# Dynamic analysis of a series resonant inverter with bidirectional switches in a half-wave operation

**A. Shenkman,** ***Senior Member IEEE,*** **B. Axelrod**
**Holon Academic Institute of Technology, Israel**

**Abstract.** In this paper the simple circuit of a series resonant inverter is analyzed. This is usually presented in textbooks and is given as an example to explain the principle of the operation of such inverters. However the simplifications usually made by its analysis are far away from the real conditions of operation. More precise analysis of this circuit shows that from a practical point of view this circuit is not operable. Examples of improving this circuit to make it operable are given.

## 1. Introduction

In section 11-2.2 of Rashids' text-book[1] a short analysis of a series resonant inverter (Fig.1) is given. This analysis examines an ideal case, i.e. the ideal switches, an ideal inductance and an ideal capacitance. In addition, which is even more important, the examined circuit does not include any load. In this ideal case it is quite correct that the output frequency $f_o$ is the same as the resonant frequency $f_r$:

$$f_0 = f_r = \frac{\omega_r}{2\pi}$$

It is then obvious, that the positive current half-wave (via thyristor $T_1$) and the negative current half-wave (via diode $D_1$), (Fig. 1) are symmetrical. Furthermore, at the instant when the negative half-wave current is finished, the capacitor voltage reaches a zero level (Fig. 2). This fact is illustrated in the numerical example[1].

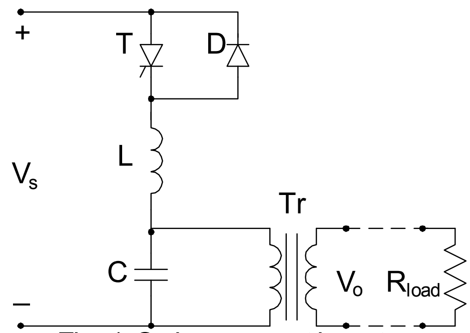



Fig. 1. Series resonant inverter

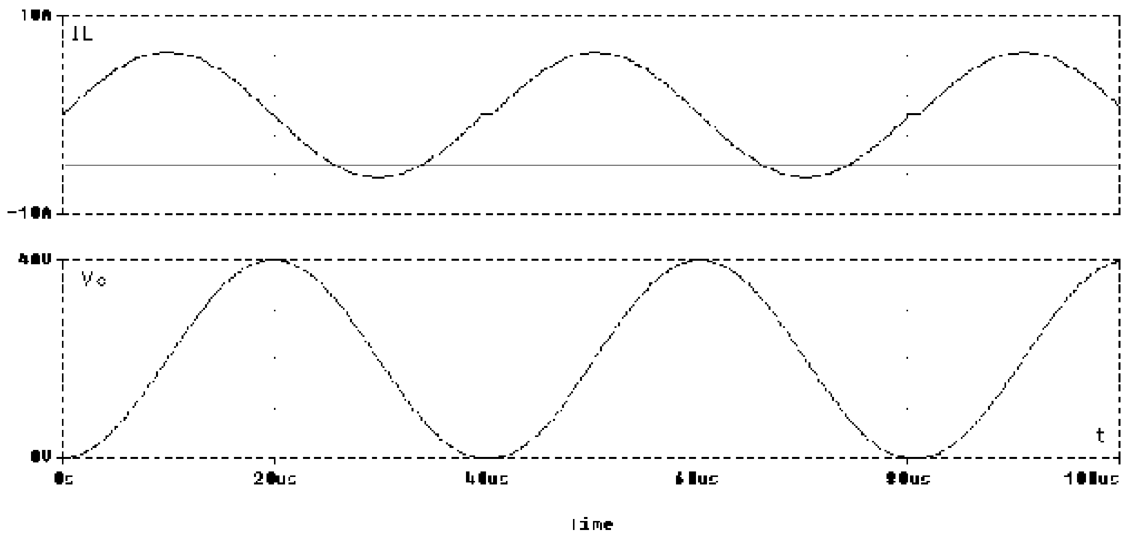



Fig. 2. The current and voltage waveforms of an unload inverter

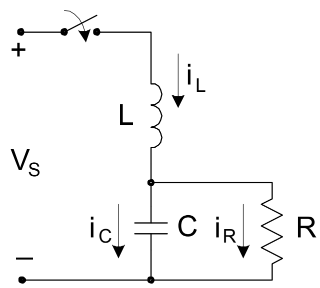



Fig. 3. The equivalent circuit of a resonant inverter.

However, making this circuit practical by adding any resistive element, for instance a resistive load, shows that this kind of inverter is not operable.

2. Full Analysis of a Series Resonant Inverter

Let us illustrate the above fact by a dynamic analysis of this circuit, taking into account in addition to the circuit elements in Fig. 1, the resistive element as a load, which is connected to the secondary side of the transformer. This resistive load may be reflected as $R_r$ into the primary of the transformer ( Fig. 3 ). Then the integro-differential equations of the circuit are given as:

$$L\frac{di_L}{dt}+v_C=V_s$$

$$\frac{1}{C}\int i_C(t)dt=Ri_R \qquad (1)$$

$$i_L=i_C+i_R$$

which results in a one second order differential equation:

$$\frac{d^2i_L}{dt^2}+\frac{1}{RC}\frac{di_L}{dt}+\frac{1}{LC}i_L=\frac{V_s}{RLC}$$

Solving this equation for the inductor current yields:

$$i_L(t)=\frac{V_s}{R}+Ae^{-\mathrm{d}t}\sin(\mathrm{w}_r t+\mathrm{y}_L) \qquad (2)$$

where $\mathrm{w}_r=\sqrt{\mathrm{w}_0^2-\mathrm{d}^2}$; $\mathrm{w}_0=\frac{1}{\sqrt{LC}}$; $\mathrm{d}=\frac{1}{2RC}$.

With the independent initial conditions $v_c=V_{C.0}$ and $i_L=0$ we may find the initial inductor voltage as $v_L(0)=V_s-V_{C.o}$ and the integration constants:

$$A=-\frac{V_s}{R\sin\mathrm{y}_L};\ \mathrm{y}_L=\tan^{-1}\frac{\mathrm{w}_r}{\mathrm{d}+\mathrm{x}_1},\quad \text{where}\quad \mathrm{x}_1=-\frac{V_s-v_C(0)}{V_s}\frac{R}{L}.$$

Therefore, the capacitor voltage will be:

$$v_C(t)=V_s-L\frac{di_L}{dt}=V_s-ALe^{-\mathrm{d}t}[-\mathrm{d}\sin(\mathrm{w}_r t+\mathrm{y}_L)+\mathrm{w}_r\cos(\mathrm{w}_r t+\mathrm{y}_L)]= \qquad (3$$
$$=V_s+Be^{-\mathrm{d}t}\sin(\mathrm{w}_r t+\mathrm{y}_C)$$

)

where

$$B = \frac{V_s \omega_0 L}{R \sin \psi_L}; \quad \psi_C = \sin^{-1}\left(\frac{R}{\rho}\frac{V_{C0} - V_s}{V_s}\sin \psi_L\right); \quad \rho = \sqrt{\frac{L}{C}}$$

Since the inductor current (2) and capacitor voltage (3) are expressed as the damped waves, after the transient response period, the oscillations will be finished and the given circuit in its steady-state will behave as a dc-circuit, i.e., the capacitor will be charged up to the source voltage and the dc-current will be flowing through the resistor (Fig. 4).

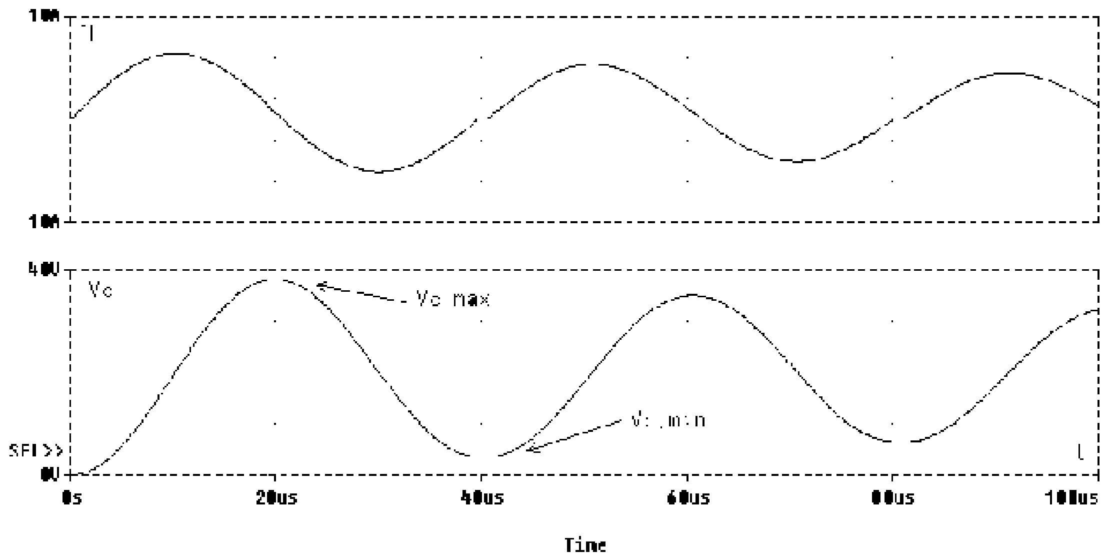


Fig. 4. The current and voltage waveforms of a loaded inverter

To understand this result we should analyze the current and voltage curves shown in Fig. 4. During the time interval starting at the second half-period, the capacitor discharges through resistor R. This discharge process depends on the time constant $\tau = RC$ and of course takes some period of time. So, at the beginning of the second period there will be some initial capacitor voltage $v_c(0)$, which was therefore taken in consideration by solving the circuit in Fig. 1. It is obvious, that at the beginning of the third period this voltage will be higher and with each further period it will increase more and more. The current curve becomes more and more unsymmetrical, so that starting at any which point the current continues to be positive and the thyristor does not turn off (point "a" in Fig. 5, in which the PSPICE simulating results are given). In other words, the inverter stops working. The waveforms confirm an analytical analysis. Checking solution (2) for the parameters of the circuit in Fig. 1 , we may find the voltages at the characteristic points (see Table and Fig. 4, which also prove the theoretical analysis of the above circuit).

Table of calculations

| Period number | $V_{C.0}$ $[V]$ | $y_C$ $[^0]$ | $B$ $[V]$ | $v_{c.max}[V]$ | $v_{c.min}[V]$ |
|---|---|---|---|---|---|
| 1 | 0 | 88.19 | -20.02 | 38.10 | 3.62 |
| 2 | 3.42 | 88.90 | -16.59 | 34.90 | 6.41 |
| 3 | 6.10 | 89.77 | -13.95 | 32.55 | 8.58 |

In the previous example the circuit parameters were taken as: $L = 20\mu H; C = 2\mu F;\ R = 50\Omega;\ V_s = 20V$.

Theoretically, the situation may be improved by shortening the capacitor before the beginning of the each working period. Another practical solution could be adding an inductor in parallel to the capacitor (Fig. 6a), i.e., in creating a parallel resonant circuit. In this case the capacitor voltage will change oscillatory even in the presence of the load resistor. The corresponding curves of the transient process are given in Fig. 6b.

3. Conclusion

A series resonant inverter with bidirectional switches in a half-wave operation is analyzed. It is shown that these kinds of inverters are not operable in any practical applications, since they stop working as soon as any load is connected to it.

In conclusion, it should be noted, that in engineering analyses [2, 3] we are allowed, of course, to make varied assumptions to simplify the solution, but not such simplifications which result in completely incorrect results. However, if such simplifications are made, it must be mentioned that an ideal case is under consideration.



*"Proceedings of the 2003 American Society for Engineering Education Annual Conference & Exposition*

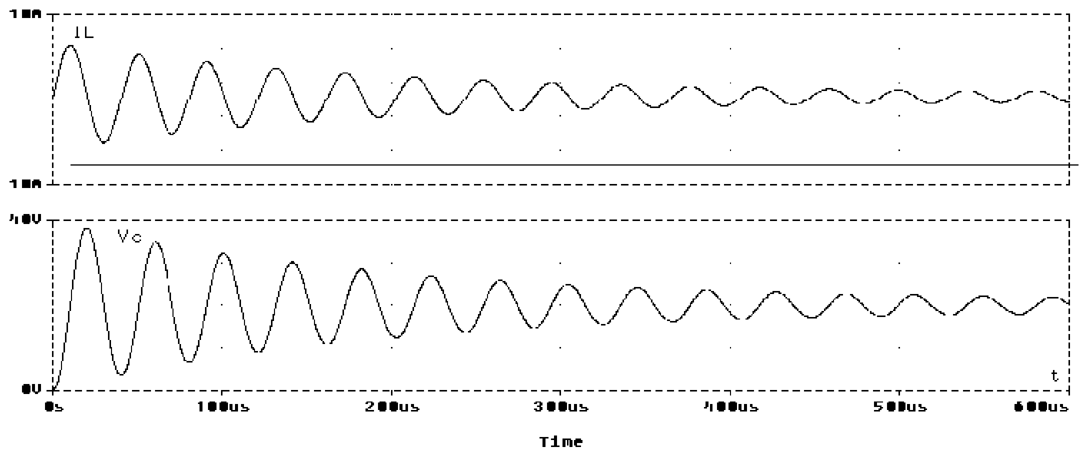
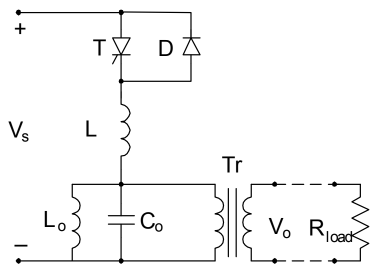
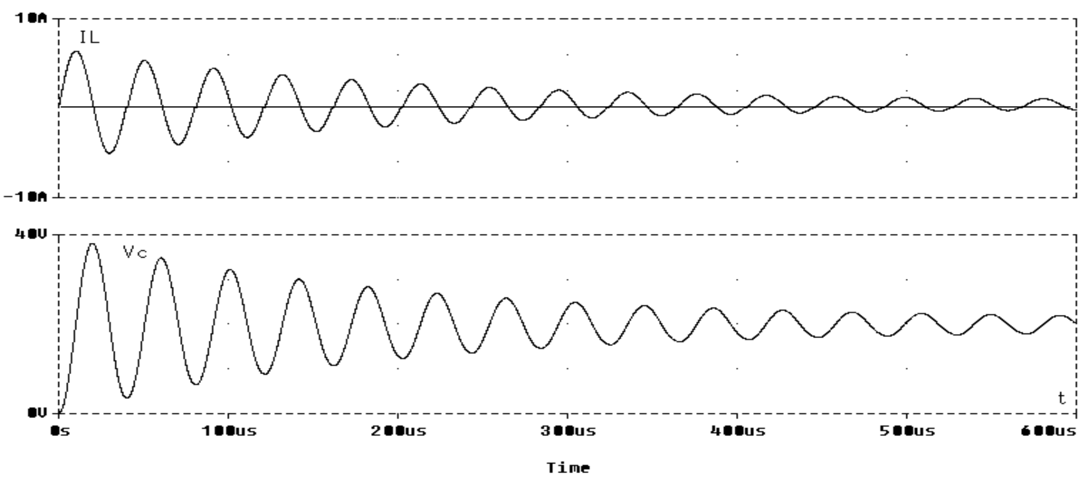



Fig. 5. The SPICE simulation results of the current and voltage waveforms.




a




b.

Fig. 6. The alyernative version of a resonant inverter (a) and its waveforms (b).

## References


1. M. H. Rashid. Power Electronics. Circuits, Devices and Applications. Second Edition. Prentice-Hall, 1993, section 11-2.2.
2. V. Chudnovsky, B. Axelrod and A. Shenkman, "An Approximate Analysis of a Starting Process of a Current Source Parallel Inverter With a High-Q Induction Heating Load," *IEEE Trans. Power Electron.*, vol. 12, pp. 294-301, March 1997.
3. A. Shenkman, B. Axelrod and Y. Berkovich, "A single-switch ac-ac converter with high power factor and soft commutation for induction heating applications", *IEE Proc.-Electr. Power Appl.*, vol. 148, No. 6, pp. 469-474, November 2001.


## Authors


**Prof. ARIE L. SHENKMAN.** (M'86-SM'87) was born in the Ukraine in 1932. He received the M.Sc. degree from the Moscow Institute of Energetics, U.S.S.R, and the Ph.D. degree (with a specialization in power systems) from the Moscow State Railway Engineering Institute in 1955 and 1965, respectively. He is a Professor of Electrical Engineering and is a Dean of the Department of Electrical and Electronics Engineering of the Holon Academic Institute of Technology. He has authored numerous publications.

**Dr. BORIS AXELROD** was born in Ukraine in 1949. He received the M.S. and Ph.D. degrees in electrical engineering in U.S.S.R., in 1972 and 1980 respectively. From 1992 he is a Lecturer in Electronics of the Department of Electrical and Electronics Engineering of the Holon Academic Institute of Technology (Israel). He has authored more than 35 published papers.