



Developing a 3D-Printed Statics Modeling Kit

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Abstract

The 3D-Printed Statics Modeling Kit was designed, tested, and finalized within three months in the summer of 2021. The Kit includes multiple 3D-printed elements with corresponding activities that are designed to model real-life problems in Statics and help students better understand the main concepts of the class. Early prototypes included LEGO and wooden apparatuses that resembled 3D-coordinate grids as well as metal pulleys and fishing line. Due to the design flaws and complexity of the build, a 3D-printed model was formed. Using SolidWorks, customizable models were created, which could be easily modified after testing for flaws. The design from SolidWorks was exported into Ultimaker Cura and printed on an Ender 3 Pro.

Introduction

For most students, Statics is the first step into any engineering-specific class. Some students start this course with little to no experience with 3D problems and vectors, which puts them a step behind from the start. For students who do not have this prior knowledge or experience, Statics becomes a huge hurdle in their engineering career and may drive them away from the field in general. The 3D-Printed Statics Modeling Kit was designed to give students a better way to start visualizing 3D problems in Statics. This skill is fundamental to any upper-level engineering class and sets them up to succeed in the years to come. This Kit gives students the chance to catch up quickly with peers and allows them to develop fundamental skills in a hands-on setting.

The creation of this kit occurred over three months, during the summer of 2021. The research process began in June, with the initial prototypes developing in early July. Over the next month and a half, a final product was refined with the students' success in mind. Hands-on experience plays a key role in a student's comprehension of a topic, as many authors have shown [1] - [14]. The overwhelming number of positive results shown reinforced this concept, and a Kit to enhance student learning in Statics was created. Group projects were also demonstrated to help students better understand certain topics in Statics as shown in [4] - [7]. These group projects allow students the chance to get feedback from their peers in a way that might spark a new understanding of a topic. This was important to incorporate into a new activity and design.

Both force and equilibrium analysis of a system play a large role in Statics and beyond. Developing a Kit or activities that help students gain confidence in these topics is very important. In order to accomplish this experimentally, a high level of precision is needed. This precision is key to the Kit's learning potential, as the mathematical answer needs to match the experimental answer as close as possible. Another important factor in the development of the Kit was simplicity. This Kit should be easy to pick up and use for any student or professor who wants to

replicate an example problem with which students may struggle. Most importantly, customizability was key to the design of the Kit and corresponding activities. Hundreds of separate activities should be able to be created using this Kit, ranging from simple force vector calculations to more complex 3D Rigid Body Equilibrium problems. Some of these activities directly model problems that are shown in the textbook used in class [15].

Product

The 3D-Printed Statics Modeling Kit includes many 3D-printed supports, beams, and pulleys, as well as metal mass hangers and corresponding masses as shown in Figure 1.

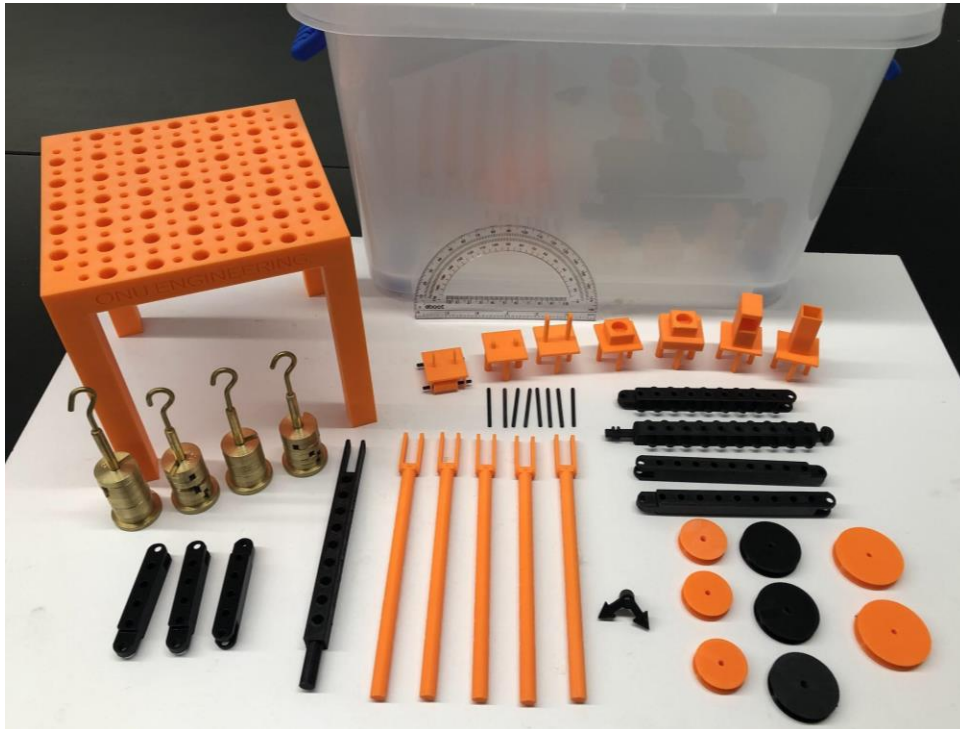


Figure 1: All 3D-printed parts and mass hangers included in a Statics Activity Kit.

String is used to create vectors and hold together the masses and beams and supports in any 3D equilibrium problem. The full Kit uses less than one kilogram of filament, which costs approximately \$25. To create the final Kit, SolidWorks was used to easily design each printed part. These files were then downloaded onto a Cura Ultimaker Ender 3 Pro 3D printer, which used filament at 70% infill to create the pieces. A full breakdown of filament usage is shown in Appendix A.

The Kit itself uses the table, pulleys and beams to demonstrate complicated textbook problems in Statics. Activities are set up using the Kit in many different ways and students are led through a series of complicated problems that are common in any Statics course. The Kit is designed to cover the topics of force vectors, dot product, particle equilibrium in 2D and 3D, as well as rigid

body equilibrium in 2D and 3D. Two sample activities given with the activity book are shown in Figure 2.

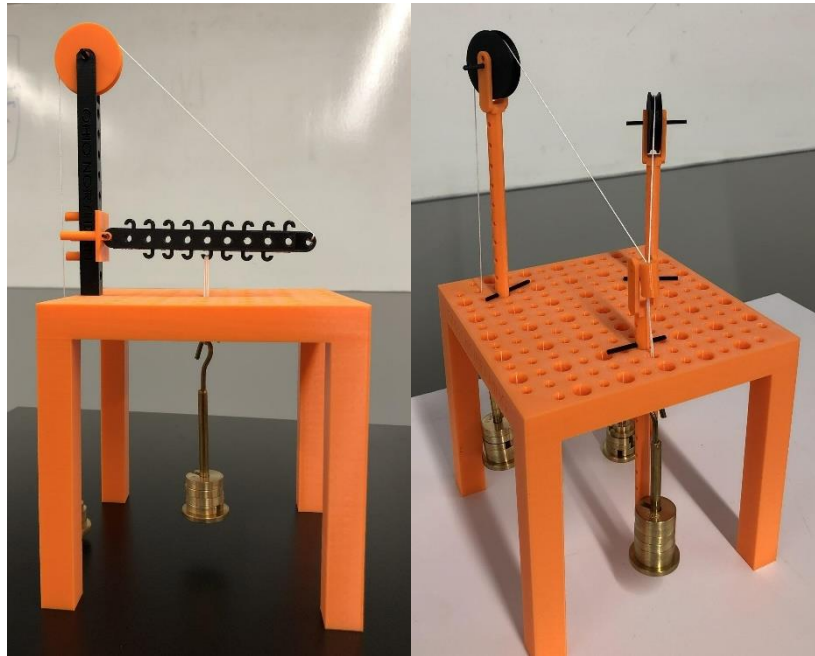


Figure 2: Demonstration of activities modeled by the Activity Kit. 2D Rigid Body Equilibrium is shown on the left, Dot Product is shown on the right.

Beginning Stages

From the beginning, a simple and easy-to-use 3D grid system was the goal. It was demonstrated before in [1] - [3] that any sort of design using a 3D coordinate grid as well as a way to demonstrate vectors is very helpful to students. The statics modeling kit described in [1] and [2] uses a plastic pegboard wall design to recreate one quadrant of a 3D cartesian grid. It uses metal hinges to make it foldable and easy to transport. Their beams and supports are made from ABS plastic using a Stratasys 3D printer. Their kit is designed to be customizable, as the supports are able to be screwed into any point on the wall. This design has advantages of transportability and customizability, but is limited to the positive axis only in the x, y and z directions. The supports and beams used also seem difficult to interchange. The activities described with this kit cover many different concepts, including unit vectors, particle and rigid body equilibrium in both 2D and 3D, as well as moment equilibrium.

In [3], a more heavy-duty apparatus used in a lab setting is introduced. They use metal for their main kit and wood for a newer, less bulky option. One activity they demonstrated with their kit was their Particle Equilibrium activity. This activity used pulleys that hung interchangeable masses below the surface of the table. This setup allowed for measurable forces in vectors and

allowed for customization and experimentation if needed. The kit described covers particle equilibrium, vector cross product, and vector dot product.

These authors show very good examples of their own kits that demonstrate equilibrium problems in many forms. For a future design, the kit would need to be accurate and easy to manufacture or replicate. The kit shown in [3] was a little more heavy duty compared to the plastic kit shown in [1] and [2], as it needed to be used in a lab setting. The use of walls added to the bulkiness, but is necessary for this type of design.

To start prototyping, a large LEGO platform was used with four individual walls set up to simulate a 3D grid system as our prototype. This gave four quadrants in the coordinate system all with a positive z component, and a grid for coordinates on each wall. At each point of the grid, small washer-like pieces were used to hold a string in place. This would give a way to hang a mass in order to hold objects as seen in a typical problem. A simple pulley was also tested with LEGOs in a different setting, as shown in Figure 3.

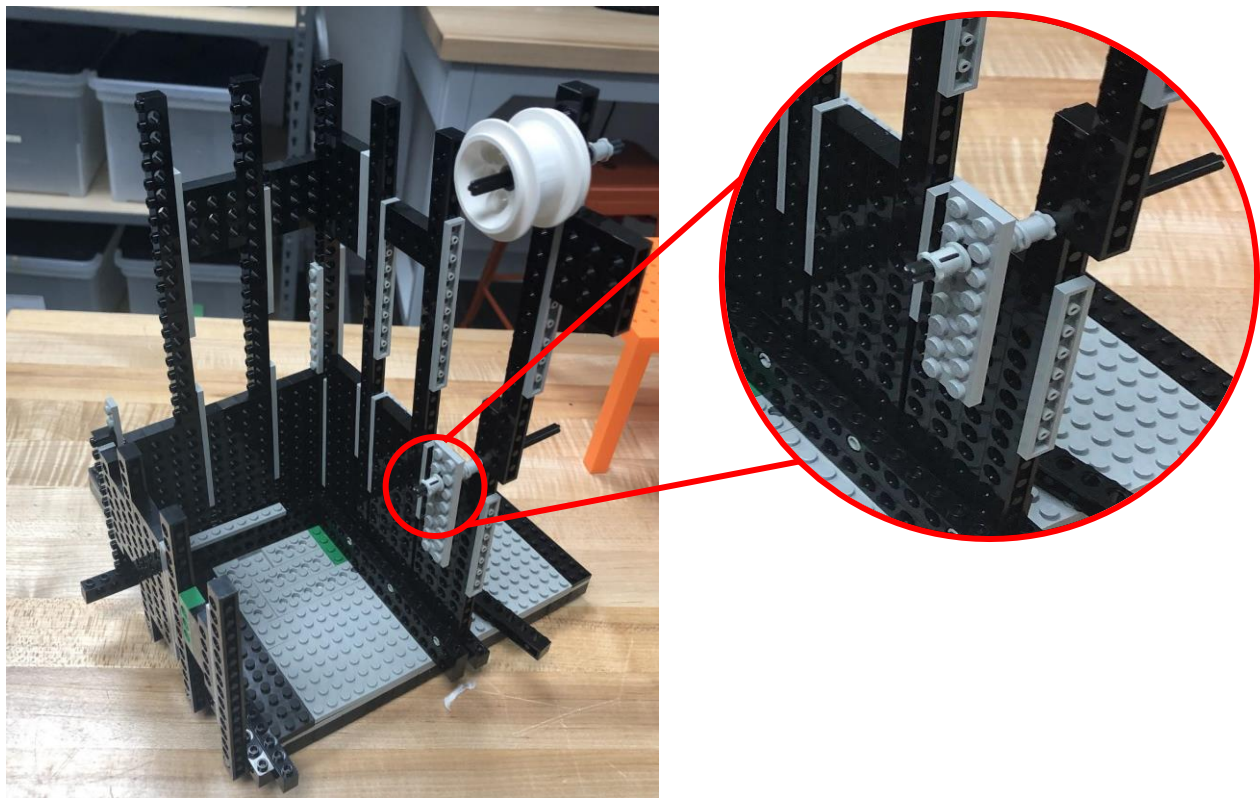


Figure 3: Demonstrates a single quadrant of the four quadrant LEGO design. An enhanced image of the washer-like piece used to keep a string in place next to a point on the grid.

This was a good first design, but still had some drawbacks. To start, it took a lot of time to put together and would be difficult to replicate perfectly. Some extra pieces that do not exist were needed in order to accurately model the coordinate system on the wall. The part needed would

have allowed us to connect the LEGOs at a central axis, keeping the walls stable and at an equal distance apart. With the rungs at mismatching distances apart, inaccurate answers were common when the prototype was tested. Also, the LEGOs could not support a lot of weight perpendicular to the axis they represented. The wall would peel from the base and was not usable for the design. The string hanging from the pulley would also come in contact with the wall, throwing off the results of the experiment. Due to these flaws, this idea was thrown out. After our first LEGO experiment, it was decided that using grid-like walls would add to the bulkiness of our design and, as useful as they were, there could be another way to replicate them.

The next idea was using a wooden table as seen in [3] along with metal pulleys to act as walls and mass hangers to provide tension in vectors. First, we determined which size hole fits the pulleys the best since they seemed to have different diameters for a base. To do this, one scrap board was used to drill various-sized holes, as seen in Figure 4.

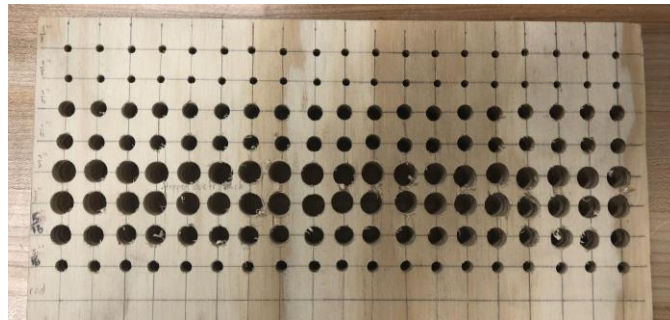


Figure 4: Various sized holes ranging from 0.25-in. diameter to 0.625-in. diameter.

The reason for a whole row of holes, as opposed to a single one for each size, was to see how the pulleys worked while in a 2D equilibrium problem. Aesthetically, the looks of rows with different size holes were examined. From the first experiment with these pulleys, it was determined that they worked as envisioned, and a full table was made to see how the Kit with wood and metal pulleys worked. This was made using 0.5-in.-diameter holes as major grid marks, with 0.25-in.-diameter holes for the minor ones, as shown in Figure 5. These diameters were determined as the pulleys fit the best in 0.5-in. holes, and the 0.25-in. holes were sufficient as a compliment to the others.



Figure 5: First full table assembly, made with scrap wood.

At first glance, the wooden assembly worked very well. It was easy to make, and could have been much easier with the use of machining rather than a drill press. The pulleys allowed for an adjustable z-axis and problems could be replicated much easier. While this was great progress, some flaws were also very noticeable. First, the pulleys could be pulled down if enough weight was added, and they would slide before equilibrium at a designated height. This made completing any problem rather difficult. It would eventually stop at a certain height due to friction, but it was not the height needed for the problem. Also, it was very difficult to get it to the correct height to begin with. A way to set the pulley at a height and keep it there no matter how much weight was added was needed. Research was done for pulleys that could be held at any height, but most of what was found clamped the shaft in only one spot. This would limit the customizability of the Kit, which is not ideal. Another solution discovered was a clamp that could be adjusted manually to any spot on the shaft. This would hold the pulley wherever it was set above the table and keep the Kit customizable; however, it was difficult to get the right height initially. A ruler would be needed to set the pulley to the correct height each time, which can be tedious as the wheel of the pulley has a curve to it in order to keep the string in the correct position.

In prior research, authors use 3D printers in order to get parts quickly and accurately [1], [2], [4], [11]. In [1] and [2], a 3D printer was used to make some supports and beams, while using a plastic pegboard as their grid-like walls. This seemed to work quite well, as any kind of rigid body could be used in a class demonstration. In [4], a truss was designed using a 3D modeling software, and a 3D printer gave them their final product. This truss' strength was tested and it was shown that it could withstand a large amount of force. In [11], the author demonstrates a simple 3D-printed beam that is flexible enough to see beam bending with a smaller sample. These ideas helped us form the initial idea for a 3D-printed pulley.

First Printed Prototype

The ability to use SolidWorks assemblies and quickly print a part was crucial to the success of the design. The first pulley was created with a radius of 0.25 in. and was around 7 in. tall. A few holes were included in the shaft in order to set a pin inside of it and hold it at a certain height. This design is shown in Figure 6. The original wheel was 0.5 in. in order to align with the holes in the table to hang a mass through it. This design worked well, as it allowed the pulley to stay at whatever height was needed. While this was great, using the pulley in the wooden table was difficult, as there was wiggle room between the pulley and the tabletop. To correct this, a small table was designed and printed in order to see what hole size would work the best.



Figure 6: Shows the holes in the pulley shaft to set a certain height.

As different hole sizes were tested, it seemed that not every hole had the same dimensions on the table. It was printed upside down on the printer because it could not print the whole table without the need for extra printed supports. This would waste filament and be more difficult to keep the design clean from leftover filament. The top face of the table ended up being a little warped as it was the bottom layer on the face of the printer. As the printer places the partially melted filament, it pushes down slightly to set it in place. This motion made the melted plastic expand outwards on the bottom few layers of the print, leaving the top layers of the hole a little smaller than the rest of it. When the holes in the table were correctly adjusted to the radius needed, the lip on the top face caused the same wiggle room as attempted to eradicate from the wooden design. An image of this table is included in Figure 7.

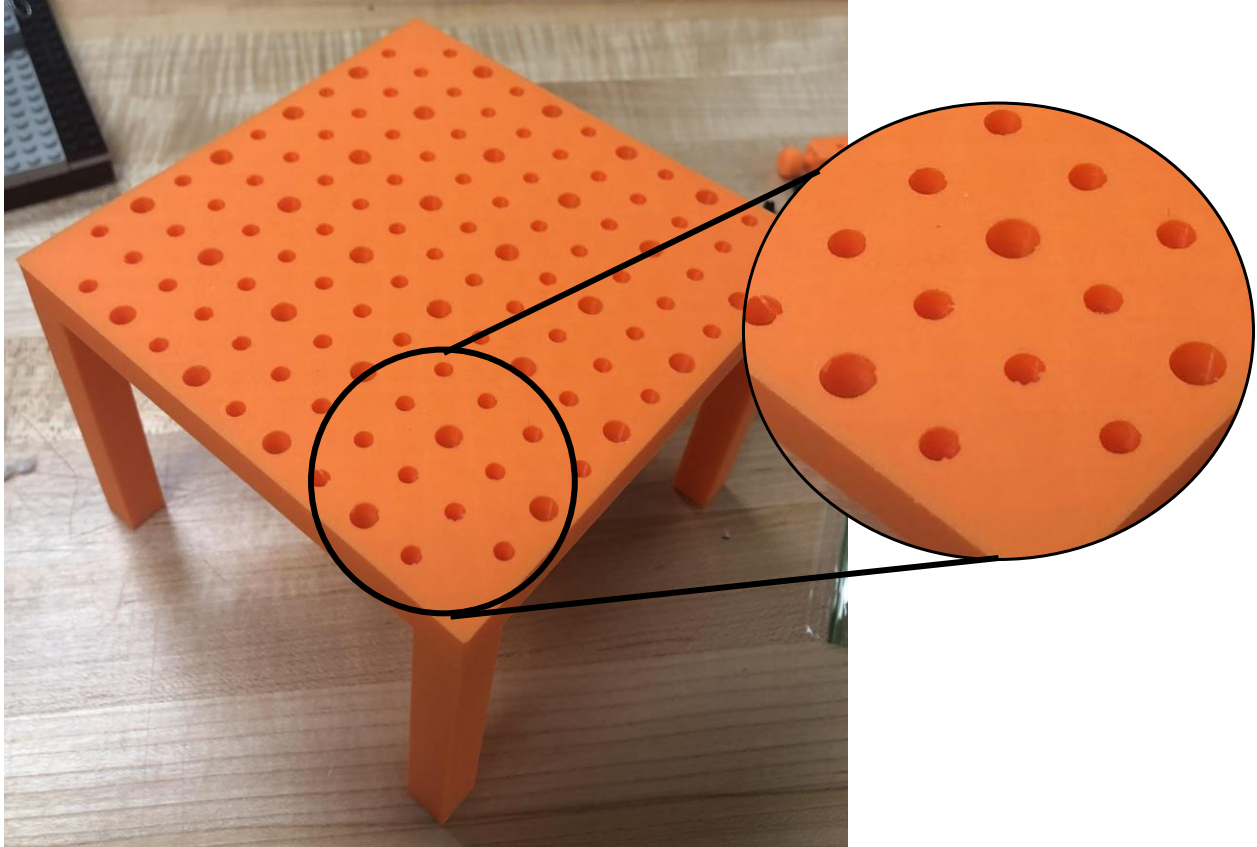


Figure 7: The first table design. Around the edges of the holes, small bits of plastic remained after pulling it off the printer as shown in the bubble.

The best solution to this problem was using a countersink in the design of the holes on the table. This allowed the lip to form, as it seemed to be part of the printing process, but kept the hole accurate to the dimension that it was designed to be. After making this adjustment and keeping the radius of the hole at 0.25 in. as originally proposed, the pulley started to work as planned and the design was tested further. Multiple tests were done using the mass set and string to replicate 2D and 3D equilibrium problems.

In these tests, the shaft of the pulley bent with any mass hung. This threw off the accuracy of the problems, as the pulley wheel was pulled closer to the origin and the string was rubbing up against the holes in the table. The initial idea to fix this was trying different printing techniques. It was advised that printing vertically would allow the shaft of the pulley to be stronger, but also more brittle. This was experimented with briefly, but printing a small and thin shaft made the print inaccurate without support and was much easier to snap than expected, even with the supported print.

The next option was to increase the infill of the shaft of the pulley in hopes of strengthening it. This would ideally cause the pulley to be more stiff. With the original diameter, the increased infill did not help much, so the next best option was to expand the radius of the shaft and holes in

the table in order to keep it more sturdy and still be able to print horizontally as originally intended. A new diameter of 0.375 in. was designed and tested in the new table. This time, the table was kept the same throughout the rest of the testing, and smaller shafts were made to get the correct size first, as shown in Figure 8.



Figure 8: Images of testing shafts used to get the correct diameter used in the final product.

It was decided that shafts with the diameters between 0.37 in. and 0.365 in. were close enough to print full pulley shafts. These six shafts were all tested and though they were all close, the best two were 0.365 in. and 0.366 in. and could be used interchangeably in each hole. Using two different pulley diameters to account for the tolerance in hole size was the best option, as some holes ended up being too loose for the 0.365 in. shaft or too small for the 0.366 in. shaft.

Kit Refinement

With the shaft and table finished, more testing was done into what can be added to make the Kit more dynamic. The current setup allowed for particle equilibrium demonstrations in 2D and 3D by tying strings together and hanging masses over the pulleys. More options were needed to look into rigid body equilibrium as well. This carried into the beam and support development.

The first two beams made were the 2.5-in. and 5-in. beams. Each beam was made with the holes through the middle at 0.5-in. increments to give the same customizability as the pulley shaft. With this, a mass could be hung at those increments in the beam and create a moment problem. Each end of the beam also had the ability to connect a pin support to make the demonstration as accurate as possible. The only downside to this design was that it was difficult to pull a string through the hole in the beam. The best option to fix this was to add hooks at the same 0.5-in. increments that make it easy to hang a mass on and keep the problem at simple increments to measure. This also allowed the mass hanger to be hung below the table as intended, as the string would then hang through each hole on the table as shown in Figure 9.

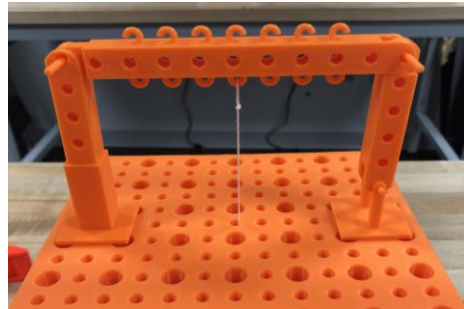


Figure 9: Image of beam demonstration with accurate hooks.

While making the 5-in. and 2.5-in. beams, the ball-and-socket beams were developed. This beam would be able to snap into place while still revolving 360 degrees. Lots of trial and error occurred while creating this, and many prototypes were printed. Some were too loose, some too tight, some were too weak and snapped. After many tests, as shown in Figure 10, the correct dimensions were found, and the socket end was left alone. The rest of the beam was then created with hooks at 0.5-in. increments.

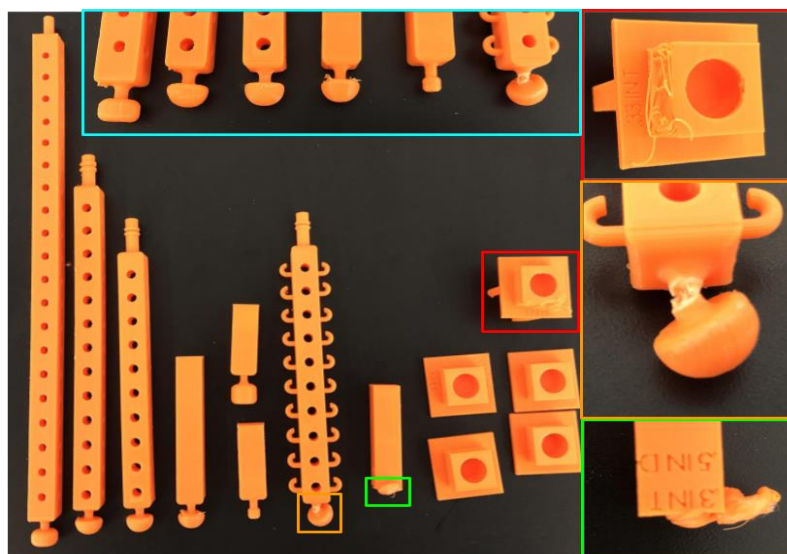


Figure 10: Examples of ball-and-socket tests with failures in order to get to the final product.

The last design was a way to get a pin or ball-and-socket support elevated enough to create a rigid body equilibrium problem with a beam attached to a wall at a certain height. This height needed to be adjustable with the ability to use pin supports at different heights. With that in mind, the peg beam was created. This beam allows supports to slide into a wall while not permanently taking up too much space on the table. The peg beam also has a pulley connection at the top end so that a 2D moment problem can be created. The peg on the bottom that slides into the table is the same size as the pulley shaft, and the beam has the same cross section as the 2.5-in. and 5-in. beams. This allowed for a quick design without too much trial and error. This trial and error is shown in Figure 11. The only issue with the holes in the peg beam is the supports became very loose if any weight was added to them, and they would fall out rather quickly.

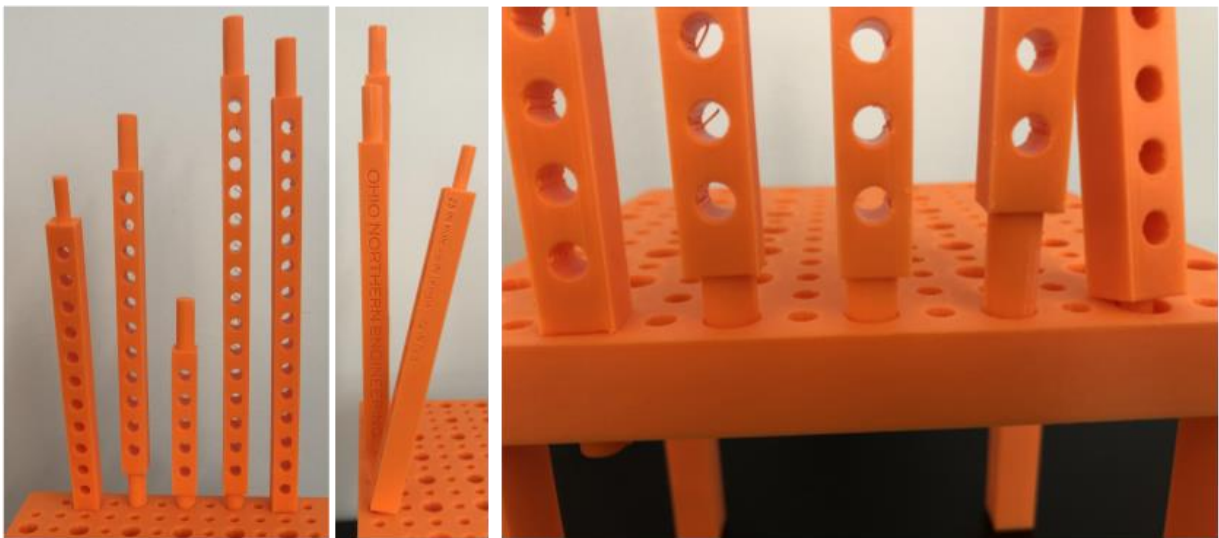


Figure 11: Issues in the beginning stages of peg-beam testing. Some pegs were too loose, some would not fit correctly.

To counteract this, additional supports were created with larger pegs on the bottom that would fit more snugly into these holes. This way, the supports needed for the peg beam were easily identifiable and the Kit was kept as accurate as possible.

A smaller part of the Kit was the axis marker. The purpose of it was to establish an origin and still keep every hole in the tabletop useful. To do this, a small peg with a 0.5-in. hole in the middle of it was made. This hole was designed to allow masses connected above the origin to still hang below the tabletop. Furthermore, the axis marker was equipped with arrows to denote the x and y axes. An arrow pointing upward as the z axis was originally added but it broke rather easily, and it was assumed that most students can recognize that the z-axis would be directly up from the point of origin. This is shown in Figure 12.



Figure 12: Axis marker.

Each design described was an important step to take while working to the final product. Table 1 shows a design matrix that describes how each step applied to the original design statement. As shown, the final 3D printed design worked the best for the desired outcome and was developed into a full scale Kit.

Table 1: Design Matrix comparing all designs to the original expectations.

Design	Customizable	Replicable	Accurate	Easy to Use
LEGO	Yes	Difficult	No	No
Wooden	Yes	Yes	Yes	No
First 3D Print	Yes	Yes	No	Yes
Final 3D Print	Yes	Yes	Yes	Yes

Activities

Six activities were created to guide students through the first five chapters of Statics as seen in the textbook [15]. The first activity covers force vectors using three pulleys in the table. Using the masses hanging from each pulley, as well as the positions of the head and tail, students were instructed to determine the force vector describing each string in Cartesian form. This was done using the setup seen in Figure 13.

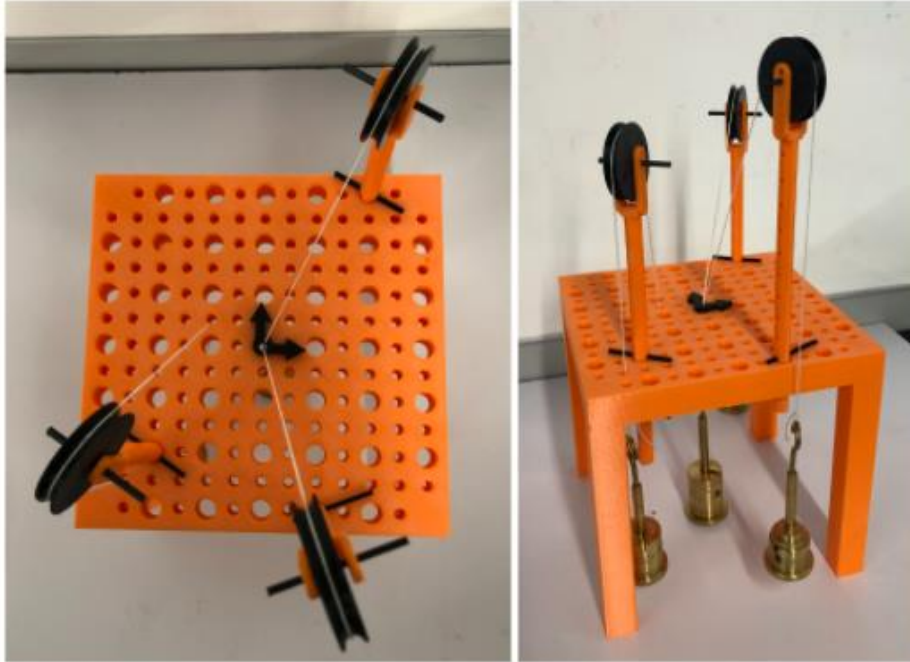


Figure 13: Images of the force vector activity setup.

The second activity guides students through dot products between two vectors. This uses two pulleys, a different pulley shaft and a single roller wheel, as shown in Figure 14, and asks students to determine the angle between the two vectors.

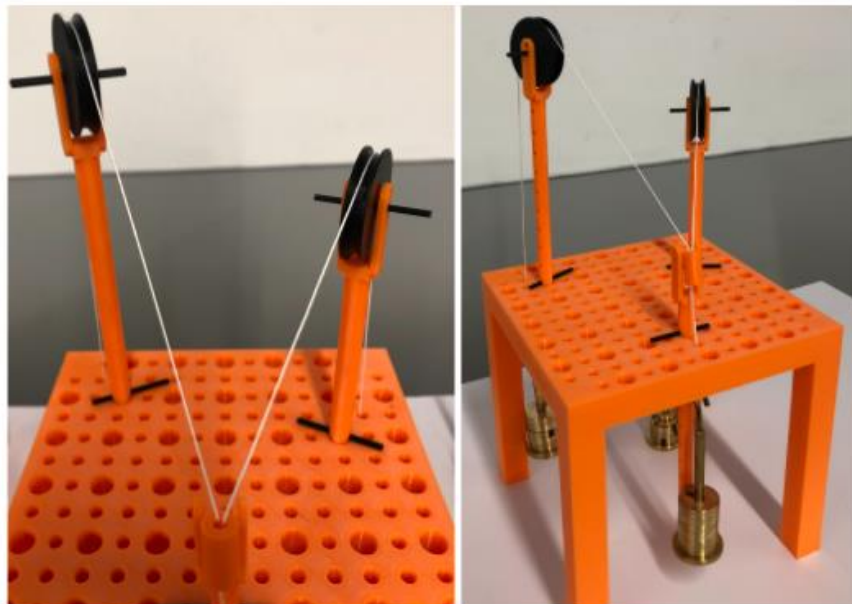


Figure 14: Images of the dot product activity setup.

The third and fourth activities lead students through particle equilibrium in both 2D and 3D. They both use pulleys and masses, as shown in Figure 15. The students are given one or two masses and need to use the equations of equilibrium to determine the remaining masses.

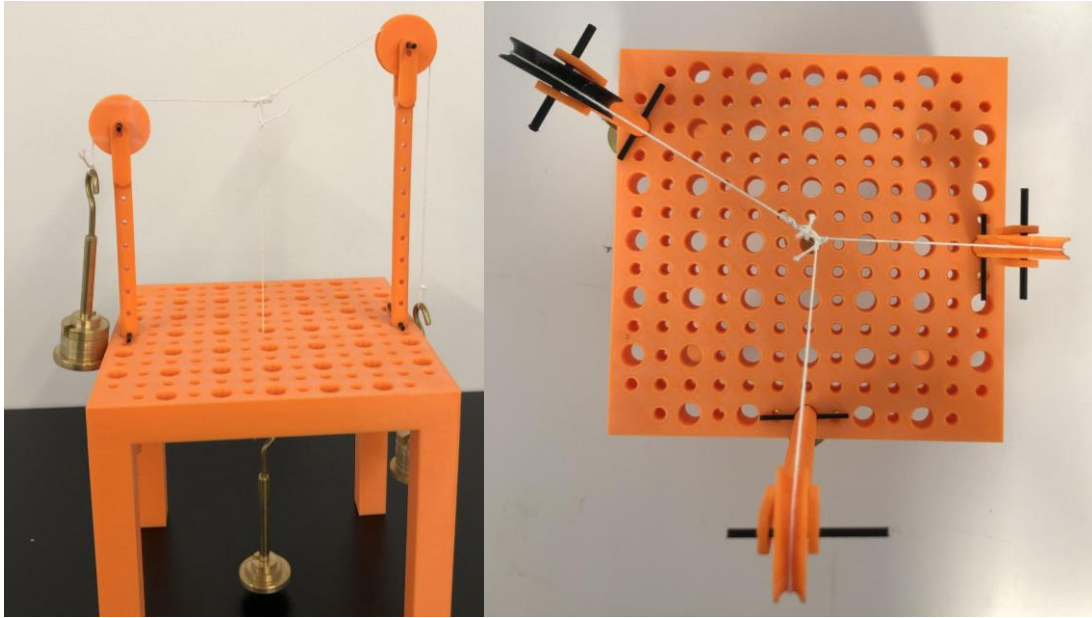


Figure 15: Images of the particle equilibrium in 2D (left) and 3D (right) setup.

The fifth activity helps students through rigid body equilibrium in 2D. It uses the peg beam, horizontal pin support, and a 5-in. beam with hooks. With a mass hanging over the wheel on the peg beam and another hanging from the middle of the beam as shown in Figure 16, the students need to use the moment about the pin to determine the mass hanging over the pulley.

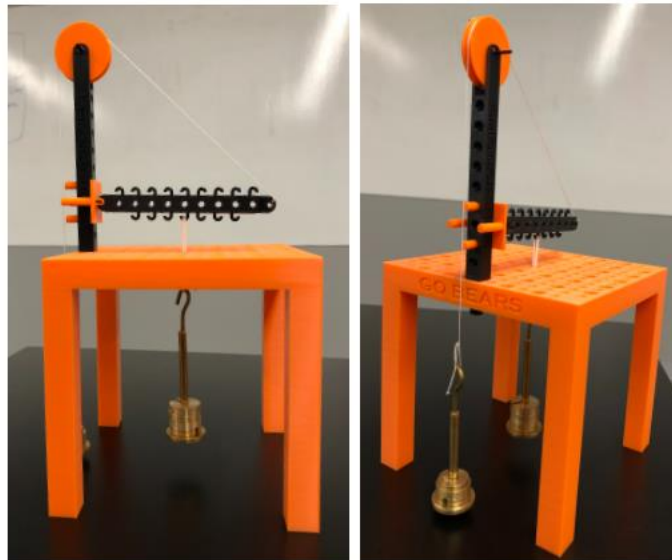


Figure 16: Images of the 2D rigid body equilibrium activity setup.

The sixth and last activity regards rigid body equilibrium in 3D. This uses the peg beam, two pulleys and the horizontal ball-and-socket beam setup shown in Figure 17. The goal of this activity is to use the equations of equilibrium to find the forces in each of the three vectors in the demonstration.

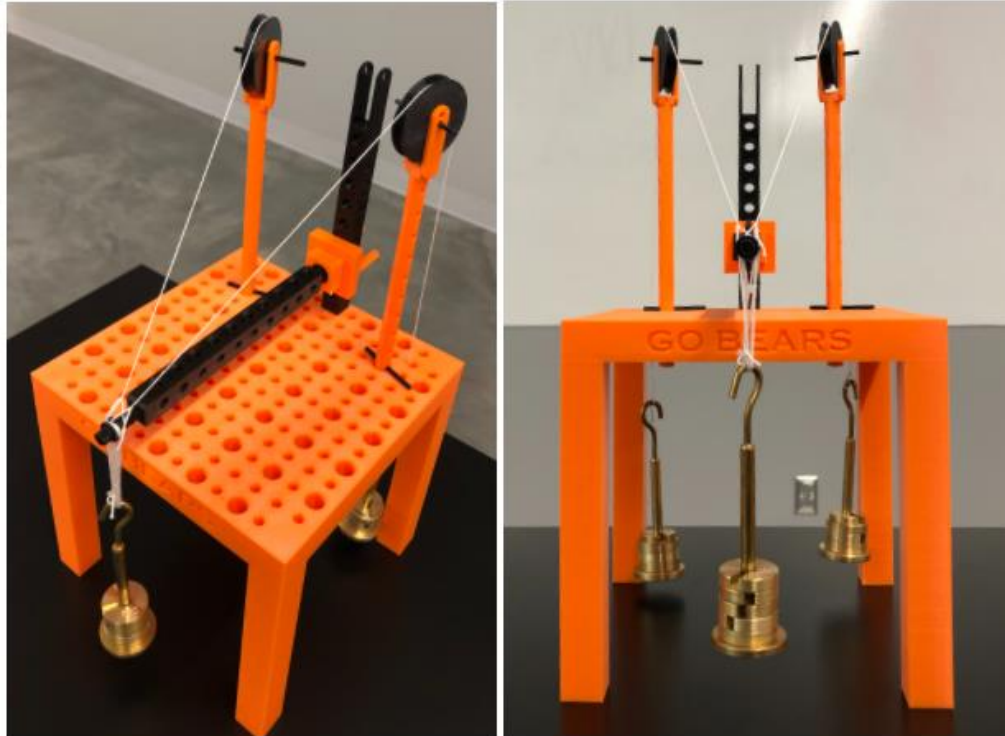


Figure 17: Images of the 3D rigid body equilibrium activity setup.

Results

The Activity Kit was used in the 2021 fall semester in all sections of the course. Compared to the two years before, students who used the Kit throughout the semester saw an average increase of 7.36% on their first exam and an average increase of 7.08% on their second exam. These two metrics strongly indicate the improvement of students' grades with the use of the Kit as the activities completed in the class only pertained to the first and second exam periods.

A survey was also given to students regarding how they liked the Kit and activities overall. For each activity, students were given statements such as "This activity helped me prepare for the exam" and "This activity helped me understand the free-body diagram". Students gave answers ranging from "Strongly Agree" to "Strongly Disagree" based on how they felt about the statement. The results of this (shown in Figure 18) show that students felt the activities helped their understanding of the topics covered and enjoyed the activities completed.

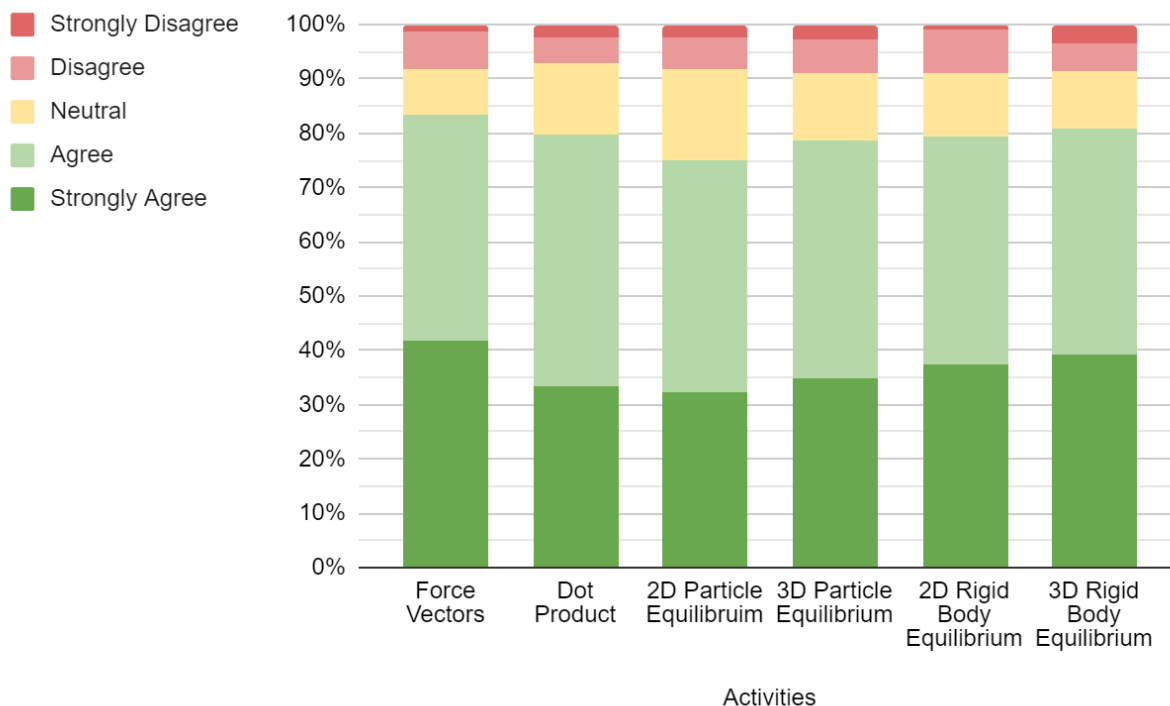


Figure 18: Breakdown of student responses from the survey given regarding the Activity Kit and corresponding activities. A question-by-question breakdown is included in Appendix B.

Conclusion

The 3D-Printed Statics Modeling Kit and corresponding activities are beneficial to students' performance and comprehension of topics in the course. The process of developing this Kit kept the students' best interest in mind, which set them up for success. SolidWorks played a large part in the design of the Kit as it allowed us to assemble parts of the Kit without wasting resources to print. Sometimes, the computer-generated models did not exactly match up to what was actually printed due to the level of accuracy needed. In this case, the use of a 3D printer made the trial and error of real-life models easy and allowed us to move forward at a fast pace. Because this Kit was designed quickly, it was able to be used in the fall semester, and students were able to grasp the topics in Statics and gain a full understanding of important concepts to build on in higher-level engineering courses.

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Appendix A: 3D Print Filament Metrics

Item	Amount Needed Per Table	Grams (each)	Grams (per table)
Axis Markers	1	2	2
Ball and Socket (Side)	1	15	15
Ball and Socket (Up)	1	14	14
Ball and Socket Beam	1	21	21
Beam (5) hooks	1	18	18
Beams (2.5)	3	7	21
Beams (5) no hook	2	15	30
Big Roller Wheels	1	3	3
Fixed Support (side)	1	16	16
Fixed Support (up)	1	17	17
Peg Beam	1	18	18
Pegs (1.5)	12	0.5	6
Pin Support (Side)	1	8	8
Pin Support (Up)	1	13	13
Pulleys	4	11	44
Roller Peg	1	5	5
Roller Wheels	2	1	2
Table	1	373	373
Wheels (.5)	4	4	16
Wheels (.65)	4	6	24
Wheels (.75)	1	8	8
	Totals:	575.5	674

Appendix B: Survey Results

Force Vectors	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
The activity helped me understand position vectors and express force vectors in Cartesian Form:	42.86%	39.29%	10.71%	7.14%	0.00%
The activity helped me visualize vectors in 3D:	53.57%	39.29%	3.57%	3.57%	0.00%
The activity helped me prepare for the exam:	28.57%	46.43%	10.71%	10.71%	3.57%
Dot Product					
The activity helped me understand the dot product and its application:	32.14%	42.86%	17.86%	3.57%	3.57%
The activity helped me visualize vectors in 3D:	42.86%	42.86%	7.14%	7.14%	0.00%
The activity helped me prepare for the exam:	25.00%	53.57%	14.29%	3.57%	3.57%
2D Particle Equilibrium					
The activity helped me understand the equilibrium of a particle in 2D:	32.14%	42.86%	17.86%	3.57%	3.57%
The activity helped me understand the free-body diagram:	35.71%	46.43%	7.14%	10.71%	0.00%
The activity helped me prepare for the exam:	28.57%	39.29%	25.00%	3.57%	3.57%
3D Particle Equilibrium					
The activity helped me understand the equilibrium of a particle in 3D:	39.29%	42.86%	7.14%	7.14%	3.57%
The activity helped me visualize vectors in 3D:	39.29%	42.86%	10.71%	7.14%	0.00%
The activity helped me understand the free-body diagram:	32.14%	42.86%	14.29%	7.14%	3.57%
The activity helped me prepare for the exam:	28.57%	46.43%	17.86%	3.57%	3.57%
2D Rigid Body Equilibrium					
The activity helped me understand the equilibrium of a rigid body in 2D:	42.86%	32.14%	17.86%	7.14%	0.00%
The activity helped me understand moment:	39.29%	39.29%	10.71%	10.71%	0.00%
The activity helped me understand the free-body diagram:	35.71%	50.00%	7.14%	7.14%	0.00%
The activity helped me prepare for the exam:	32.14%	46.43%	10.71%	7.14%	3.57%
3D Rigid Body Equilibrium					
The activity helped me understand the equilibrium of a rigid body in 3D:	39.29%	42.86%	7.14%	7.14%	3.57%
The activity helped me visualize vectors in 3D:	39.29%	46.43%	7.14%	3.57%	3.57%
The activity helped me understand moment in 3D systems:	46.43%	35.71%	7.14%	7.14%	3.57%
The activity helped me understand the free-body diagram:	39.29%	42.86%	10.71%	3.57%	3.57%
The activity helped me prepare for the exam:	32.14%	39.29%	21.43%	3.57%	3.57%

Appendix C: Activity Book Instructors Copy

Activity 2 (Force Vectors)



Materials

- Three Pulley Shafts
- Six Pegs
- Three Wheels
- Four Mass Hangers
- Weights as needed
- Three Strings
- Axis Marker

Set Up

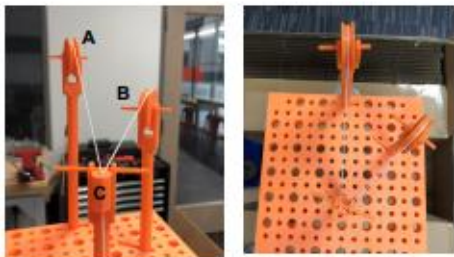
- Make sure the Axis marker is placed at (0,0)
- Set up three pulleys with wheels
 - Pulley A will be placed at (1,-3) at a height of 6.5 inches or the last hole (5 wheel)
 - 0.5 Wheel through the top hole on the pulley
 - Other end through the origin
 - Add 75 grams to make 125g total
 - Pulley B will be placed at (2,3) at a height of 5 inches or the 4th hole (.65 wheel)
 - 0.65 wheel through the bottom hole on the pulley
 - Other end through the origin

- Add 45 grams to make 95g total
 - Pulley C will be placed at (-2,-2) at a height of 5 inches (.65 wheel)
 - 0.65 wheel through the bottom hole on the pulley
 - Other end of the string goes through the point (-1,0.5)
 - Add 100g to make 150g total
- Mass hangers should be attached at all ends and the third pulley will need to be attached to a peg underneath the table to hold the string there.
- Set Up Time
 - 10 Minutes
- Activity Time
 - 13-15 minutes

Answers

- Vector DA
 - Tail Coordinate : 0,0,0
 - Tip Coordinate : 1,-3,6.5
 - Position Vector : $1\mathbf{i}-3\mathbf{j}+6.5\mathbf{k}$
 - Unit Vector : $0.138\mathbf{i}-0.415\mathbf{j}+0.899\mathbf{k}$
 - Mass on Hanger : 125g
 - Force Vector : $17.3\mathbf{i}-51.9\mathbf{j}+112.4\mathbf{k}$
- Vector OB
 - Tail Coordinate : 0,0,0
 - Head Coordinate : 2,3,5.5
 - Position Vector : $2\mathbf{i}+3\mathbf{j}+5.5\mathbf{k}$
 - Unit Vector : $0.3\mathbf{i}+0.46\mathbf{j}+0.84\mathbf{k}$
 - Mass on Hanger : 95g
 - Force Vector : $29.9\mathbf{i}+43.3\mathbf{j}+79.45\mathbf{k}$
- Vector EC
 - Tail Coordinate : -1,0,5.0
 - Head Coordinate : -2,-2,5
 - Position Vector : $-1\mathbf{i}-2\mathbf{j}+5\mathbf{k}$
 - Unit Vector : $-0.176\mathbf{i}-0.44\mathbf{j}+0.88\mathbf{k}$
 - Mass on Hanger : 150g
 - Force Vector : $-26.4\mathbf{i}-66.0\mathbf{j}+132.0\mathbf{k}$

Activity 3 (Dot Product)



Materials

- Two Pulley Shafts
- Four Pegs
- Two Wheels
 - Both 0.65 diameter
- Three Mass Hangers
- Weights as needed
- Two Strings
- Roller Wheel
- Axis Marker
- Protractor

Set Up

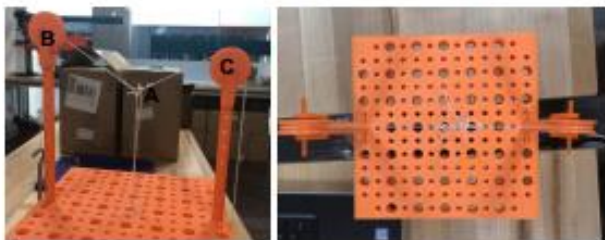
- Place the Axis Marker at (0,0)
- Place Three Pulleys
 - Pulley A is at (0,3) at the maximum height (6.5 inches)
 - 0.65 Wheel through the bottom hole
 - Pulley B is at (2,2) at 5 inches (4th hole from the bottom)

- 0.65 Wheel through the bottom hole
 - Pulley C is at (0,-1) at the 5th hole from the top (3.5 inches no peg)
 - This will have no wheel but still have a peg
 - Peg will be in the top hole of the pulley
- The strings should meet and be fed through the roller wheel at the top of Pulley C
- Each side of the pulley should be attached to a mass hanger
- Give each table a protractor
- Set Up Time
 - 5 Minutes
- Activity Time
 - 10 Minutes

Answers

- Vector CA: $3\mathbf{i}+4\mathbf{j}+3\mathbf{k}$
- Vector CB: $2\mathbf{i}+3\mathbf{j}+1.5\mathbf{k}$
- Unit Vector CA: $0\mathbf{i}+0\mathbf{j}+0\mathbf{k}$
- Unit Vector CB: $5/12\mathbf{i}+7/6\mathbf{j}+1/4\mathbf{k}$
- $CA \cdot CB = 9+12+4.5k = 16.5$
- $|CA| = 5$
- $|CB| = 3.9051$
- $\theta = 32.33$
 - Measured 32

Activity 4 (2D Equilibrium)



Materials

- Two Pulley Shafts
- Four Pegs
- Two Wheels
 - Both 0.5 Diameter
- Three Mass Hangers
- Assortment of weights
- Three Connected Strings

Set Up

- Set two pulleys into the table
 - Pulley B should be at (3,0) at Max Height
 - 0.5 Wheel through the top hole on the pulley
 - Pulley C should be at (-3,0) at 5 inches (4th)
 - 0.5 Wheel through the top hole on the pulley
- Use the three connected strings to assemble the vectors like shown.
 - Each should have Mass Hangers attached to the ends
 - Central Mass Has 65g total (15g added)
 - Pulley B has 95 g total (45g added)
 - Pulley C has 80 g total (30g added)

- Point A should be at (0,0,4.5) but will be considered the origin
- Set Up Time
 - 5 Minutes
- Activity Time
 - 10 Minutes

Answers

- Check FBD
- Angle Between AB and X axis: 33.7
- Angle Between AC and X axis: 9.5
- Force Vector AB: $95(\cos 33.7^\circ)\mathbf{i} + 95(\sin 33.7^\circ)\mathbf{j}$
 - $79\mathbf{i} + 52.7\mathbf{j}$
- Weight @ B: 95g
- Weight @ C: 80g
- Weight : 65g
- All should be within a gram or two

Activity 5 (3D Equilibrium)



Materials

- 3 Pulleys
- 3 Wheels
 - 2 0.5 diameter wheels
 - 1 0.65 diameter wheels
- 6 Pegs
- 4 Mass Hangers
- 4 Connected Strings
- Axis Marker

Set Up

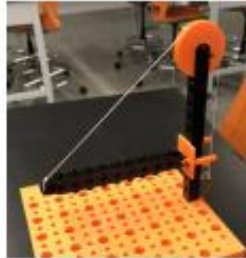
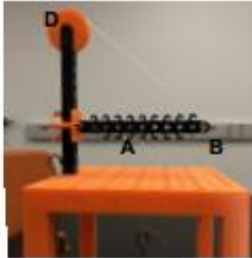
- Set the Axis Marker in the origin
- Place three pulleys with wheels
 - Pulley B should be placed at (3,0) at a height of 3.5
 - 0.5 Wheel through the top hole on the pulley
 - Pulley C should be placed at (0,-3) at a height of 6.5
 - 0.5 Wheel through the top hole on the pulley
 - Pulley D should be placed at (-3,2) at a height of 4.5
 - 0.65 wheel through the bottom hole on the pulley
- Place weight on each mass hanger
 - 170g total at B
 - 145g total at C
 - 210g total at D

- 150g total at the Central Mass
- Adjust for equilibrium (String should be in the middle of the hole at E with the point A equal in height to B)
- Set Up Time
 - 10 Minutes
- Activity Time
 - 15 Minutes

Answers

- AB in Cartesian: $-3\mathbf{i}, 0\mathbf{j}, 0\mathbf{k}$
 - Mass: 170
 - Force Vector: $-170\mathbf{i}$
- AC in Cartesian: $0\mathbf{i}, 3\mathbf{j}, 2.5\mathbf{k}$
 - Unit Vector: $0\mathbf{i} + .77\mathbf{j} + .64\mathbf{k}$
- AD in Cartesian: $3\mathbf{i}, -2\mathbf{j}, 1\mathbf{k}$
 - Unit Vector: $.5\mathbf{i} - .53\mathbf{j} + .27\mathbf{k}$
- Weight in Cartesian: $-1\mathbf{k}$
 - Unit Vector: $-1\mathbf{k}$
- Calculated Masses should be within 2 grams of given Masses
 - Weight: 150
 - AB: 170
 - AC: 145
 - AD: 210

Activity 6 (Rigid Body Equilibrium in 2D)



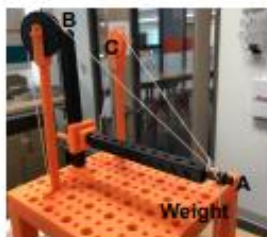
Materials

- Peg Beam
- 2 Pags
- Horizontal Pin Support
- 5 inch Beam with Hooks
- .75 Wheel

Set Up

- Set the Peg Beam in an edge hole
 - Place the .75 Wheel into the slot at the top of the beam
- Set the horizontal pin connection 2 inches from the table (pins in holes 3 and 5)
- Connect a 5 inch beam with hooks in the pin connection like shown
- Tie a string into the end of the beam at point B and connect it around the pulley wheel and around the back to connect to a mass hanger
 - Add 50g to the hanger to make a total of 100g
- The second string should connect to the hook 1.5 inches from the peg beam and connect to the mass hanger straight down under the table.
 - 70g were added to make equilibrium
- Add weight to make equilibrium.

Activity 7 (Rigid Body Equilibrium in 3D)



Materials

- Peg Beam
- Two Pulleys
- Two (.85) Wheels
- Ball and Socket Horizontal Peg
- Ball and Socket Beam
- 4 Pags

Set Up

- Set the Peg Beam in a central edge hole
- Place the ball and socket peg 2 inches above the table
 - The Ball of the Ball and Socket beam inside the peg will act as the origin
- Set the ball and socket beam into the peg as shown
- Set up two pulleys
 - Pulley B is one inch left of the Peg Beam at maximum height
 - .85 wheel through the bottom hole on the pulley
 - 70g is added to make 120g total

- Set Up Time
 - 10 Minutes
- Activity Time
 - 5-10

Answers

- Check FSD
- Weight in BD should be around 100g +/- 2
 - Must Use Moment about A

- Pulley C is one inch right of the Peg Beam at maximum height
 - .85 wheel through the bottom hole on the pulley
 - 70g is added to make 120g total
- Both Pulleys will connect to the peg at the far end of the Ball and Socket Beam in the ring.
- Add one more string directly down from the ring of the peg
 - 50g is added to make 100g total
- Set Up Time
 - 10 Minutes
- Activity Time
 - 10 Minutes

Answers

- Point A: (0,0,0)
- Point B: (-1,-2,3.5)
- Point C: (-1,2,3.5)
- Weight at AB: 120g
- Force Vector AB: $-103.99i - 29.71j + 82.395k$
- Unit Vector AC: $-.8695i + .2476j + .51996k$
- Weight Unit Vector: $-j$
- AC Mass should be around 120g +/- 5
- Weight should be around 150g +/- 5