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Engineering Connections between Math, Physics, & Music (Strand 2)

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Introduction

As a math teacher at the School for Creative & Performing Arts, a performing arts magnet school in Cincinnati Public Schools, I participated in the Summer Institute for Teachers at the University of Cincinnati designed to help me create and teach instructional units connecting math concepts to engineering design. The Summer Institute was part of the Cincinnati Engineering Enhanced Math and Science Program (CEEMS), which is a Math and Science Partnership grant funded by the National Science Foundation, DUE-1102990. In one instructional unit taught to approximately 30 students in my honors pre-calculus class, I made mathematical concepts more relevant to students' everyday lives by presenting them with the connections between math, music, engineering, and science. The study of sound through music is an excellent way to introduce the mathematics of waves. When presented with trigonometry, students are often confused and sometimes turned off by the subject. My approach, especially since I teach in a performing arts school, helped my students more easily grasp sine and cosine functions when presented in Algebra II, Pre-Calculus, or Physics. In fact, this unit could address both math and physical science standards, as it involved modeling periodic phenomena with trigonometric functions[2] and using math representations to support a claim regarding relationships among frequency, wavelength, and speed of waves traveling in various media.[5] This particular paper documents the results of my unit when taught in a pre-calculus setting.

Literature review

Although the connections between music, physics, and engineering have been documented in literature, there are few settings where students have the opportunity to participate in hands-on projects that clearly connect the three disciplines.[6] Tufts University offers an interdisciplinary program in Music Engineering, which was started in 1998 and formerly named the Musical Instrument Engineering Program.[4] It started as a way to teach the fundamentals of engineering to undergraduate students in an engaging and enjoyable way by having students construct musical instruments.[3] Today, Tufts students enrolled in either the School of Engineering or the School of Arts and Sciences can obtain a Minor in Music Engineering.[4] However, in perusing the literature, I could not find any other examples of *high school* students constructing musical instruments in order to better understand mathematical concepts.

Unit implementation

Complete details of this unique unit, including a video and poster summary, can be found on the CEEMS website. I started the unit through an open-ended class discussion about sound, music, and sound waves. I used props, such as wind chimes, boom whackers, and a standing wave generator to grab students' attention. Then, a demonstration of an audio editing program

Audacity was used to examine the wave form of sound from popular music. This started a discussion of amplitude and frequency. Students were able to see the visual view of the waveform of the music on the screen, which looked very choppy and did not seem to have much structure. Choosing a small portion of a song, I then zoomed in so students could see a brief piece of the wave and asked the open-ended question: "What can you tell me about this wave?" I then elaborated on students' answers to point out the amplitude or height of the wave and its period (how long it takes the wave to repeat), and how period is related to frequency. Next, I wanted the students to examine a purer wave form. I used a percussion instrument called a Vibratone to produce a sound and then asked the same questions as the students viewed two periods of this purer sound in Audacity. These tools concretely and visibly demonstrated concepts such as sound waves, amplitude, and frequency as well as their connections to math and music. An accompanying worksheet helped students see the connection between waves and sine and cosine functions.

Next, students produced the sine and cosine wave from the unit circle. On the surface, the concept of the unit circle may seem quite trivial, but the subtlety and importance in the realm of mathematics is essential. One of the crucial steps involves understanding how the unit circle relates to trigonometry. The x axis and y axis can be assigned to any variables we desire. The angle, although it is determined by the x axis and the terminal side of the angle, can be put horizontally along the x axis. The y axis can then be either the x coordinate of the unit circle or the y coordinate of the unit circle. Generally, one has great power to choose what numbers go on the x or y coordinate.

I used a hula hoop, white board, markers, a projector and graphing calculators to visually teach this lesson on the unit circle. Please see **Figure 1** for a visual representation. Starting with the hoop, I said, "Let's assume the radius of this circle is one. Does everyone know what the radius is?" Proceeding to roll the hoop on the floor, I asked the following questions, "How can we mathematically describe what is going on here with the rolling circle? How can we describe the angle? How can we identify the x and y coordinates? How can we graph 3 things when we only have two axes?" Next, placing an x and y axis on the board, I inquired, "What shall we put on the x-axis?" My students correctly identified that the angle degrees belonged along the x-axis. I then followed up with the question, "What shall we put for the y – axis?" I pointed out that it doesn't matter whether the students choose the x or y coordinate. By choosing the x coordinate, they will graph the cosine wave. By choosing the y coordinate, they will graph the sine wave. To further describe this phenomena, I added, "Let's keep it simple at first and we will only focus on the coordinates on the two axes, that is (1,0), (0,1), (-1,0), (0,-1). Graph the angle as x and the other chosen coordinate as y." Then, I added, "Let's then look at some of the other simple values, that is, where we can graph a ½ or negative ½. Graph the angle as x, and the other chosen coordinate as y. Will it be nice and smooth in between, or will it be 'choppy'?" The students correctly replied that it will be smooth. I then connected the dots, nice and smoothly, and produced either the sine or cosine graph, depending on whether each bell chose the x or y coordinate. A worksheet formally assessed whether the students understood the concept. This activity provides an exploration into the subtle transition between the unit circle and the sine and cosine waves.

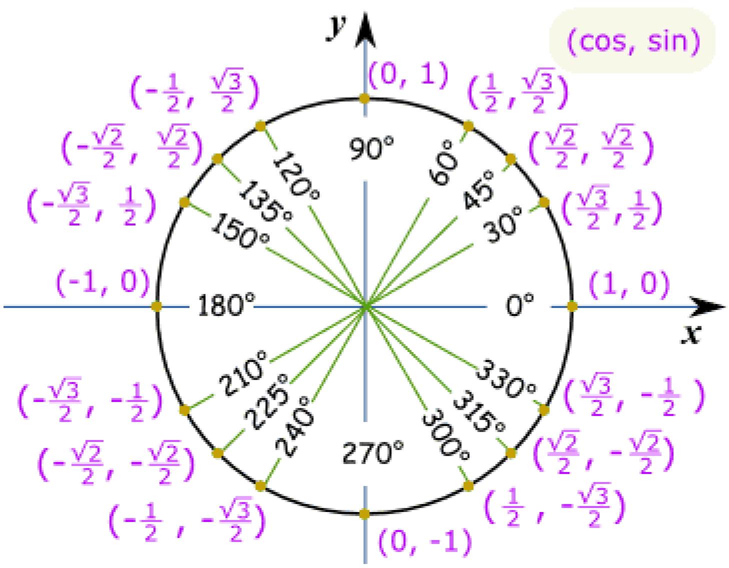

Figure 1. Unit Circle

The connection to music is introduced as students learn to calculate the lengths of pipes, bars, or strings which will produce a musical scale. There are different musical scales from around the world, but they all have some commonalities. The first relationship is the octave. If one tensions a string to produce a note, and then frets the string in half, it creates a relationship called the octave. It increases the frequency by a factor of two. In between going from 1 to ½, you have lots of choices for fractions. Our more familiar music uses the scale in **Figure 2**.

C	D	E	F	G	A	B	C'
$\frac{1}{1}$	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

Figure 2. Musical Scale

To demonstrate this, I played a lower frequency note on the left side of the xylophone. I asked one of my music majors to locate the octave to the note I just played and then the measured the length of the two pipes, which is why conducting this discovery with a xylophone is helpful. One should see that the pipe producing the higher note is half the length of the pipe producing lower note. This discovery is essential for the students as they design and build their own instruments.

Next, students viewed an inspirational video, *Landfill Harmonic*[1] about a youth orchestra program in a third world nation where volunteers construct instruments for the students using materials from a local garbage dump. An engineering challenge for this unit was for student teams to build, in teams, a musical instrument out of recycled materials that can be tuned to play at least 8 notes on the musical scale. To do so, they needed to work through the engineering design process.

First, I provided his students with a variety of websites explaining how to construct musical instruments so the groups could research different options. A variety of materials were provided to the students, including PVC tubing, medal rods, wooden dowels, wooden pins, string, wire, fishing line, various types of boxes, and duct tape. Students also had access to hammers, saws, drills, screwdrivers, glue guns, and compressed air. They were provided with safety procedures, in addition to goggles and gloves, which were required when using power tools. That being said, a graduate assistant provided by the CEEMS program, students from my school's Technical Theater Department, or I would do the difficult cutting or drilling for student groups that felt uncomfortable with the process. Students could also bring in materials from home if different items were needed. Knowing what they had to work with and armed with helpful websites, each group of 3-4 students needed to brainstorm various instrument designs and decide as a group which prototype to try first. After brainstorming, each group drew a scale model of their proposed design from two different perspectives prior to building. Most groups' instruments went through multiple iterations. For example, the group that made the xylophone originally wanted to make a drum, but then quickly realized they would have to make 8 drums to play a scale. The thumb piano group first built a box, but then found out the wood used would not support the bobby pins and simplified the plan by using just a piece of plywood. Please see **Figure 3** for examples of instruments designed and created by student groups.

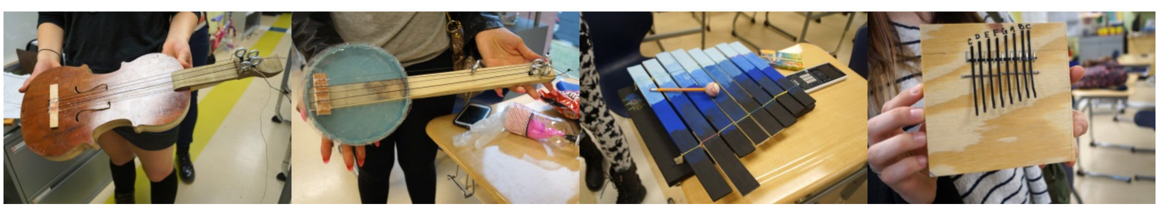

Figure 3. Examples of Student Designed Instruments

Via the Audacity program, students tested their instruments to see if they were in tune. I would have the program up and running when the students came into class. After playing the instrument and importing it into Audacity, students would take the cursor and highlight a selection from the song, and zoom in until they can see the wave form. Often, they would have to magnify to thousandths of a second to measure the length of one cycle of the wavelength. To determine if the instrument was in tune to within 5 hertz of 440 hz, it was necessary for them to work through the following procedures:

1. Using the Audacity sound recording program, play the note 'A' and record the sound.

2. Zoom in on the wave form until you can see one wave form clearly. Sketch a graph of the waveform on graph paper. Indicate the period of time for one wave and the amplitude.
3. How much time does it take for your note to go through one cycle? That is, the length of one period?
4. What is the reciprocal of that number, that is, 1/period? This is the frequency.
5. Is this within 5 units (5 hz) of 440 hz? If not, how can you alter your instrument somehow and redo this note?

Because the students were highly engaged in the project, the groups were mostly self-driven. I gave the students five class days to complete their instruments. One particularly ambitious group designed and built a violin, as shown in **Figure 3**. Student groups worked hard during class and many groups stayed after school to complete construction on their musical instruments. As a culminating activity, all the instruments played "Mary Had a Little Lamb" together as an ensemble.

Results

There seemed to be a higher degree of student understanding of sine and cosine functions as a result of connecting those concepts to music and sounds prior to teaching the math in more detail. Prior to the unit, students averaged a score of 47.13% on the pre-test. On the post-test, the average was 66.67%, which represented a statistically significant difference in a two-tailed t-test. Student survey data verified what I observed: there was a tremendous amount of student interest and engagement in this unit. As performing arts students, they appreciated the non-traditional method in which these mathematical concepts were presented. By connecting the content to music, I was able to capture their attention thus resulting in better understanding and retention of the material. I acknowledge that part of the success of this venture may have been due to both my and my students' interest in and knowledge of music, as well as utilizing the Tech department to produce some of the more intricate construction pieces. Overall, this was a very engaging and fulfilling unit to teach, as well as an effective way to introduce a difficult math concept.

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Bibliography

1. Case, D. (Producer), &. (2012). *Landfill harmonic.* [Video/DVD] You Tube: https://www.youtube.com/watch?v=sJxxdQox7n0.

2. Common Core State Standards Initiative. (2014). Retrieved from http://www.corestandards.org/

3. Jones, J. N., Racz, L., & Rogers, C. (2001). Musical instrument engineering program at Tufts University. *Journal of Engineering Education, 90*(1), 101-103. doi:10.1002/j.2168-9830.2001.tb00574.x

4. Music engineering. Retrieved from http://engineering.tufts.edu/academics/undergradprograms/musicEngineering.htm

5. Next generation science standards. (2014). Retrieved from http://www.nextgenscience.org/

6. Olson, H. F. (1967). *Music, physics and engineering* [Musical engineering] (2d ed.). New York: Dover Publications.