Session IIIA4

Demonstrating a Technique for Estimating the Constant of Proportionality in a Commonly Occurring Variation Problem in College Algebra Textbooks

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ABSTRACT

In the engineering component of LaPREP, a nationally acclaimed intervention program in engineering, math and science for high-ability middle and early high school students held at LSU-Shreveport, a hands-on activity was developed to estimate the constant of proportionality in a commonly occurring variation problem. In a typical variation problem with n unknowns found in College Algebra texts, students are given values of the n unknowns, and use them to determine k. They are then given another set of values for n-1 of the unknowns and, using the value they found for k, they can determine the value of the remaining unknown.

Occasionally, one of the better students will inquire about k and how it is determined. To address this issue, an experiment was conducted in which several estimates of the constant of proportionality were made for a specific variation problem: the load supported by a cylindrical column of known height and diameter. To make these estimates, the students constructed paper cylinders, and using steel shot for a load, determined the weight which the paper cylinder would support before it began to crumple. The formula $L = kd^4/h^2$ was used to find k. This experiment was repeated using cylinders of varying dimensions and the average of the estimates was calculated. Students then used this average to predict the load that a paper cylinder of different dimensions would support.

This paper gives a brief history and some of the accomplishments of LaPREP as well as specifics of the experiment and the several problems encountered in conducting it.

LaPREP Program

LaPREP (Louisiana Preparatory Program) is a two-summer enrichment program which identifies, encourages, and instructs competent middle and early high school students, preparing them to complete a college degree program, in math, science or engineering[1]. With emphasis on abstract reasoning, problem solving, and technical writing skills, participants attend seven weeks of intellectually demanding classes and seminars, interspersed with field trips and recreation.

Cass assignments, laboratory projects and scheduled exams are integral parts of LaPREP.

Each summer 30 first-year participants join with approximately 25 returning second year participants for a summer of intellectually stimulating work and fun on the LSUS campus. Students successfully completing the first summer session with a 75% or better average are eligible for the second summer session in 2003.

The topics studied over two summer sessions include:

- Engineering
- Logic
- Algebraic Structures
- Probability and Statistics
- Problem Solving
- Technical Writing
- ACT Preparation
- Medical Career Preparation
- Drug, Alcohol, and Gang Awareness and Prevention

Other features include: field trips to local industries, visiting lecturers and minority speakers, college and career awareness, swimming, basketball, ping pong, pool and other recreation.

Since a significant number of LaPREP students come from low-income families, LaPREP charges no tuition or fees. LaPREP provides free transportation to and from the program site via Sportran bus passes, free lunches in the University Center, books and other materials needed for classes, and cost-free field trips.

LaPREP Accomplishments

LaPREP will begin its twelfth annual summer session on the campus of LSUS in June of 2002. Over the past eleven sessions, approximately three hundred students have completed at least one summer of LaPREP. Approximately 80% of the participants have been minority students and almost 60% have been female.

Evaluations contributed by the participants of the program, their parents, and by local and state officials who have visited the program have shown the program to be highly successful. Participant interest in attending college and majoring in math or science has greatly increased. No former LaPREP participant has dropped out of high school, and 84% of exiting participants have indicated LaPREP has increased their desire to study math and science. Moreover, the first 68 LaPREP graduates, who became eligible, enrolled in college, and almost 90% of them responding to a survey indicated they were majoring in engineering, math, or science.

LaPREP has received honors both locally and nationally. Dr. Carlos Spaht, LaPREP founder and director, has received prestigious awards resulting from his work with LaPREP: The Jefferson Award for outstanding contribution to public service; the Jacqueline Kennedy Onassis Award, the highest public service award offered nationally; and the White House Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring. At the state and regional level, Dr.Spaht has been awarded the Carnegie Foundation's Louisiana Professor of the Year award and the LA/MS Mathematical Association of America's award for Distinguished Teaching of Mathematics and the Governor's Award of Excellence, all due in part to his work with LaPREP.

LaPREP has been recognized by the National Science Foundation, listing it in its directory of enrichment programs and the Mathematical Association of America, praising it for its contribution to mathematics in Louisiana. In addition, the Shreveport City Council passed a resolution applauding LaPREP "for making a positive impact on the lives of young people and for contributing to the future prosperity of the community and the nation."

The Cylindrical Column Problem

Many College Algebra texts contain a section on variation and accompanying exercises in which students must determine the constant of proportionality based on given information before they can complete the problem. As examples, one customarily finds problems about spring compression, volume occupied by a gas, amount of illumination produced by a light source, and the load supported by a cylindrical column. This last problem, somewhat simplified as found in Algebra for College Students (Lial and Hornsby, 2000)[2] is stated as follows: "It is shown in engineering that the maximum load a cylindrical column of circular cross-section can hold varies directly as the fourth power of the diameter and inversely as the square of the height. If a column 9 feet high and three feet in diameter will support a load of 8 tons, how great a load will be supported by a column 12 feet high and 2 feet in diameter?"

Of the typical exercises in a variation section in a College Algebra text, this last one seemed to be a good choice to do in a classroom setting because it combined the elements of relevance to real world engineering/construction issues, participation by all students, the use of mathematics, and the low cost of the required materials.

The problem as presented to the LaPREP students, however, was not one in which the constant of proportionality was determined by plugging in values for the height and diameter of the cylinder and then solving for k, but one in which they were to estimate the value of k through experimentation. The activity was designed so that the experiment would be repeated under conditions as identical as possible, then vary the dimensions of the cylinder and repeat the experiment. Since some variation in the value of the constant of proportionality was expected, it was decided to take the arithmetic mean of the estimates to obtain a final working value for k.

As an introduction, the use and importance of cylindrical columns in commercial and highway/bridge construction were discussed. Students were asked to consider the consequences if columns used in such applications were not manufactured according to specifications, especially with regard to the load supported by the column.

At the beginning of the experiment, each student was given three pieces of paper of different dimensions and asked to form cylinders using tape to join the edges. Two pieces were from the same ream of 8.5” x 11” 20-pound acid-free paper manufactured by Georgia Pacific, with one of these pieces being cut to 6” x 11”. The third piece of paper was from a ream of

11” x 14” 20-pound acid-free paper from the same manufacturer. Students were instructed to make three cylinders: (1) h = 6”, c = 11”; (2) h = 8.5”, c = 11”; (3) h = 11”, c = 14 using ordinary cellophane tape to join the edges. To reduce experimental error, specific instructions were given including how long the tape should be, on which side of the paper to place the tape, and how to join the edges to form the cylinder.

Each cylinder was examined for visual construction by some of the students playing the role of “quality control” inspectors. Cylinders with creases, excessive or insufficient amount of tape, incorrect placement of tape, and serious misalignment of paper edges were considered to have critical nonconformities, and were excluded from the testing.

From the cylinders that passed inspection, three of each size were randomly selected for inspection. Once selected, each was set on a horizontal (or near-horizontal) surface. A plastic cup was inserted in the top of the cylinder, and steel shot was slowly and carefully poured into the cup until deformation of the cylinder was observed. Again, students played the role of inspectors, with each student inspector being assigned an area of the paper cylinder to monitor. Once deformation was observed, no more steel shot was added. The cup and its contents were weighed on a Cardinal Detecto digital scale that measures in 0.05-ounce increments.

Results and Problems Encountered

The results of the experiment and the corresponding values for k follow:

Test #	Height	Diameter	Load	k
1.	6	3.5	57.9	13.9
2.	6	3.5	72.7	17.4
3.	6	3.5	70.2	16.8
4.	8.5	3.5	53.0	25.5
5.	8.5	3.5	54.0	26.0
6.	8.5	3.5	54.2	26.1
7.	11	4.5	41.7	12.3
8.	11	4.5	34.0	10.0
9.	11	4,5	34.5	10.2

After the data was collected and the values of the constant of proportionality k determined, the results were discussed with the students. Of particular interest was the variability in the values for k (see table) and the reasons for that variability. Under ideal conditions, the values would have been closer, as the same weight paper was used in the construction of each cylinder. Two sources of experimental error were identified -- nonconformities (defects) in the cylinder which were introduced during the actual construction of the cylinder and observer error.

In the paper cylinder construction phase of the activity, each student took a piece of tape and applied half of it, sticky side down, to one of the "height" sides of the rectangular piece of paper. The cylinder was formed by curving the other "height" side around to meet and align with the taped side, then pressing it to the exposed tape. Students quickly discovered that it was not easy to construct a perfect paper cylinder, and very few, if any, were without nonconformities. Some cylinders had critical nonconformities and were therefore excluded from the second phase of the activity.

Although cylinders with non-critical nonconformities remained in the pool, any nonconformity would be expected to have an effect on the load supported by the cylinder. Some of the most frequently occurring nonconformities found in cylinder construction included the following:

1. Paper edges (height sides) met, but were not in alignment at top and bottom of cylinder. These cylinders were, therefore, not perpendicular to surface of the table.
2. Paper edges (height sides) were in alignment at top and bottom of cylinder, but do not meet, leaving a gap between the edges filled in by the tape.
3. Excessive amount of tape used in connecting edges. When this occurred, the excess tape was not snipped off, but bent over one or both of the cylinder ends and stuck to the outside of the cylinder resulting in an "unauthorized" re-enforced (and stronger) seam.
4. Too little tape used in connecting edges, resulting in one or more cylinder ends subject to tearing when subjected to load.
5. Indentations, creases, or bends in paper prior to or during construction of cylinder. Cylinders had a tendency to begin pre-mature buckling at these sites when subjected to load.

The second potential source of experimental error, that due to the observer (inspector-student), includes the following:

1. Cylinders with one or more of the nonconformities listed above may have been accidentally " passed" by the inspectors.
2. The inspectors may not have observed buckling at the moment it began, resulting in an inflated load on the cylinder before the test was halted and an artificially high value for k.
3. The pouring rates for the steel shot may not have been the same from test to test. A faster pouring rate would result in more steel shot added after the inspector observed buckling as well as an artificially high value for k.

Summary and Conclusion

In summary, students were active participants in an experiment to estimate the constant of proportionality k in a variation problem as found in a typical college level algebra textbook. By being involved in cylinder construction and the quality control end of the project, students were able to see in a short time a considerable portion of the manufacturing process.

In addition, after the discussion on the possible sources of variability, the arithmetic mean of the values of k was found and used in a subsequent class exercise to estimate the load supported by a cylinder of given height and diameter.

References

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