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Embedding Mathematics in Engineering Design Projects

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Embedding Mathematics in Engineering Design Projects

Abstract

For many years, students and faculty at the University of Virginia have been developing materials to teach pre-college students about engineering. These materials, *Engineering Teaching Kits (*ETKs), introduce and reinforce concepts from mathematics and science in the context of engineering design challenges. Age-appropriate mathematics are embedded in all of our ETKs, but we do not explicitly teach it. Rather, we use inductive learning principles via project-based learning challenges that lead students through exercises involving experimentation and measurement; data collection, analysis, and display; estimation and prediction; and budgeting and making trade-offs. For example, data representation and computation are essential skills for engineering problem solving. In working through the challenges, students gain practice and comfort in applying the mathematics, logic, and problem solving skills needed to solve engineering design problems. We provide examples of how mathematics are embedded in ETKs after providing an introduction to the kits.

Background

Since 2002, we have been developing and testing Engineering Teaching Kits (ETKs) to introduce middle school students to engineering concepts and techniques as well as underlying mathematics and science concepts.[10, 11, 12] ETKs emphasize the engineering design approach to problem solving. Topics are identified from science, mathematics, and technology that have interesting engineering applications, and then lessons are constructed to help students learn mathematics and science in the context of engineering design. Our purpose is to introduce pre-college students to engineering and show them what engineers do. We also hope to convey the fun and excitement of engineering and stimulate interest in the field. These efforts are part of a national emphasis on pre-college engineering education that has developed over the last two decades.[6]

Although initially targeted for middle school audiences, ETKs have also been used in high school and elementary school classes. Over 60 ETKs have been developed to date on a range of engineering design challenges, and many are in frequent use in precollege and professional development learning environments. ETK topics include alternative energy (*RaPower* – solar cars and *Harness the Wind* – wind turbine blade design), buoyancy and propulsion (*Under Pressure* – submersible vehicles, *HoverHoos* – hovercraft design, and *Sail Away* – sail design), heat transfer principles (*Save the Penguins* – radiation, convection, and conduction as well as climate change), structures and loads (*Bridges to Engineering* – balsa wood / craftstick bridge building), forces and motion (*This is How We Roll* – relationship between ramp height and distance a Lego car travels and *Movin' Along* – types of motion and translational mechanisms), and projectile motion (*Catapults in Action* – comparing the results from delivering payload using varying arm angles). Other ETKs address water filtration and purification, aerospace engineering, rockets, electricity and magnetism, magnetic levitation, and fluid flow.

The ETKs are developed by teams of fourth year Mechanical Engineering students as their capstone project. These teams work with local teachers to insure that our ETKs fit into existing curricula and meet state and national educational standards. An ETK typically includes five days of lessons. Three days are devoted to explanation and demonstration of concepts in mathematics and science related to the challenge – including identification and remediation of misconceptions – and two days are devoted to designing, building and testing an object, structure, or system. Mirroring professional practice, students work in teams to conduct experiments, test components, and meet a design challenge.

In the first test of an ETK, the UVA student authors initially teach the material, conduct the demonstrations, and supervise the design challenge on site in the classroom. The classroom teacher is always present and will assist as needed. However, we encourage the teacher to observe and critique what we do in the classroom. Many teachers give us detailed comments and suggestions. This feedback helps shape the final version of the ETK. We expect that having seen us conduct the lessons and design challenge, the teacher can use this unit without our presence. Most in fact can and do use our materials without our help. We have distributed the ETKs to teachers around the country who have used them successfully.

Our lessons are structured so that the pre-college students experience the material and learn concepts and methods in a dynamic collaborative environment. They are challenged to devise techniques to accomplish specified tasks and make informed choices. We guide them through the lessons, but do not direct them what to do. In the design challenge, the teams are very creative and devise many unique solutions to the defined problem.

RaPower [12, 13]

In the *RaPower* ETK**,** pre-college student teams design and build model solar cars. Through a series of hands-on activities, they learn how solar cells and motors work: they conduct experiments and take measurements, analyze data, and draw conclusions. Figure 1 shows the types of solar cells used in this experiment. They determine which solar cells, motors and tires would work best for their car. In one experiment, they use multimeters to measure the voltage produced by three solar cells under four lighting conditions (ambient, incandescent, halogen, and solar). Each team constructs a 3x4 matrix and enters their measured values. All teams display their results on the blackboard, and the class discusses them. Although every team has different numerical values, the matrices all display the same patterns. Each team picks the same best solar cell and preferred lighting conditions.

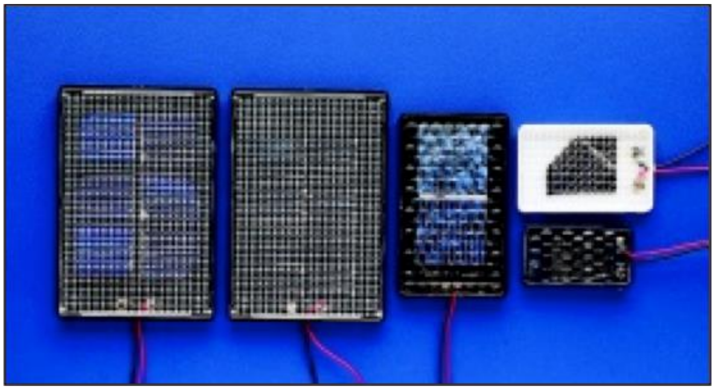

Figure 1: Various Solar Cells

In a second experiment, the teams have to determine which of three motors will pull the most weight. They are given three motors and a battery pack and asked to design an experiment to determine which motor can pull or lift the most weight (Fig. 2). Various solutions are possible. One configuration is shown in Figure 3. Again, all teams find the same ranking for the motors and eventually select the same motor as best. The teams have to focus on lifting or pulling capability and ignore rotational speed.

Figure 2: Types of Motors Used in Solar Cars

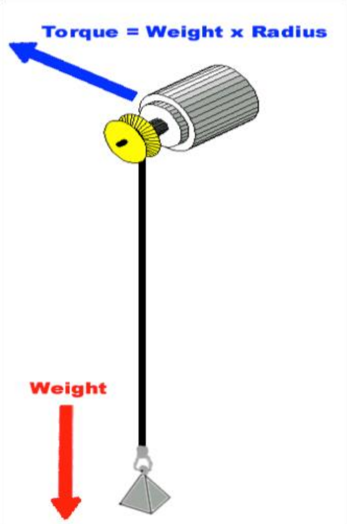

Figure 3: Lifting Experiment

Finally, the teams must determine which of four materials – wax paper, fine sandpaper, coarse sandpaper or rubber – produces the greatest friction as measured using spring scales; see Figure 4. These materials may be used in addition to or in place of tires on the solar cars.

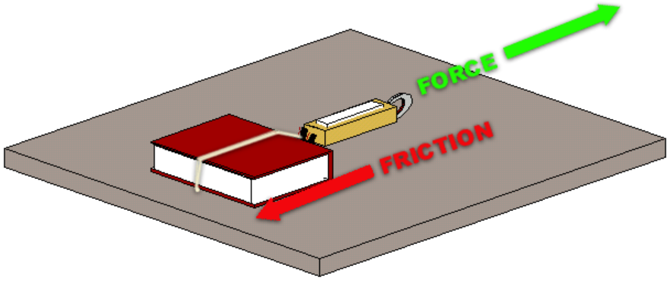

Figure 4: Friction Experiment

After these experiments, each team knows the optimal components for building their car. However, they have a materials budget and must buy the key components for their car. The solar cells, motors, and tires have costs associated with them, and the teams must make tradeoffs to stay within budget. They compute the costs to produce various configurations, and are often very creative in satisfying the constraints in their final designs. Appendix 1 shows the costs associated with each part.

Then the teams design and build their model solar car. The key components are limited by the budget but they can use all the Lego blocks, gears, and pieces they want free of charge; see Figure 5. Their final car must be able to pull a cart loaded with weights. In the final competition, we load the cart with increasing amounts of weight on each trial; see Figure 6. The team whose car pulls the most weight wins.

Figure 5: Assembling a Solar Car

Figure 6: Pulling a Load with a Solar Car

Each team prepares a summary sheet showing a sketch of their design, a bill of materials (the parts they used with the cost of each), the total cost of their design, and how much weight their car was able to pull. After the competition, the entire class reflects on the results and discusses what worked and what did not.

After completing this ETK, the students have learned about solar cells, motors, and tire materials, but they have also learned about the engineering design process, and how to construct a vehicle to perform a task. They also learned how to measure the values of variables, the importance of consistent procedures for making measurements, how to compute statistics such as mean and variation, how to use a budget to address limitations and make tradeoffs; and the role of math in engineering design and analysis.

This is How We Roll: Forces and Motion [3, 4]

Over the past three years at three elementary schools in Albemarle County, Virginia, teams of first grade students have participated in *This is How We Roll*. The initial design challenge is to build simple cars using a standard set of Lego parts and then to test their cars on a ramp elevated at four different angles. The ramps are elevated by one to four 6” wooden blocks placed under one end of the ramp; see Figure 7. The teams would start by predicting how far they expected their cars to go and then tested those predictions. They measured the distance their car traveled using index cards to meet the non-standard measurement Virginia Standard of Learning for first grade mathematics; see Figure 8. On worksheets, they entered their data using a histogram format, and were able to see how the slope of ramp determined how far their car would go. They also learned to improve their predictions of distance traveled as they continued their testing.

Figure 7: Testing Cars with Ramps at Different Slopes

Figure 8: Measuring Distance with Index Cards

The next day, the teams redesigned their cars using different (and more) Legos and conducted the same tests again; see Figure 9. Most redesigned cars performed much better. Finally, the teams were allowed to add weights to their cars and test again. After these experiments, the students reflected on the results and explained their understanding of the concepts of force and motion.

Figure 9: An Example of More Elaborate Car Designs

These young students have very limited math skills, yet they intuitively understand the relation between the slope of the ramp and the distance their car travels. In subsequent trials, they incorporate a series of other variables such as weights into their designs. We believe these interactions with physical systems help prepare the students for developing a sense of numbers and relationships between variables.

The Bridge to Engineering [7]

At a local all-girl's middle school, student teams designed and built bridges to support various loads. The lessons covered the history of bridges, types of bridges, their geometry and structural characteristics, and construction materials. The students learned how different geometric patterns such as triangles and truss designs affect the strength and structural integrity of structures. The teams used the West Point Bridge Building software to experiment with multiple geometric configurations before selecting their final designs (http://bridgecontest.org/resources/download/).[8,9] For their construction, they were limited to a certain number of craft sticks and a jar of wood glue.

Figure 11 shows an example bridge design. Testing involved suspending the bridges between two tables (see Figure 12) and loading it with weights (see Figure 13). All six teams produced bridges that supported 100 pounds hanging from the deck without breaking. The following week we brought in additional weights and broke four of the six bridges. Two were still intact at 200 pounds. The students' understanding of the impact of geometry on the strength of their structures allowed them to exceed the loads held by many other groups' previous constructions over the years.

Figure 11: A Final Bridge Design

Figure 12: Suspending a Bridge between Two Tables

Figure 13: Bridge Loaded with Everything in Sight

Catapults in Action

Our ETKs can be used in the classroom in a variety of ways. In most classes, the student experience is defined by our lessons, demonstrations, and the design challenge. Occasionally, teachers will place our materials in a wider context. An example is *Catapults in Action*, which was originally designed for a 9th grade physics class. The primary goal was to teach about projectile motion. A design for a catapult with an adjustable arm was developed for this kit; see Figure 14. Student teams experimented with different placements of the components, and computed how to achieve two goals on different trials. One goal was to accurately hit a target, such as an archer on a turret window in a castle; the other goal was to send the projectile as far beyond the castle wall as possible.

A local middle school math teacher invited us to try this ETK with his younger students. We ran two successful sessions with his classes. Figure 15 shows a student about to launch the projectile. At the end of our scheduled visits to his school, we were asked if we could leave all our materials and supplies for a few weeks. Several weeks later, we learned that four teachers had developed and taught an integrated curriculum. The teachers' areas were English, History, Science and Mathematics (Algebra). The students learned about the history of medieval warfare, the science and engineering involved in building different types of structures and weapons, the math required to determine how to aim the projectiles and maximize the distance traveled, and they conducted research and wrote reports on their results.

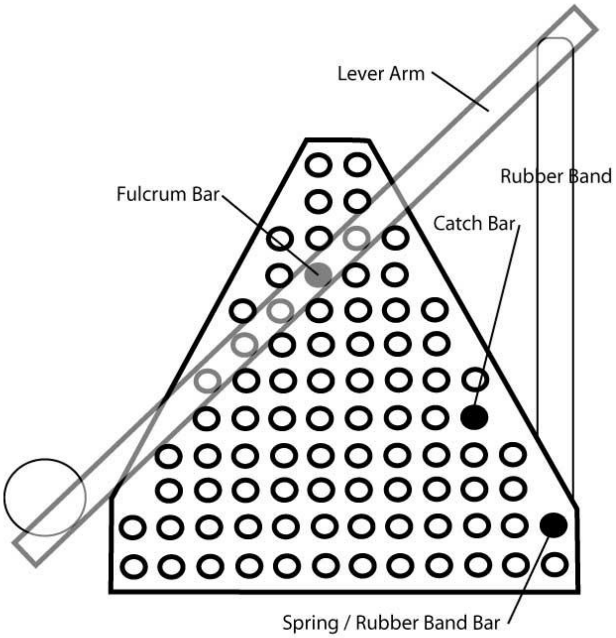

Figure 14: Standard Catapult Design

Figure 15: Catapult in Action

Summary/Conclusions

The ETKs featured in this paper are good examples of how we demonstrate to students the relevance of mathematics to engineering problem solving. They are required to use mathematics during experimentation, measurement, analysis, decision-making, optimization, and design iteration. Mathematics is embedded in all ETKs: measurement and statistics in *RaPower*; projectile motion in *Catapults in Action*; geometry and optimizing structures for *Bridges to Engineering*, *Sail Away*, and *Harnessing the Wind*; and many others. We use math without explicit instruction, but the students see how essential it is to engineering by working through the design challenges.

Although originally designed for middle school science and math classes, we have successfully implemented ETKs in all levels of pre-college education. The appropriate mathematics varies with grade (age) levels; as does the amount of guidance (scaffolding) the student teams need. Once the students understand algebra, we can introduce realistic equations and deeper design challenges. With the solar car ETK, 6th graders cannot make systems of gears work; by 8th grade they can. Younger students can gain an appreciation of the role of numbers in representing design variables (slope) and performance criteria (speed, distance). High schoolers can use equations for estimation and prediction.

In discussing the ETK lessons with the pre-college students, we emphasize the importance of representation (sketching, modeling and visualization) [5, 15] and quantification[1, 2, 14] (measuring, analyzing, and logical reasoning). We have them make drawings of their designs, document the results of their tests, and record their performance on the design challenge. They also reflect on the outcomes of the challenge and consider how another design iteration might lead to improved performance.

One of our goals is to increase interest in mathematics, science, and especially engineering, and motivate the students to continue their study of these subjects. ETKs appeal to students with different learning styles due to our project-based orientation, and they make learning fun! They demonstrate how, and why, mathematics is used by engineers.

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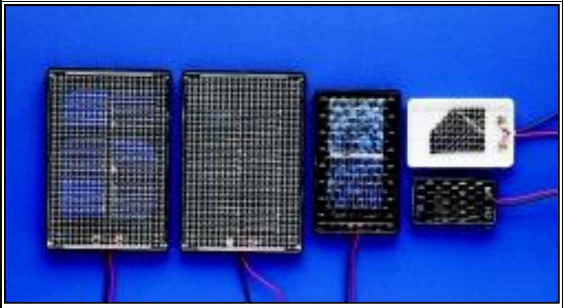
Appendix 1: Official RaPower Space Agency (RPSA) Materials Catalog

SOLAR CELLS



From: Edmund's Scientific

Type	Cost (each)
1.5 V / 200 mA	$100.00
3 V / 100 mA	$300.00
6 V / 50 mA	$450.00

SUPER-TORQUE MOTORS

From: Edmund's Scientific

Type	Cost (each)
115RPM, 0.8 in.-oz Torque	$100.00
30RPM, 2.4 in.-oz Torque	$300.00
16RPM, 5.0 in.-oz Torque	$450.00

WHEEL MATERIALS

Type	Cost (to cover 4 wheels)
No Material	
Wax Paper	$50.00
Sand Paper	$200.00
Type 1 Rubber	$300.00
Type 2 Rubber	$400.00

RPSA provides you with $1,000 and 1 Body Kit (50 pieces).