



Computer-Aided Design (CAD) of Recursive/Non-Recursive Filters

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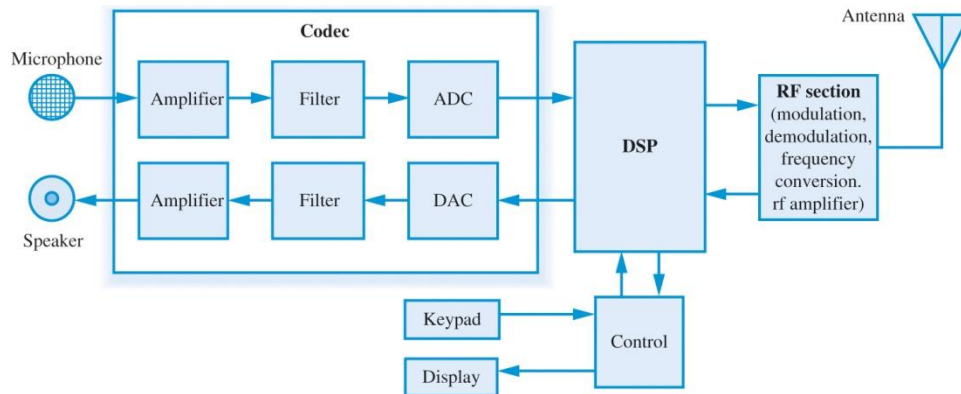
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Abstract. Computer Tools are integral part of many engineering design courses, they should be used in the right place, right time. Courses in the Digital Signal Processing/Filter areas (including speech, image and video processing) have been traditionally viewed by students to be fairly mathematical subjects including many abstractions (e.g., spectrum, analysis/design methods in time/frequency domains, SNR, bandwidth, white/pink noise, various transforms, etc.) The pedagogical value of this work is that, with the help of modern engineering tools, engineering educators can better help students visualize these apparently difficult (but important) concepts. We focus on the subject of designing digital filters, both with and without feedback, as well as analog prototypes. In addition to the use of popular software packages like MATLAB & MATHCAD, our filter course uses a tool accompanying the textbook that can help students with various designs of these digital, analog systems and at the same time (in case of analog “prototype”) come up with PSpice circuit elements as well as providing frequency response plots, pole/zero diagram.

Introduction. Engineering, computing curricula include courses on signals & systems as well as analysis/design of frequency selective circuits/systems called filters. The subject of filters (both analysis and design) are usually covered in several classes. Even at introductory level courses (e.g., circuits & systems, see, for example, [1]) whenever we are dealing with alternating current (AC) topics students are being introduced to simple forms of frequency selective structures. As example, in first order circuits (RC or RL) if we include AC excitations we are really dealing with filters: low-pass, high-pass, band-pass or band-stop (reject) devices. These simple circuits are examples of passive circuits & systems. With the introduction of operational amplifiers (Op-Amps) integrated circuits, we start description of so-called active filters. Studying the input/output (I/O) models of these systems, students are gradually introduced to using computer-aided design, CAD, or analysis tools that can be beneficial in their work. For example after “digitizing” the differential equation of the system, they can solve the I/O model (linear, constant-coefficient difference equation) of the system. The digital I/O models include both recursive and non-recursive systems. For the CAD design there is a need for utilizing window functions, one important class of windows require the use of high-level mathematical functions for which an algorithm is introduced so that students can come up with their final design and again use the software tool for the verification/validation step. Whenever possible students are given opportunity to make presentation in the class where more results can be discussed and a term paper/report submitted to receive credit toward their grades. Since the required subjects/topics are very complex by nature, these tools can be very beneficial for both learners and engineering educators. Finally there are cases that a regular design employing the so-called classical system approach may not be practical, however it will be possible to design the system using, for example, non-recursive, digital systems.

The subject of digital signal processing (DSP) is covered in most computing and electrical engineering programs, as example see the textbook by Lathi & Green, [6]. We next show an important application of DSP in mobile computing, e.g., smartphones in simplified block diagram form, [2]:



As can be seen several of the system components are related to the subject of DSP. Design of FILTERS is a major topic in DSP including analog filter design (passive, active electronic circuits), classical filters, and digital filters. There are two main kinds of digital filters used in practice: filters with feedback (also known as IIR, ARMA filters) and filters without feedback (or FIR, MA filters.)

IIR Filters. Analog filter design, a mature subject studied by several investigators and well-documented (as example see [3]), is the starting point for design of IIR (infinite impulse response) digital filters. Basically we are dealing with K-th order differential equation input/output (I/O) model, with $x(t)$ being the input (excitation) and $y(t)$ the response or output:

$$\sum_{k=0}^K a_k \frac{d^k}{dt^k} y(t) = \sum_{l=0}^L b_l \frac{d^l}{dt^l} x(t)$$

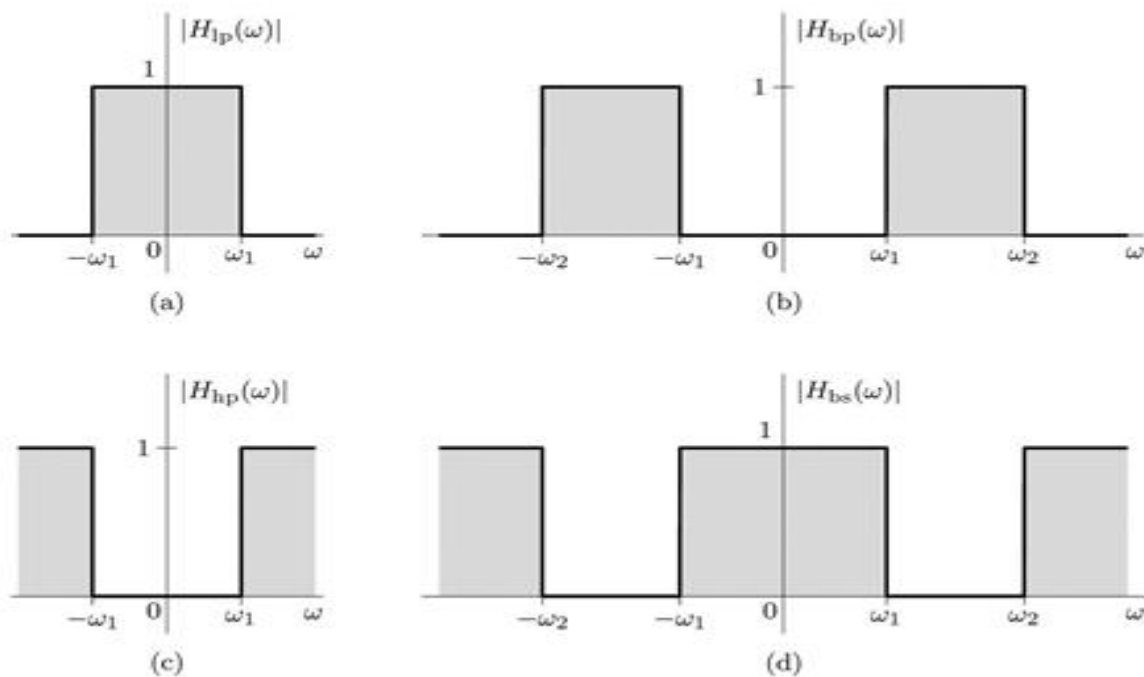
Digital filter is then obtained by using several transformations as developed in the DSP literature:

- ❖ $H(s) \rightarrow H(z)$ (**mapping** from the s-plane to the z-plane), Laplace transforms, z-transforms
- ❖ Backward Difference Approximation, $s \rightarrow (1/T) [1 - z^{-1}]$ where $1/T$ is the sampling frequency of DSP system
- ❖ Impulse Invariance, $h(t) \rightarrow h(nT) = h[n] \rightarrow H(z)$ (also step invariance)
- ❖ Bilinear Transformation, $s \rightarrow (2/T) (z - 1) / (z + 1)$
- ❖ CAD tools, e.g., MATLAB, MATHCAD, WFilter

Like in any engineering design problem there is a trade-off in the design process. The most optimum method of designing IIR filters is using the bilinear transformation, however this transformation creates a non-linear relationship between analog and digital frequencies. We need to use pre-warping techniques to avoid distortion in the final design. Impulse invariance method

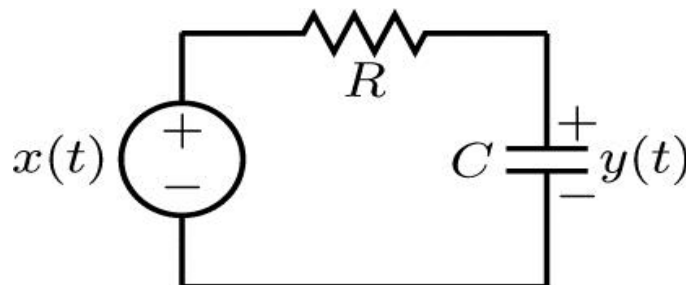
also requires using a high sampling rate to avoid aliasing, this method is only used to design low-pass and band-pass filters. From the stability point of view we need a transformation that results in a stable, causal system (filter.) While all methods mentioned above result in a stable final filter (the so-called forward difference transformation may result in unstable filter), the backward difference transformation is not widely used because pole locations do not occupy all the space within the unit circle (u.c.) in the z-plane but a sub-set inside u.c.

To design digital filters with feedback we first introduce students to the subject of analog filter design, a difficult subject but most engineering curricula in affected disciplines do include courses (for example second electric circuit class) on this subject. Signals & systems courses also cover filters especially when studying system frequency response. In our school the recent book by Lathi & Green [6] is used in the digital signal processing senior, first-year graduate classes. Practical filter design discussion starts with the fact that ideal filters cannot be realized in practice. As example if we look at following ideal LPF, BPF, HPF and BSF frequency responses, we can show that their impulse response, $h(t)$, will not represent a causal system.



In case of LPF with cutoff frequency ω_c and assuming zero phase response, it is easily seen that inverse Fourier transform gives:

$h(t) = (A\omega_c/\pi) \text{Sa}(\omega_c t)$; $-\infty < t < \infty$, where sampling function is defined as $\text{Sa}(x) = \sin(x)/x$. On the other hand a simple RC LPF circuit shown below:



will result in the causal impulse response:

$$h_{RC}(t) = (1/RC) \exp(-t/RC) u(t)$$

which is the inverse Fourier transform of frequency response:

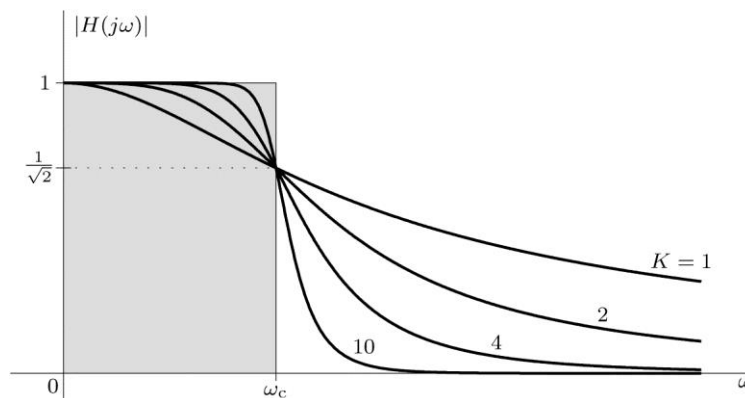
$$H(\omega) = 1/(1 + j\omega / \omega_c), \text{ where } \omega_c = 1/(RC) \text{ is the so-called 3-dB (radian) cutoff frequency.}$$

Many practical filters are defined by their magnitude response squared function, $|H(\omega)|^2$. For this simple RC filter we have:

$$|H_{RC}(\omega)|^2 = 1/[1 + (\omega / \omega_c)^2]$$

In the classical filter literature (see, for example, [3]), these filters are known as Butterworth filters, which for order K are defined as:

$$|H_B(\omega)|^2 = 1/[1 + (\omega / \omega_c)^{2K}]$$



Once analog filter is designed to specifications, one gets the digital filter by using transformations (mappings) mentioned earlier. We show an example next.

Example, IIR filter design. Design a digital Butterworth LPF to satisfy the following specifications:

3-dB cutoff frequency of $f_c = 3$ kHz, the minimum stopband attenuation of 23 dB for frequencies $f \geq 6$ kHz. Use a sampling frequency of $f_s = 20$ KHz in your digital design utilizing the bilinear transformation mapping.

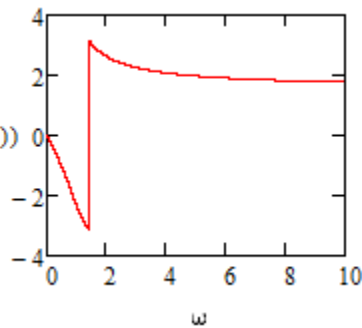
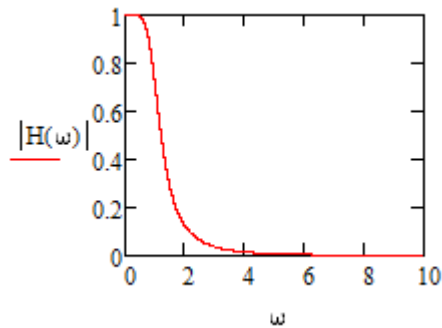
Using the design statement, a Butterworth filter of order $K = 3$ will satisfy the design requirements after applying the so-called pre-warping operation needed in the bilinear transformation.

$$H(s) = 1/[(s + 1)(s^2 + s + 1)] \quad (\text{note that pre-warped cutoff frequency is actually } 2 \tan(0.15\pi) \text{ which is approximately } 1 \text{ r/s.})$$

We show a MATHCAD simulation of this analog filter next.

$$\omega := 0, 0.001.. 10$$

$$H(\omega) := \frac{1}{(j \cdot \omega + 1) \cdot (-\omega^2 + j \cdot \omega + 1)}$$



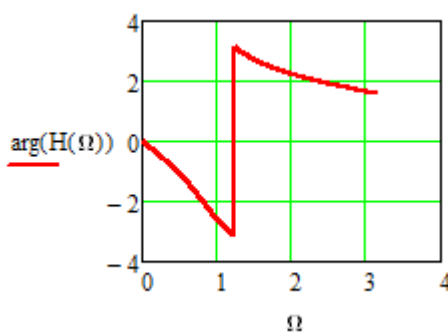
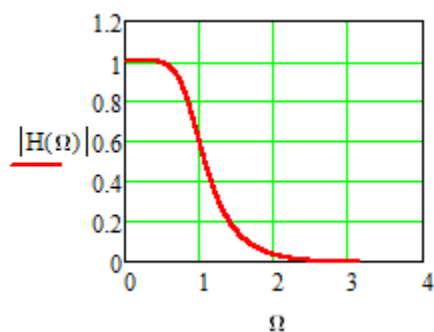
Once bilinear transformation is applied we replace s by $2(z - 1) / (z + 1)$ to find the digital filter transfer function:

$H(z) = (z + 1)^3 / [(3z - 1)(7z^2 - 6z + 3)]$. The simulation of digital filter is shown next (note that Ω represents digital frequency in radians).

$$\Omega := 0, 0.001.. \pi$$

$$z(\Omega) := \exp(j \cdot \Omega)$$

$$H(\Omega) := \frac{(z(\Omega) + 1)^3}{(3 - z(\Omega) - 1) \cdot (7 \cdot z(\Omega)^2 - 6 \cdot z(\Omega) + 3)}$$



There are simple transformations in the s -domain to design other types of filters. For example simple replacement of s by $1/s$ will transform a LPF into a HPF. Table look-ups can be used for other filter types starting with a prototype LPF.

FIR Design. These digital filters are also popular in practice because they do not require the design of an analog prototype first. Also these filters can be designed to have linear phase but the designed filter order is much higher as compared to IIR filters. Another strength of FIR filters is that they can be designed to implement any given desired frequency response including designing multi-band filters, Hilbert transformers, digital differentiators, and comb filters. We show a couple of examples of FIR designs used in the class next.

Example, multi-band FIR design. Design an FIR 111-tap (order) linear phase digital filter approximating the desired frequency response:

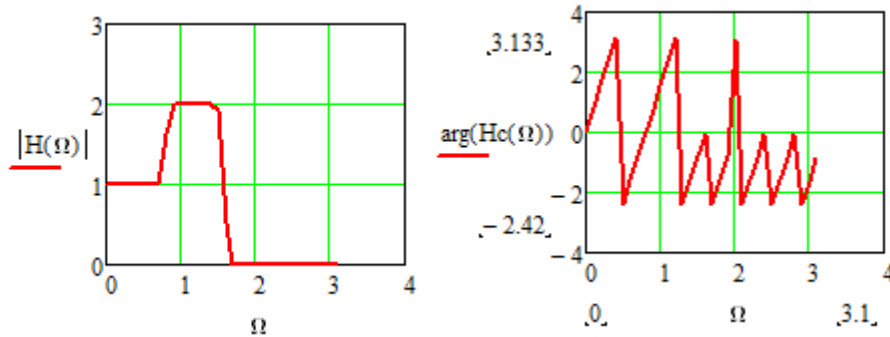
$$H_d(\Omega) = 1 ; -\pi/4 \leq \Omega \leq \pi/4 \quad \& \quad H_d(\Omega) = 2 ; \pi/4 \leq |\Omega| \leq \pi/2 \quad \& \quad H_d(\Omega) = 0 ; \pi/2 \leq |\Omega| \leq \pi .$$

We use the Hanning window function in our design:

$$w[n] = 0.5 + 0.5 \cos(\pi n/55) ; -55 \leq n \leq 55 , w[n] = 0 ; \text{otherwise.}$$

Simulation of this filter is shown next.

$$\begin{aligned} n &:= 1, 2, \dots, 55 \\ \Omega &:= 0, 0.1, \dots, \pi \\ hd(n) &:= 2 \cdot \frac{\sin\left(n \cdot \frac{\pi}{2}\right)}{n \cdot \pi} - \frac{\sin\left(n \cdot \frac{\pi}{4}\right)}{n \cdot \pi} \\ w(n) &:= 0.5 + 0.5 \cdot \cos\left(n \cdot \frac{\pi}{55}\right) \\ hw(n) &:= hd(n) \cdot w(n) \\ H(\Omega) &:= 0.75 + 2 \cdot \left[\sum_n (hw(n) \cdot \cos(n \cdot \Omega)) \right] \\ Hc(\Omega) &:= H(\Omega) \cdot \exp(-j \cdot 55 \cdot \Omega) \end{aligned}$$



Example, FIR Hilbert transformer. This filter is actually a phase shifter (-90°) system:

$Y(\Omega) = -j X(\Omega)$ for $0 < \Omega < \pi$ and $Y(\Omega) = j X(\Omega)$ for $-\pi < \Omega < 0$, or using the so-called signum (sgn) function, sgn(Ω), filter's desired (ideal) frequency is given by:

$$H_d(\Omega) = -j \operatorname{sgn}(\Omega)$$

This kind of transformation has application in communication systems, [4], [5], for example, in single-sideband modulation. In DSP this is just another given frequency response to be implemented. The impulse response is given by:

$$h_d[n] = [1 - \cos(n\pi)] / (n\pi) , h_d[n = 0] = 0.$$

Using an appropriate window function $w[n]$, $h_w[n] = h_d[n] w[n]$, and designed frequency response is:

$$H(\Omega) = -2j\{h_w[1] \sin(\Omega) + h_w[2] \sin(2\Omega) + h_w[3] \sin(3\Omega) + \dots\}.$$

We show simulation result next.

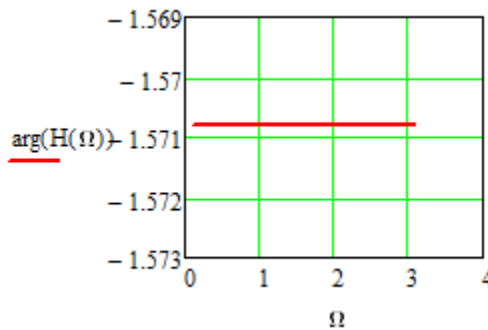
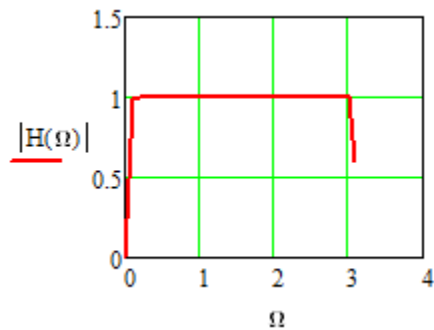
$$n := 1, 2.. 50$$

$$\Omega := 0, 0.1.. \pi$$

$$h_d(n) := \frac{2 \cdot \sin\left(n \cdot \frac{\pi}{2}\right)^2}{n \cdot \pi}$$

$$w(n) := 0.5 + 0.5 \cdot \cos\left(\frac{n \cdot \pi}{50}\right)$$

$$H(\Omega) := -j \cdot 2 \cdot \left[\sum_n (h_d(n) \cdot w(n) \cdot \sin(n \cdot \Omega)) \right]$$



Conclusions. In this paper we have shown how computer-aided design tools can be utilized in design of digital filters. These frequency selective systems have widespread applications in many engineering disciplines including computing, electrical/computer engineering, telecommunications and biomedical signal processing. From engineering educators' perspective, there is a right place, right time for introducing modern tools to enhance student learning. Overall students find these tools useful for their design work as well as helping them in visualizing some difficult DSP concepts such as frequency response, various filter design methods, linear phase, multi-band filter characteristics, z-transform and Laplace transform applications. Enrollment in the DSP course in our school has increased over the years despite the fact that, by nature, these subjects are considered by many students to be highly theoretical and with difficult concepts to understand. We have tried to include many MATLAB/MATHECAD examples throughout our lectures so that students are not bored with mathematical details. This semester we are trying a software laboratory component for the DSP course. More examples and student satisfaction results will be presented at the Annual Conference.

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