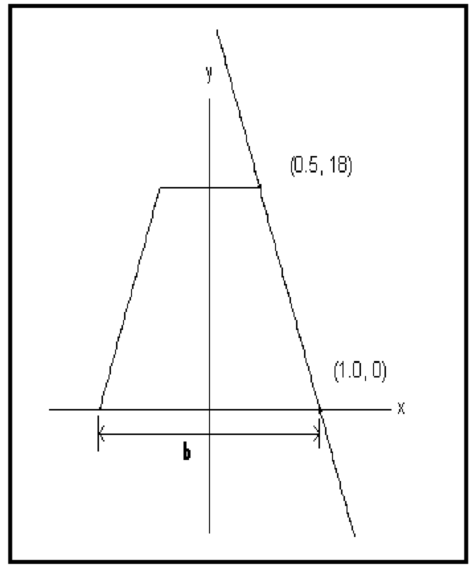

Figure 5. Mapping the Geometry of the Tapered Cantilever on a Coordinate System.

In our case, L is equal to $18 - x$, which is the moment arm of the force; b is the base of the cross-section of the cantilever. In the case of the prismatic cantilever, $b = 1.5$ and does not vary, while, in the case of the tapered cantilever, b varies following the linear relationship:

$$b = -\tfrac{1}{18}x + 2.0\,.$$

This relationship is obtained by mapping the cantilever shown in Figure (5) on a Cartesian coordinate system. Note that the line created by the taper of the cantilever is ½ the distance of the base, b. Knowing this, by obtaining the equation of the tapered line (using the point-slope method and multiplying it by 2), it is possible to create an equation for the base as a function of the beam's length.

Therefore, for the tapered and prismatic cantilevers respectively, we may use equation **(Ω)** to express strain as a function of position on the length;

Prismatic: $$\varepsilon(x) = \frac{6F}{t^2 E} \times \frac{(18 - x)}{1.5}$$

Tapered: $$\varepsilon(x) = \frac{6F}{t^2 E} \times \frac{(18 - x)}{-\frac{1}{18}x + 2.0}$$

With these relationships, theoretical strain can be calculated as the position along the cantilever varies for both the prismatic and the tapered beams. These theoretical strains can be compared to see how the behavior of strain (as well as its relationship to the magnitude of stress) is affected by tapering. These values will follow the trend depicted in Figure (6); a graph of strain versus position for two different weights of 10 lb and 50 lb. [Although both beams can handle this range of loads, such magnitude loads should not be applied to the proposed beams].

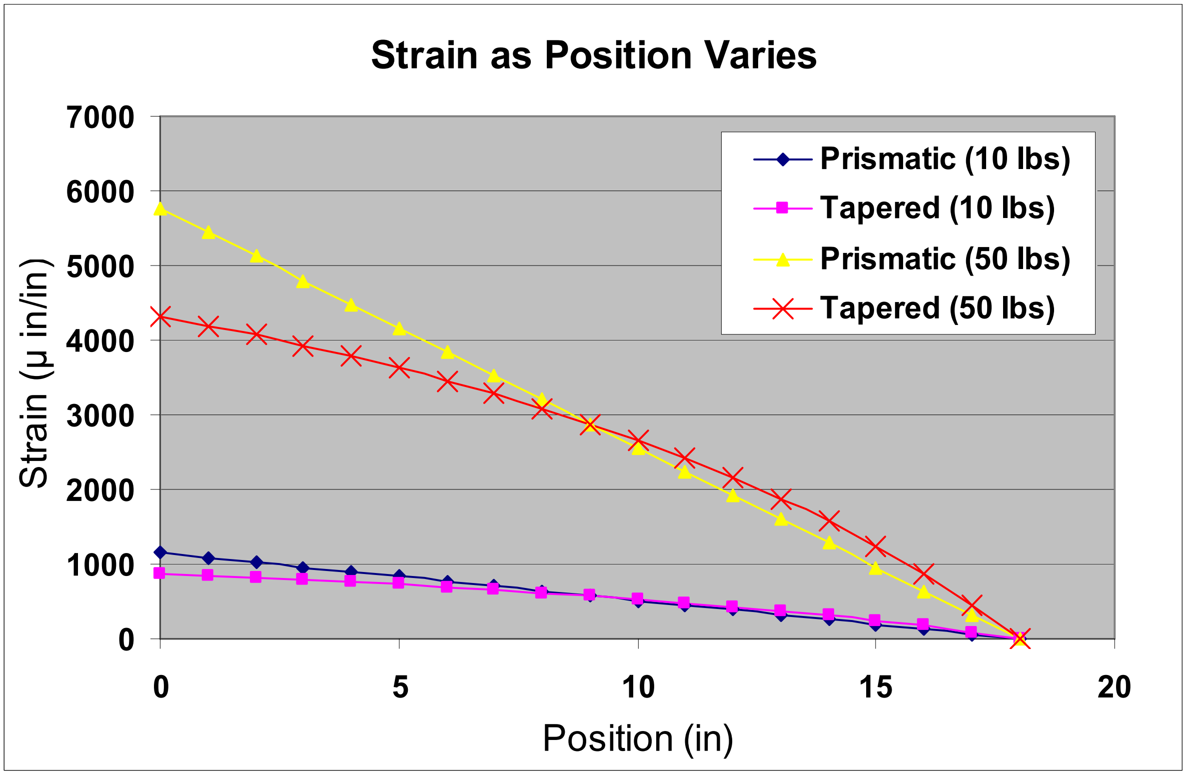

Figure 6. Comparison of Strain versus Position of the two Cantilevers for Two Loads.

The graph pictured above clearly depicts that at the support location (when the moment arm is maximum), the strains in all cases are at their maximum levels. Conversely, at the point of application of the load, the strains are zero. Comparison of the graphs reveals that the prismatic beam produces a linear relationship between strain and position, while the tapered beam produces a 2nd degree relationship. Perhaps the most interesting observation to make is that at (almost) half the effective length (9 in), the strain for both beams is equal. This is because at this location, in addition to having the same thickness (t = h), the bases of the sections of the two beams are equal as well ($b=1.5$). As a result, the moments of inertia of the two beams will be the same at this (break-even) section[6] *[* $\boldsymbol{I = (1/12)\, bh^3}$ *].*

V - Manual Calculations

Hand calculations were performed for both beams to obtain the values of strain at the four points of interest. To get a realistic theoretical value, distributed loads were applied over the entire length of the beams in order to simulate their corresponding weights. For the prismatic beam, this load was a constant 0.03675 lb/in; for the tapered beam, the equation for the weight at any point had to be determined in order to get the load. For both beams, a *1.5 lb* point load was applied to the free end.

VI - The ANSYS Alternative

If desired, the experiment may call for an additional component using ANSYS. If so, the two beams should be modeled in Pro-Engineer, and then imported into ANSYS as an IGES file[7]. To simulate a cantilever beam, the supported end of the beam is modeled such that all degrees of freedom are set to zero. A 1.5 lb load (in our case) was applied to the other end of the beam in the negative Y-direction. In order to simulate the weight of the beam in ANSYS, under preferences, FLOTRAN CDF[8] is selected, and under, Preprocessor, FLOTRAN Set Up, Flow Environment, Gravity, set gravity in the Y-direction equal to 386.4 in/sec^2. For this to work the density of the material also has to be defined in the material properties; in this case, for aluminum, 0.000254 slug/in^3. In this setup, a cubic meshing shape works better than the default tetrahedral shape, so in order to mesh the beam, volume sweep should be used. After the piece is meshed, it is possible to list all nodes with their coordinates in order to find which nodes are to be looked at for the strain results. After the loading situation is solved, the nodal solution for every node is listed. The results at the nodes of the four points of interest are recorded and compared with the hand results in the following table (1). Although the percent differences (Δ) are within acceptable range, interesting questions may be raised about the potential assumptions / sources contributing to the differences.

Table 1. Comparison of the Manual and ANSYS Calculated Strains

XXX	**Prismatic Beam**			**Tapered Beam**		
Distance from fixed end (in)	ANSYS μ strain X- direction	Hand	Percent **Δ**	ANSYS μ strain X- direction	Hand	Percent **Δ**
2.0	182.6	183.7	0.60	139.9	142.6	1.94
5.5	138.4	138.4	-0.03	120.1	119.6	-0.43
9.0	95.95	95.93	-0.03	95.36	93.81	-1.65
12.5	56.38	56.36	-0.03	63.13	63.67	0.84

To make a comparison of the measured value with those obtained using ANSYS, the same loading was physically applied to the two beams. Four (4) tests were performed on each of the beams and the average of the (4) strain readings are being used for the comparison. Extra care was exercised in the calibration of the indicators [Micro-Measurements' P-3500 and SB-10 Units]. The results and the percent differences (Δ) are shown in Table (2).

Table 2. Comparison of the Measured and ANSYS Calculated Strains.

XXXXXXXX	Prismatic Beam			Tapered Beam		
Distance from fixed end (in)	*ANSYS μ strain X- direction*	*Measured **	*Percent Δ*	*ANSYS μ strain X- direction*	*Measured **	*Percent Δ*
2.0	182.6	185	+1.3	139.9	141	+0.79
5.5	138.4	139	+0.43	120.1	119.5	-0.50
9.0	95.95	96	+0.05	95.36	94	-1.4
12.5	56.38	55.5	-1.6	63.13	64	+1.4

* Average of the Measurements obtained from four (4) Tests.

It is clear that there is (nearly) complete agreement between the *Analytical* and the *Experimental* results. Again, at the break-even section, the values of strain for both beams tend to be (nearly) the same. Figure (7) shows the distribution of strain of the two beams due to application of a single *1.5 lb* load at the free ends. Evaluation of the results tabulated in table (2) and the graphs of figure (7) strongly support the fact that tapered beams have a considerably better strength to weight ratio. However, students should be alerted to the fact that the manufacturing costs of tapered sections are considerably higher than prismatic section. This parameter must be taken into account when it comes to finalizing design decisions.

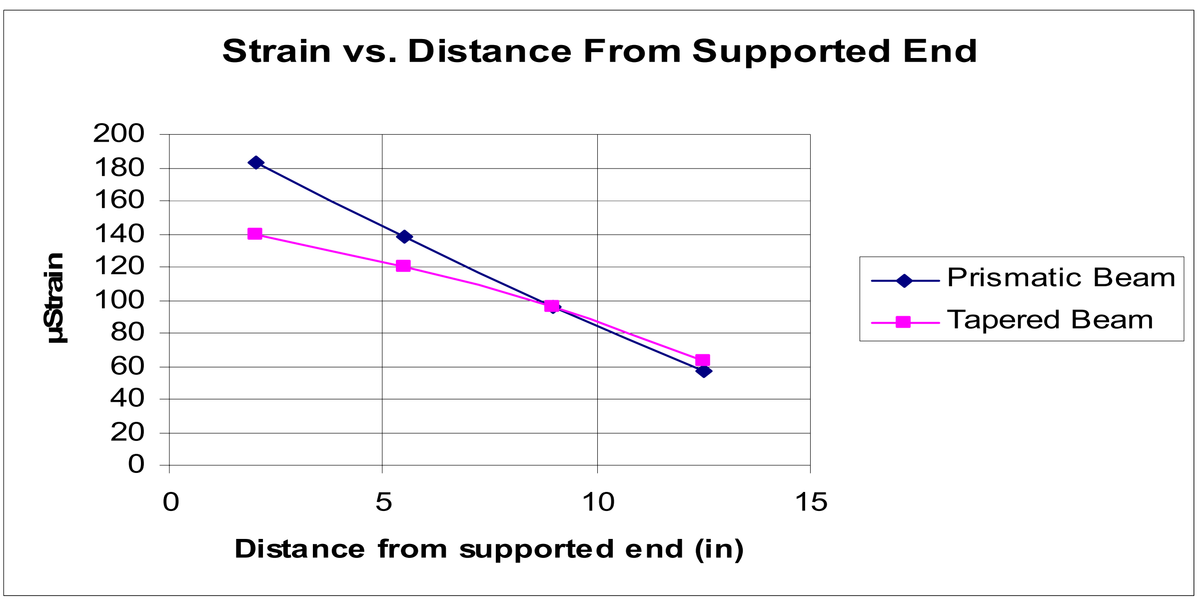

Figure 7. Superposition of the Strain vs. Distance from Supported Ends of both Beams.

VII – Summary and Conclusions

The major objectives listed in *section II* have been achieved in this project. It is believed that in comparison with the commercially available counterparts of the proposed design and experiment, an alternative solution is offered to those colleagues who may be interested in adopting this proposed experiment and apparatus. This approach is beneficial for all parties involved; the researching/collaborating student(s), underclassmen who would benefit from such experiments and the enthusiastic instructors/laboratory coordinators who may be fighting with budgetary issues. The experiment and the associated apparatus return excellent results and enable the undergraduate engineering students to *Validate Theories of Mechanics of Materials through Experimental and Computer Aided processes*.

Acknowledgements

The authors express special thanks to Alexander Michalchuk (department senior technician and machinist) for his continuous support of and dedication to the project. They also thank the former collaborating students (Daniel Salman, Greg Reese, and John Martin) for their contributions. They certainly made a difference.

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Appendix A:

Parts List and Breakdown of the cost

Part	Part No.	Quantity	Price ($)
¼ x 2 x 48 6061-T6 Certified Aluminum**	B0001	1	51.00
Strain Gauges	B0002	8	55
Gauge Installation Package	B0003	1	120
Hardware	B0004	-	19
		Total Cost:	245 / (1 Set) 385 / (2 Sets)

****Sample Data:**
Material: Aluminum 6061-T6 with certification.
The (average) Ultimate Strength, S_U = 47.34 kpsi,
The (average) Yield strength, S_Y = 44.16 kpsi,
Modulus of Elasticity, E = 10,000 kpsi,

XXXXXXXXXXXXXX	**The Set of the Two Beams**	**Beam supports (2)**	**Frame (1)**
I - Average Machining	2	10	NA
II - Above Average Machining	1	4	NA
III - Assembly of Frame and Components	-	-	4-5
IV -Installation of the Gauges and Wiring	4	NA	NA

1. Overall Cost of the Materials and Components ≤ $ 250
2. Frame ≤ $ 1050
3. Beam supports (2) ≤ $ 150
3. Required Machining and Assembly Time:

I - Average Machining: About 12 hours (20 for two units)
II - Above Average Machining: About 5 hours (8 for two units)
III - Assembly of Frame and Components: About 4-5 hours

Appendix: B

Laboratory Handout
for
Stress Distribution in Tapered VS Prismatic Beams

Introduction & Objectives

The purpose of this experiment is to contrast the stress distribution on a rectangular cantilever and a tapered (trapezoidal) cantilever, both made of the (same stock of) 6061-T6 aluminum alloy. Each of the beams has four strain gauges attached to at exactly identical locations. Using elementary equations, the strain values would lead to establishment of the corresponding stress values. A comparison of the distribution of stress on the two beams will show that the tapered beam has a higher *Strength to Weight Ratio*. Distribution of strains (or stresses) of both beams can be plotted on the same graph to visually examine the performance of the two beams.

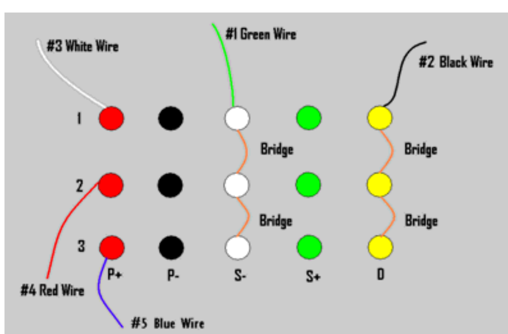
Equipment and Supplies

- Frame and Cantilever support assembly/mount
- High-strength prismatic aluminum alloy beam, ¼ x 1.5 x 18 in - Gauged
- High-strength tapered aluminum alloy beam, $t = ¼$, $w_1 = 2$, $w_2 = 1$, $L = 18$ in - Gauged
- Laboratory weights for loading cantilever beams
- Multi-port Strain Indicator

Set up and Procedure

1. Mount the cantilever beam in the support assembly.

2. With reference to the diagram below, connect the lead wires from the strain gauges to the corresponding posts on the *Strain Indicator*. **Note: take care to set up the indicator system properly in order to get correct readings.**



3. Using an accurate scale, measure the distance from the centerline of each strain gauge grid to the point of load application at the free end of the beam.

4. Measure the width and thickness of the beam with a micrometer/dial caliper.

5. With the beam unloaded, set the gauge factor on the strain indicator.

6. Zero the channel in use. **Note: do not adjust the balance control again for the remainder of this portion of the experiment.**

7. Apply small loads in several increments, totaling about 1,500 – 2,000 grams max.

8. For each of the gauges, record the indicated strain.

9. Add an additional load and repeat step 8.

10. Tabulate the results of steps 8 and 9 in the following table,

Load (g)	**Gauge #**	**Strain (ε) Prismatic – increasing load**	**Strain (ε) Prismatic – decreasing load**	**Strain (ε) Tapered – increasing load**	**Strain (ε) Tapered – decreasing load**
	1				
	2				
	3				
	4				
	1				
	2				
	3				
	4				

Table 1. Strain Distribution on the two beams

Analysis

Using the data collected in Table (1), apply the necessary equations to generate the corresponding stress distributions on the two beams. Plot the distribution of strains (or stresses) of both beams on the same graph to visually examine the performance of the two beams. Correlate the distribution of strain as a function of length for both beams. Comment on how the shape of the two beams has affected their corresponding distribution of strain/stress.

Report

Prepare a brief report, describing in your own words the purpose of the experiment, the equipment and setup used, and the procedure followed. Tabulate the results obtained and include experimental data and the expected theoretical values for comparison and calculation of percent differences. Discuss probable sources for error and their relative effects on the accuracy of the values that have been determined.