Effects of Variable Mix on Student Mathematics Performance

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Abstract

Engineering and Engineering Technology students encounter a wide variety of variables in their coursework. To prepare for courses in their specific majors, these students usually take a common core of mathematics classes, which are typically taught using x and y. The authors wished to determine whether students' mathematical performance using other variables was on a par with their ability to manipulate x and y. A ten problem quiz was designed in two versions: one using only x and y, the other using a wider range of variables. The problems were identical in all other respects. Since the population to be tested included first-year students, none of the problems was calculus-based. One of the questions did not use any variable at all. This was included as a control question. The quiz was given to one hundred twenty-four students in first, second, and third year courses at Georgia Southern University. The sample population is a mix of Engineering and Engineering Technology majors, including native and transfer students. Students in several classes were given the quiz with the two versions randomly distributed within each class. Students taking the xy version of the quiz scored significantly higher than students taking the mixed-variable quiz. Students with higher class rankings (juniors and seniors) showed a lower difference between the quiz versions than did students classified as freshmen / sophomores. Classification of students' rankings was based on individual mathematics course histories as reported by the students. This paper presents the detailed results of the study, along with suggestions for further research into this topic. These results may be useful in indicating areas where review is needed.

Introduction

On presenting a set of technology-based problems for teaching mathematics to engineering students, Klebanoff and Winkell[1] noted the compartmentalization that exists in which students see little substantive relationship between math, science and engineering. Although they speculated that the type of symbolic manipulations that students are asked to perform in mathematics classes does not prepare them for applying mathematical concepts in science or engineering contexts, they did not further explore the lack of perceived relationship.

In a study of the knowledge and application of College Algebra, Conway Link[2] asked students to solve for the radius of a sphere using the sphere's volume formula. He found that only 34.6% of the surveyed students were able to produce a correct answer, but that the number of mathematics courses taken after College Algebra seemed to increase the likelihood of a correct answer. He ascribed this to students' encountering an increasing number of examples and problems with "non-traditional" variables and formulas as they progress through their coursework.

On discussing the problems encountered in teaching mathematics to engineering students, Sazhin[3] noted that changing the basic equation

$$x = b/a$$

to the slightly more complex

$$\alpha \phi^2 x + B x = \gamma$$

prevented most students from solving it for *x*. He ascribed this to students' tendency to memorize equations and their manipulation in a particular notation.

The disconnect between mathematics teaching with *x* and *y* as preferred variables and the use of more varied and descriptive names in engineering and technology courses may explain why students find it difficult to solve mathematically simple problems in an engineering application. This disconnect has apparently not been investigated before this study.

A survey was conducted to determine whether the variables used in solving problems affected students' mathematical performance. Students enrolled in sophomore and junior level engineering and engineering technology courses at Georgia Southern University were given a ten-problem survey. The students were enrolled in

(A) Mechanism Design, a junior level course in Mechanical Engineering Technology,

(B) Statics, a sophomore level course for Civil, Electrical, and Mechanical Engineering Technology majors,

(C) Statics and Dynamics, a sophomore level course for engineering majors, and

(D) Electrical Devices and Measurements, a sophomore level course for Electrical and Mechanical Engineering Technology majors.

The survey included two sections. The first section collected information about the students' mathematics background. Questions in this section addressed which courses had been completed, what grades had been achieved, and the amount of time elapsed since each course was completed. The students' names and the course in which they were enrolled were also collected. (See Fig. 1)

The second section of the survey was a set of ten problems. Six problems involved algebraic solution, two problems involved trigonometry, and one problem involved graphical analysis. A tenth question, not involving any variables, was included as a control question.

The survey instrument was prepared in two versions, one using the variables *x* and *y*, the other version using the variables *g, h, m, n, p, Q, q, r, s, t,* and *r*. The questions on the two versions were identical except for the variables used. (See Figures 2 and 3)

The survey was conducted on an unannounced basis. Calculators were allowed. The grade history was completed first, after which fifteen minutes were given for the problem-solving portion.

One hundred twenty five survey forms were completed; of these, four were multiple responses from the same individuals, due to enrollment in more than one of the courses surveyed. These multiple responses are not included in the analysis which follows.

Survey Instrument

Standard Assessment of Variable Preference

Name (Please print) ______________________________

Class ____

Although this quiz does not affect your grade, it is important to do your best.

If you took a class but cannot remember your grade, or CLEPed the course, mark X for grade.

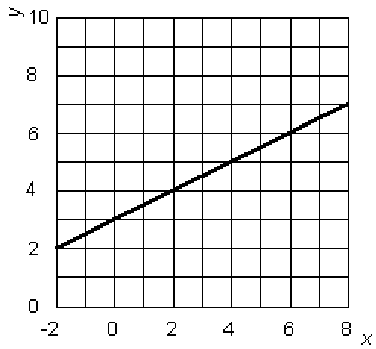
College Algebra Grade:	_____	□ 1 year ago	□ 2 years ago	□ 3 years or more ago
Pre-Calculus Grade:	_____	□ 1 year ago	□ 2 years ago	□ 3 years or more ago
Trigonometry Grade:	_____	□ 1 year ago	□ 2 years ago	□ 3 years or more ago
Calculus I Grade:	_____	□ 1 year ago	□ 2 years ago	□ 3 years or more ago
Calculus II Grade:	_____	□ 1 year ago	□ 2 years ago	□ 3 years or more ago

Time limit: 15 minutes. Do not turn the paper over until you are told to do so.

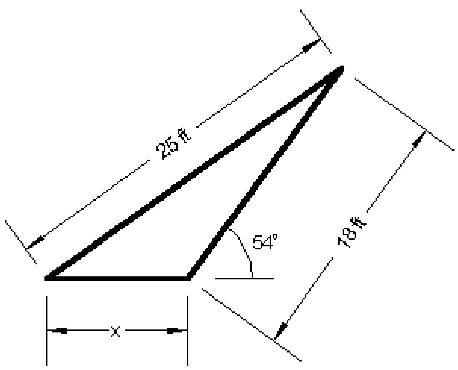
Instructions: Work as many of the problems as you can, in any order. You may use scratch paper.

Figure 1

1. Find the maximum value of $y = -x^2 + 1$
2. Factor: $x^2 + 7x + 12$
3. Multiply: $(x + 4)(x - 9)$
4. Evaluate: $\log(y^2) - \ln(x)$, given $y = x = 3$
5. Evaluate: $10^{(-x)} / e^{(3x)}$, given $x = 2$
6. Express as a single fraction: $13/16 + 7/12$
7. Find the roots of $2x^2 + 10x + 8 = 0$
8. Express the function shown in slope-intercept form.



9. Find the distance x in the diagram shown.



10. Find the lengths of line segments x and y.

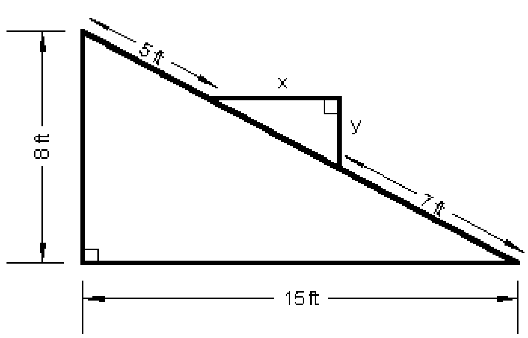
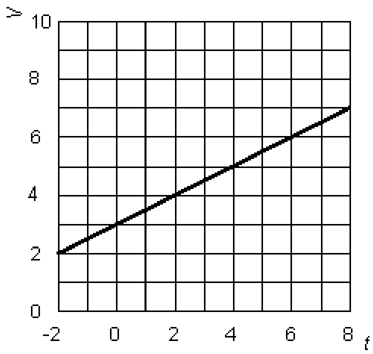
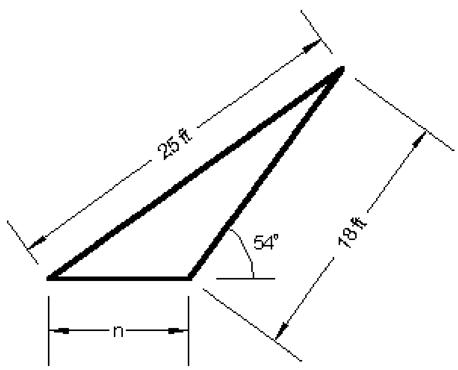


Figure 2

1. Find the maximum value of $Q = -t^2 + 1$
2. Factor: $s^2 + 7s + 12$
3. Multiply: $(p + 4)(p - 9)$
4. Evaluate: $\log(h^2) - \ln(g)$, given $h = g = 3$
5. Evaluate: $10^{(-m)} / e^{(3m)}$, given m = 2
6. Express as a single fraction: 13/16 + 7/12
7. Find the roots of $2q^2 + 10q + 8 = 0$
8. Express the function shown in slope-intercept form.



9. Find the distance *n* in the diagram shown.



10. Find the lengths of line segments *r* and *t*.

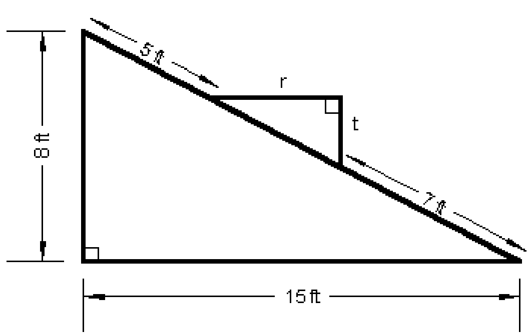


Figure 3

Results

The overall average score was 4.63 / 10. Average for the *xy* version was 4.69 / 10 while the average for the multivariable version was 4.56 / 10. Normalizing these figures to the average raw score, the *xy* version showed 101.4% of average while the multivariable version showed 98.5%.

Removing the control question (number 6), the *xy* version performance was 102.0% of the nine-question average and the multivariable version 97.9%, a difference of over four percentage points.

A significant difference between the two versions is shown by students who correctly answered the control question: 108.1% normalized score for the *xy* version versus 99.1% for the multivariable version. In contrast, students who missed the control question actually performed worse on the *xy* version: 76.6% as compared to 91.9%.

Separating the results by course enrollment, the students taking Mechanism Design (MET juniors) showed the best performance on ten questions (5.04 / 10) and on the nine questions involving variables (4.24 / 9). They also showed the smallest spread between the two versions: 100.2% for *xy* and 99.8% for multivariable (nine-question normalized results).

Sophomore Engineering students taking Statics and Dynamics had overall averages of 4.79 / 10 and 3.91 / 9. The spread between the two versions was also small for this group: 100.7% vs. 99.3% of the nine-question average (*xy* higher).

Students in Electrical Devices and Measurements (EET and MET majors, mostly sophomores) averaged 4.55 / 10 and 3.72 / 9. While the difference between versions (*xy* 102.7% / multivariable 96.2% of nine-question average) is the second widest spread seen in the different course sub-populations, the students in this course who answered the control question correctly showed an even wider spread: 116.3% *xy* as compared to 96.2% multivariable, a spread in the nine-question normalized average of 20.1%.

Students in the Statics course (mostly MET and CET sophomores) averaged 4.24 / 10 and 3.47 / 9, with a spread of 10.1% between versions (*xy* 105% , multivariable 94.9%). This was the highest spread shown by any of the sub-populations separated by course enrollment.

Possibly the most interesting disparity between sub-populations is seen when performance by students who began their collegiate study of mathematics with College Algebra, Trigonometry, Pre-Calculus, or some combination of these course is compared to performance by students who went directly into calculus courses. The first group scored an average of 101.2% on *xy* and 98.7% on multivariable versions, while the students who went directly into Calculus I appear to have been the group most severely affected by the change in variables: 119.2% for *xy* compared to 85.7% for the multivariable version, a spread of 33.5%.

Analysis of the sub-population with all mathematics grades "B" or better showed only a 1% spread.

Students answering the control question correctly also scored higher on the remaining nine questions (4.12 / 9 *xy*, 3.78 / 9 multivariable) than did students missing the control (2.92 / 9 *xy*, 3.50 / 9 multivariable). 18% of the sample population missed the control question.

Average performance on the individual questions was: Q1,36.4%; Q2, 86.8%; Q3, 91.7%; Q4, 43.8%; Q5, 40.5%; Q6, 81.8%; Q7, 38.8%; Q8, 23.3%; Q9, 9.9%; and Q10, 10.7%. Question 3, involving algebraic multiplication, showed the highest results, while question 9 and 10, involving trigonometry, showed the lowest.

Summarizing the results, students do not perform as well when they encounter unfamiliar variables. The degradation in performance is small but significant. Some groups of students were significantly affect, most noticeably students who had been placed directly into calculus courses (see Fig. 4).

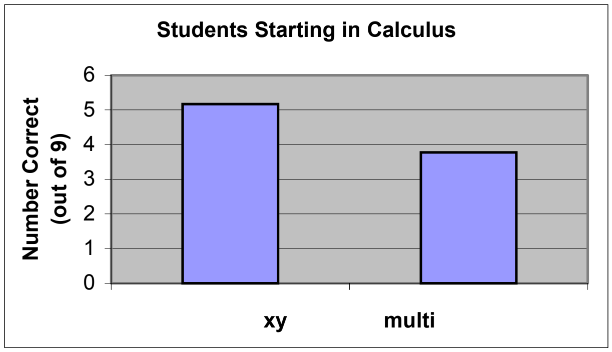


Figure 4

Discussion

Students seem more comfortable with variables with which they are familiar. Setting the same problems in the context of an unfamiliar set of variables appears to affect students' mathematical performance adversely, some groups being more adversely affected than others. Students missing the control question seem less affected than students answering the control correctly. One possible explanation for this is that their overall mathematical performance is poorer to begin with, as indicated by their overall scores.

Students in the group least affected by variable mix were Juniors. These students have completed more technical courses than sophomores, and these technical courses have exposed them to a far wider variable mix than the rest of the sample population.

Suggestions for Further Research

The disparity (in terms of the effects of variable mix) between students starting in Calculus courses and students starting in other courses presents no obvious explanation. Further investigation into this question is indicated. Further investigation is also indicated in the area of performance as a function of the age (time since taken) of students' mathematics courses.

Conclusions

Students are taught mathematics in the context of a limited set of variables. When the students in Engineering or Engineering Technology programs begin to apply their mathematics background, their performance is hindered by unfamiliarity with the new variables they encounter. A wider mix of variables throughout mathematics courses, both in lecture and drill settings, would be of great benefit to these students.

In Engineering and Engineering Technology courses, a review of mathematics skills using the new variables to be found in the course would be helpful if conducted before the new concepts of the course are introduced.

The effects of making these small changes seem well worth the effort. Students' comprehension in technical courses would not be clouded by confusion over unfamiliar variables. Better and faster comprehension, improved test scores, and increased student confidence could be expected.

References

1 Klebanoff, A.D. and Winkel, B.J., Technology-based problems in calculus from science and engineering, Proceedings of the 1996 ASEE Conference , June 1996, Session 1265.

2 Conway Link, W., The definition of a function – Do post college algebra students know it and can they apply it?, Proceedings of the 1996 ASEE Conference, June 1996, Session 2465.

3 Sazhin, S.S. Teaching mathematics to engineering students, International Journal of Engineering Education, Vol.14, No2, 1998, 145-152.

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